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where $V(x)$ is any of the singular functions encountered in the text, $\psi(x\cdot P)$ is the matrix element between the vacuum and one-particle state of any of the bilocal operators, and $f(x)$ is a suitable test function. A similar procedure is applied for the two-particle case. In this connection see H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970).

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PHYSICAL REVIEW D VOLUME 6, NUMBER 9 1 NOVEMBER 1972

Asymmetries of Multiplicity Cross Sections*

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(Received 26 June 1972)

Front-back asymmetries of multiplicity distributions are shown to discriminate among pictures of multiparticle reactions.

Considerable emphasis has been placed of late on the study of multiplicity fluctuations in high-energy collisions.[1] Particular stress has been laid on the fact that the energy dependence of moments of multiplicity distributions can provide a means of discriminating between the so-called "independent emission" and "fragmentation" pictures. In this note we call attention to a mode of data presentation which permits one to distinguish the alternatives in a single experiment at one energy, in which momenta of secondaries need not be measured if the experiment is performed with colliding beams of equal energy.

The experiment we envisage consists of a measurement of the cross section for production of n_R particles in the right hemisphere (forward in the c.m. system) and n_L particles in the left hemisphere (backward in the c.m. system), which cross section we denote by $\sum(n_L, n_R)$. For fixed total multiplicity $n = n_L + n_R$, plot

$$P(n;n_L)\equiv \sum(n_L, n-n_L)/\sum(\tfrac{1}{2}n, \tfrac{1}{2}n) \qquad (1)$$

as a function of n_L. In proton-proton collisions this distribution is necessarily symmetric about the point $n_L=\frac{1}{2}n$ since we may write

$$P(n;n_L)=\sigma(n_L)\sigma(n-n_L)/[\sigma(\tfrac{1}{2}n)]^2 . \qquad (2)$$

Its behavior near the symmetry point is a sensitive indicator of the shape of the multiplicity cross sections $\sigma(n_L)$ within each hemisphere.

In a fragmentation picture, with the possibility of large multiplicity fluctuations within each hemisphere, it is usual to assume $\sigma(n_L)\propto(n_L)^{-2}$ for large n_L. This leads to a distribution

$$P_{\text{fragmentation}}(n;n_L)=(\tfrac{1}{2}n)^4(n_L)^{-2}(n-n_L)^{-2} \qquad (3)$$

which is minimal for $n_L=\frac{1}{2}n$, as shown in Fig. 1. Thus *in a fragmentation picture, asymmetric events,* with unequal numbers of particles produced in the right and left hemispheres, are the rule.

In a simple multiperipheral (or independent-emission) model, the multiplicity cross sections follow a Poisson distribution in the variable $(\frac{1}{2}n_L)$, i.e.,

$$\sigma(n_L)=(\tfrac{1}{2}\langle n\rangle)^{(n_L/2-1)}/(\tfrac{1}{2}n_L-1)!\ .$$

This in turn leads to a distribution

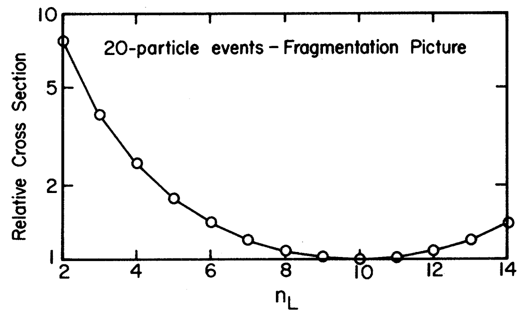

FIG. 1. The relative cross section $P(n;n_L)$ given by Eq. (3) for a simple fragmentation model, for 20-particle events; asymmetric events are favored.

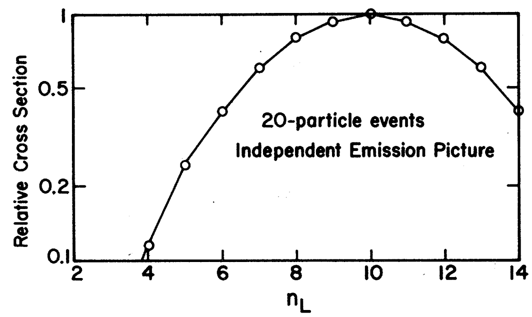

FIG. 2. The relative cross section $P(n;n_L)$ given by Eq. (4) for a simple multiperipheral model, for 20-particle events; symmetric events are favored.

$$P_{\text{multiperipheral}}(n;n_L)=\frac{[(\frac{1}{4}n-1)!]^2}{(\frac{1}{2}n_L-1)![\frac{1}{2}(n-n_L)-1]!}\,, \tag{4}$$

illustrated in Fig. 2, which is maximal for $n_L=\frac{1}{2}n$. Hence *in a multiperipheral model, symmetric events are favored.*

From definition (2) it is easy to show that if $\sigma(n_L)$ decreases faster (slower) than an exponential for $n_L=\frac{1}{2}n$ the distribution $P(n;n_L)$ is maximal (minimal) at the symmetry point. It is unfortunately true that present thinking along fragmentation or multiperipheral[2] lines does not lead to precise predictions for the shapes of the multiplicity distributions $\sigma(n_L)$. The procedure we suggest should therefore be more informative than the usual exercise of fitting data to specific but ill-motivated formulas.

It is not impossible that nature allows particle production both by independent-emission mechanisms and by fragmentation processes. In this eventuality, fitting $\sigma(n_L)$ to particular expressions may be even less enlightening than in the simple situations for which we gave examples. The shape of $P(n;n_L)$ will still reveal whether events of given total multiplicity n are dominantly produced by one mechanism or the other.[3]

We have suggested a technique for inferring the character of multiple production processes from the front-back asymmetry of multiplicity distributions. Data of the required kind are particularly accessible at storage-ring facilities where right and left hemispheres can be defined without recourse to momentum measurements. In addition to their implications for the fragmentation vs independent-emission issue, such data will test directly the symmetry assumption underlying the Castagnoli method[4] and thereby shed light on the credibility of cosmic-ray results.

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*Work supported in part by the NSF under Grant No. GP32998X.

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