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Muon Pair Production in Electron-Positron Annihilation and the Bjorken-Johnson-Low Limit

M. A. Ahmed
Physics Department, University of Khartoum, Khartoum, Sudan

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Electron-positron annihilation leading to the production of a muon pair and a system of hadrons is investigated in the Bjorken-Johnson-Low asymptotic limit. The equal-time commutators are calculated from light-cone expansions and expressions for differential cross sections obtained. Comparison with the work of Gross and Treiman is made.

In a recent article Gross and Treiman[1] proposed new experiments that probe further the properties of products and commutators of electromagnetic (e.m.) currents near the light cone. Specifically they consider the process

$$e^+ + e^- \to \mu^+ + \mu^- + X_1 + X_2 + \cdots, \tag{1.1}$$

where $\{X\}$ is any system of hadrons, and the process

$$e + p \to e + \mu^+ + \mu^- + X, \tag{1.2}$$

where the hadron system is denoted by X for short. The process (1.1) involves two timelike photons while in (1.2) the incident photon is spacelike and the outgoing one is timelike. We shall limit ourselves here to a discussion of processes (1.1).

To lowest order in electromagnetism, two types of Feynman diagrams are relevant and these are shown in Figs. 1 and 2. Figure 1 describes hadron production in states that are even under charge conjugation while Fig. 2 corresponds to the production of states that are odd under charge conjugation. There is no interference between the contributions arising from these two sets to the inclusive cross section and following Gross and Treiman we restrict our attention to processes of the type shown in Fig. 1.

The Bjorken-Johnson-Low[2] (BJL) asymptotic limit for the process (1.1) is accessible physically and in fact one is probing a new kinematical region for these types of processes. For the equal-time commutators (ETC) that arise in the BJL expansion Gross and Treiman[1] use the quark-gluon model. On the other hand, operator-product expansions for short[3] or lightlike[4] distances have been offered on rather general grounds. These expansions of course determine the ETC's.[3] In this note we treat the process (1.1) with the ETC's calculated from the general framework provided by light-cone expansions. The latter have proved valuable in understanding scaling behavior, and since the ETC's that involve time derivatives of current components or that involve space components are necessarily model-dependent, it seemed to us desirable to extract the properties of these objects, in particular the Schwinger terms, from the light-cone expansions. In this way direct contact between ETC's and the bilocal operators that characterize light-cone expansions is made.

Let l_+, l_- be the momenta of the incident electron and positron pair, k_+, k_- be those of the outgoing μ^+ and μ^-, respectively, and let P be the momentum of the hadron system. Define

$$l = l_+ + l_-, \quad k = k_+ + k_-, \quad l^2 = s, \qquad Q = \tfrac{1}{2}(l+k), \tag{1.3}$$

and note that $P = l - k$. The graph of Fig. 1 involves the amplitude

$$M_{\mu\nu} = i\int d^4x\, e^{iQ\cdot x}\langle X|\, T^*(J_\mu(\tfrac{1}{2}x)J_\nu(-\tfrac{1}{2}x))\,|0\rangle, \tag{1.4}$$

with J_μ being the e.m. current. The BJL limit is $Q_0 \to \infty$ with $\vec{Q}$ and all hadron momenta held fixed. In the c.m. frame of the incident electron-posi-

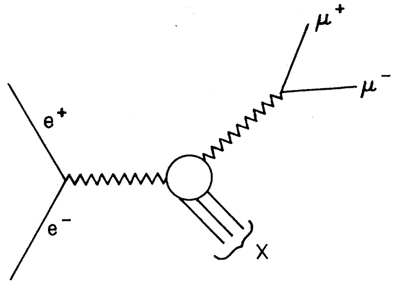

FIG. 1. The process (1.1) with final states even under charge conjugation.

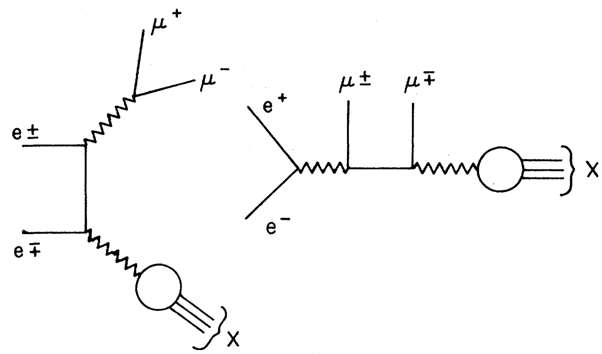

FIG. 2. The process (1.1) with final states odd under charge conjugation.

tron pair we have

$$l_+ = (E, \vec{l}), \quad l_- = (E, -\vec{l}),$$

and we write $P=(P_0, \vec{p})$. As $Q_0 = 2E - \frac{1}{2}P_0$ in this frame, the BJL limit is achieved by $E \to \infty$ with $\vec{Q} = -\frac{1}{2}\vec{p}$ fixed.

Following Brandt and Preparata[4] we write the following expression for the commutator of two e.m. currents at unequal times:

$$[J_\mu(x), J_\nu(0)] \triangleq (\partial_\mu \partial_\nu - g_{\mu\nu}\Box)V_0(x)R_0(x,0) + (g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu}g_{\beta\nu}\Box)V_2(x)R_2^{\alpha\beta}(x,0) + \epsilon_{\mu\nu\alpha\beta}\partial^\alpha V_1(x)R_1^\beta(x,0)\,. \tag{1.5}$$

The symbol $\triangleq$ denotes equality in the vicinity of the light cone. The singular functions $V_0(x)$, $V_2(x)$, and $V_1(x)$ are given by

$$V_0(x) = V_1(x) = 2\pi i \epsilon(x_0)\delta(x^2)\,, \tag{1.6}$$

$$V_2(x) = 2\pi i \epsilon(x_0)\Theta(x^2)\,. \tag{1.7}$$

The bilocal operators appearing in Eq. (1.5) have the general form

$$R_0(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1}\cdots x^{\alpha_n} R_{0,\alpha_1\cdots\alpha_n}(0)\,, \tag{1.8}$$

$$R_2^{\alpha\beta}(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1}\cdots x^{\alpha_n} R^{\alpha\beta}_{2,\alpha_1\cdots\alpha_n}(0)\,, \tag{1.9}$$

$$R_1^{\beta}(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1}\cdots x^{\alpha_n} R^{\beta}_{1,\alpha_1\cdots\alpha_n}(0)\,. \tag{1.10}$$

Here $R_{0,\alpha_1\cdots\alpha_n}(0)$, $R^{\alpha\beta}_{2,\alpha_1\cdots\alpha_n}(0)$, $R^{\beta}_{1,\alpha_1\cdots\alpha_n}(0)$ are higher-rank local tensor operators. For the case $\mu = i$ ($i = 1, 2, 3$) and $\nu = 0$ the expansions (1.5) and (1.8)–(1.10) determine the ETC as follows[5]:

$$[J_i(\vec{x}, 0), J_0(0)] = 4\pi^2 i R_0(0)\partial_i \delta^3(\vec{x})\,. \tag{1.11}$$

Thus the Schwinger term is proportional to the local scalar operator $R_0(0)$ of lowest dimension that occurs in the expansion of the bilocal operator $R_0(x,0)$ given in Eq. (1.8).

We now turn to the construction of the covariant or T^* product of two components of the e.m. current. Given the structure of ETC's a general procedure for this construction was given by Gross and Jackiw.[6] With the notation $T^*(J_\mu(x)J_\nu(0)) \equiv T^*_{\mu\nu}(x,0)$, etc. one writes

$$T^*_{\mu\nu}(x,0) = T_{\mu\nu}(x,0;n) + C_{\mu\nu}(x,0;n)\,, \tag{1.12}$$

where n is a unit timelike vector. The appearance of n in the T product $T_{\mu\nu}(x,0;n)$ is indicative of its noncovariance or frame dependence. $C_{\mu\nu}(x,0;n)$ is the so-called seagull term which depends on n in such a manner that when summed with the n-dependent T product yields a covariant (n-independent) T^* product. In the case at hand the Schwinger term appears solely in the ETC (1.11) and one finds the following expression for the seagull term:

$$C_{\mu\nu}(x,0;n) = 4\pi^2 i R_0(0)(n_\mu n_\nu - g_{\mu\nu})\delta^4(x)\,. \tag{1.13}$$

The T^* product so constructed satisfies the gauge-invariance condition

$$\partial^\mu T^*_{\mu\nu}(x,0) = 0\,. \tag{1.14}$$

In particular with a specific choice for the unit vector n one can write

$$T^*(J_\mu(x)J_\nu(0)) = T(J_\mu(x)J_\nu(0)) + 4\pi^2 i(g_{\mu 0}g_{\nu 0} - g_{\mu\nu})R_0(0)\delta^4(x)\,. \tag{1.15}$$

Having determined the T^* product we can now envisage the limit $Q_0 \to \infty$, with $\vec{Q}$ and all hadron momenta held fixed, of the covariant amplitude $M_{\mu\nu}$.

The limiting behavior is given by the BJL expansion as

$$M_{\mu\nu} = 4\pi^2(g_{\mu\nu} - g_{\mu 0}g_{\nu 0})\langle X|R_0(0)|0\rangle - \frac{1}{Q_0}\int d^3x\, e^{-i\vec{Q}\cdot\vec{x}}\langle X|[J_\mu(\tfrac{1}{2}x), J_\nu(-\tfrac{1}{2}x)]|0\rangle_{x_0=0}$$
$$+ \frac{i}{2Q_0^2}\int d^3x\, e^{-i\vec{Q}\cdot\vec{x}}\langle X|\{[\partial_0 J_\mu(\tfrac{1}{2}x), J_\nu(-\tfrac{1}{2}x)] - [J_\mu(\tfrac{1}{2}x), \partial_0 J_\nu(-\tfrac{1}{2}x)]\}|0\rangle_{x_0=0} + \cdots . \quad (1.16)$$

The first term in the expansion, which arises from the seagull term, if nonvanishing then controls the asymptotic behavior.

We now turn to a discussion of specific final-state products of the reaction (1.1). First consider the production of a single scalar particle [e.g., $\epsilon(750)$]. Working in the c.m. frame of the incident lepton pair we find the leading behavior

$$M_{ij} \to 4\pi^2 g_{ij}\langle X|R_0(0)|0\rangle, \quad (1.17)$$

where only $\mu = i$ and $\nu = j$ enter since the e.m. current is conserved. If we write for the matrix element of the bilocal operator $R_0(x, 0)$ between the scalar particle state X and the vacuum the following form,

$$\langle X(P)|R_0(x, 0)|0\rangle = \phi_X(x\cdot P) + O(x^2), \quad (1.18)$$

we then see that

$$M_{ij} \to 4\pi^2 g_{ij}\phi_X(0). \quad (1.19)$$

The differential cross section $d\sigma$ in the limit $s \to \infty$ has the following form:

$$d\sigma \underset{s\to\infty}{\sim} \frac{e^4}{16 s^3}|\phi_X(0)|^2(1+\cos^2\theta)\frac{p^2}{P_0}\,dp\,d\Omega, \quad (1.20)$$

where θ is the angle between $\vec{p}$ and the momentum vector of the positron. A measurement of this cross section then entails a measurement of the off-diagonal matrix element of $R_0(x, 0)$, albeit only at zero argument, between one scalar particle state and the vacuum.

Next we consider the case when $X = \pi^0$. Here the matrix element $\langle \pi^0|R_0|0\rangle$ vanishes and the amplitude M_{ij} is determined by the second term in the BJL expansion (1.16) which involves the ETC $[J_i(\vec{x}, 0), J_j(0)]$. Defining now

$$\langle \pi^0(P)|R_1^\beta(x, 0)|0\rangle = P^\beta\chi(x\cdot P) + O(x^2), \quad (1.21)$$

we readily compute the ETC from Eqs. (1.5) and (1.8)–(1.10) as[7]

$$\langle \pi^0(P)|[J_i(\vec{x}, 0), J_j(0)]|0\rangle = -4\pi^2 i\epsilon_{ijk}P^k\chi(0)\delta^3(\vec{x}). \quad (1.22)$$

The function $\chi(0)$ is proportional to the matrix element of the first operator, $R_1^\beta(0)$, in the expansion of $R_1^\beta(x, 0)$. The differential cross section is then given by an equation like (1.20) with $|\phi_X(0)|^2$ replaced by $|\chi(0)|^2$ and s^{-3} by s^{-4}. The vertex π^0-γ-γ has been considered earlier in Refs. 8 and 9. The authors of Ref. 9 propose to identify the operator $R_1^\beta(0)$ with the third component of the isotopic axial-vector current. This identification is suggested by the gluon model which provides the starting point for Gross and Treiman.[1] If one is willing to make this identification then our considerations for this case will coincide with those of Ref. 1. However, it is clear from our discussion that a measurement of the differential cross section for $e^+ + e^- \to \mu^+ + \mu^- + \pi^0$ entails a measurement of the matrix element of the bilocal operator $R_1^\beta(x, 0)$, at zero argument, between the one-pion state and the vacuum.

We now turn our attention to the case of two-pion production. Here the leading contribution comes from the first term in the BJL expansion. Defining

$$\langle \pi(P_1), \pi(P_2)|R_0(x, 0)|0\rangle = \phi_{2\pi}(x\cdot P_1, x\cdot P_2, P^2) + O(x^2), \quad (1.23)$$

where we have made explicit the dependence of the matrix element on the invariant mass P^2 of the dipion, we readily obtain for the cross section

$$d\sigma \sim \frac{e^4}{32(2\pi)^3}\frac{(1+\cos^2\theta)}{s^3}|\phi_{2\pi}(0, 0, P^2)|^2 \times \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}. \quad (1.24)$$

We thus see that the differential cross section for the 2π channel behaves like s^{-3} as $s \to \infty$ in contrast to the behavior of s^{-5} found by Gross and Treiman.

We now discuss the situation when the leading light-cone singularity is absent. Namely, suppose that the singularity that appears before the covariant $(\partial_\mu\partial_\nu - g_{\mu\nu}\Box)$ in Eq. (1.5) is absent. Then the Schwinger term will also be absent and the asymptotic behavior of $M_{\mu\nu}$ as $Q_0 \to \infty$ is governed solely by equal-time commutators. To discuss this case assuming canonical dimensionality[4] it follows that the next-to-leading singularity in the light-cone expansion is proportional to $\ln(-x^2+i\epsilon x_0)$ so that it manifests itself in the unequal-time commutator (1.5) as the additional term

$$(\partial_\mu\partial_\nu - g_{\mu\nu}\Box)V_2(x)H(x,0), \tag{1.25}$$

where $V_2(x)$ is given by Eq. (1.7). This term arises both from the nonleading contributions of the original operator satisfying $\dim R_{\alpha_1\cdots\alpha_n}=n+2$ and from leading contributions of an additional operator satisfying $\dim R_{\alpha_1\cdots\alpha_n}=n+4$. We have denoted the sum of these operators by $H(x,0)$.

Let us work out the cross sections in this case. Firstly when X is a single scalar hadron the first nonvanishing contribution comes from the third term in the BJL expansion which involves one time derivative of the e.m. current. Defining the function $h(x\cdot P)$ by

$$\langle X(P)|H(x,0)|0\rangle = h(x\cdot P)+O(x^2), \tag{1.26}$$

and the functions $g_1(x\cdot P)$ and $g_2(x\cdot P)$ by

$$\langle X(P)|R_2^{\alpha\beta}(x,0)|0\rangle = g^{\alpha\beta}g_1(x\cdot P)+P^\alpha P^\beta g_2(x\cdot P) + O(x^2), \tag{1.27}$$

we obtain for the relevant ETC

$$\langle X(P)|[\partial_0 J_i(\vec{x},0),J_j(0)]|0\rangle = (A g_{ij}+BP_iP_j)\delta^3(\vec{x}), \tag{1.28}$$

with

$$A = 16\pi^2 i[-h(0)+2g_1(0)+P_0^2 g_2(0)], \tag{1.29}$$

$$B=16\pi^2 i g_2(0). \tag{1.30}$$

The differential cross section now reads

$$d\sigma \sim \frac{e^4}{32\pi s^5}\{|A|^2(1+\cos^2\theta)-(AB^*+BA^*)p^2 + |B|^2(1-\cos^2\theta)^2p^4\}\frac{p^2dpd\Omega}{P_0}, \tag{1.31}$$

i.e., it behaves like s^{-5} as $s\to\infty$. For the single-pion channel the second term in Eq. (1.16) continues to provide the leading contribution and $d\sigma$ is the same as before. Finally we come to the 2π channel. Again the first nonvanishing contribution comes from the third term in Eq. (1.16). We define the scalar functions $f_a(x\cdot P_1, x\cdot P_2, P^2)$, $a=1,\ldots,4$, by

$$\begin{aligned}\langle\pi(P_1),\pi(P_2)|R_2^{\alpha\beta}(x,0)|0\rangle &= f_1(x\cdot P_1,x\cdot P_2,P^2)g^{\alpha\beta}+f_2(x\cdot P_1,x\cdot P_2,P^2)P^\alpha P^\beta\\ &\quad+f_3(x\cdot P_1,x\cdot P_2,P^2)\Delta^\alpha\Delta^\beta\\ &\quad+f_4(x\cdot P_1,x\cdot P_2,P^2)(P^\alpha\Delta^\beta+\Delta^\alpha P^\beta)+O(x^2),\end{aligned} \tag{1.32}$$

where $\Delta=P_1-P_2$. We also define the function $h_2(x\cdot P_1, x\cdot P_2, P^2)$:

$$\langle\pi(P_1),\pi(P_2)|H(x,0)|0\rangle = h_2(x\cdot P_1,x\cdot P_2,P^2) + O(x^2). \tag{1.33}$$

We then compute the ETC between the vacuum and the two-pion state and find[7]

$$\begin{aligned}\langle\pi(P_1),\pi(P_2)|[\partial_0 J_i(\vec{x},0),J_j(0)]|0\rangle = 16\pi^2 i\{&[2f_1(0,0,P^2)+f_2(0,0,P^2)P_0^2+f_3(0,0,P^2)\Delta_0^2\\ &+2f_4(0,0,P^2)P_0\Delta_0-h_2(0,0,P^2)]g_{ij}+f_2(0,0,P^2)P_iP_j\\ &+f_3(0,0,P^2)\Delta_i\Delta_j+f_4(0,0,P^2)(P_i\Delta_j+\Delta_iP_j)\}\delta^3(\vec{x}).\end{aligned} \tag{1.34}$$

One can compute the differential cross section but we shall not pause to do so here. We merely note that as in the scalar channel case the cross section for the two-pion channel behaves like s^{-5} as $s\to\infty$.

In conclusion we have seen how measurements of cross sections for the processes (1.1) involve off-diagonal matrix elements, at zero argument, of the bilocal operators that characterize the light-cone expansion. In the absence of the leading singularity that appears before the covariant $(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)$, specification of the ETC $[\partial_0 J_i, J_j]$ by the contribution of the term proportional to $R_2^{\alpha\beta}$ alone is incomplete and one has to specify the form of the nonleading singularity because it gives rise to a nonvanishing contribution as we have seen. Finally in electroproduction the situation, $F_L(\omega)=0$ if exactly true would imply the absence of the most leading singularity, if one discards the circumstance that the matrix element of each of the operators $R_{0,\alpha_1\cdots\alpha_n}(0)$ vanishes between spin-averaged proton states. This then would entail the vanishing of q-number Schwinger terms. We have proposed here two further tests for the q-number nature of the Schwinger term, namely the high-energy behavior of the cross sections in Eqs. (1.20) and (1.24) for scalar and 2π production, respectively. These questions will be settled both by more precise data on electroproduction and by possible future measurements of these cross sections.

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$$E(f,t)=\int d^4x\,\delta(x_0-t)f(x)V(x)\psi(x\cdot P)\,,$$

where $V(x)$ is any of the singular functions encountered in the text, $\psi(x\cdot P)$ is the matrix element between the vacuum and one-particle state of any of the bilocal operators, and $f(x)$ is a suitable test function. A similar procedure is applied for the two-particle case. In this connection see H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970).
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Asymmetries of Multiplicity Cross Sections*

S. Nussinov,† C. Quigg, and Jiunn-Ming Wang
Institute for Theoretical Physics, State University of New York, Stony Brook, Long Island, New York 11790
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Front-back asymmetries of multiplicity distributions are shown to discriminate among pictures of multiparticle reactions.

Considerable emphasis has been placed of late on the study of multiplicity fluctuations in high-energy collisions.[1] Particular stress has been laid on the fact that the energy dependence of moments of multiplicity distributions can provide a means of discriminating between the so-called "independent emission" and "fragmentation" pictures. In this note we call attention to a mode of data presentation which permits one to distinguish the alternatives in a single experiment at one energy, in which momenta of secondaries need not be measured if the experiment is performed with colliding beams of equal energy.

The experiment we envisage consists of a measurement of the cross section for production of n_R particles in the right hemisphere (forward in the c.m. system) and n_L particles in the left hemisphere (backward in the c.m. system), which cross section we denote by $\sum(n_L, n_R)$. For fixed total multiplicity $n=n_L+n_R$, plot

$$P(n;n_L)\equiv \sum(n_L, n-n_L)/\sum(\tfrac{1}{2}n, \tfrac{1}{2}n) \qquad (1)$$

as a function of n_L. In proton-proton collisions this distribution is necessarily symmetric about the point $n_L=\frac{1}{2}n$ since we may write

$$P(n;n_L)=\sigma(n_L)\sigma(n-n_L)/[\sigma(\tfrac{1}{2}n)]^2\,. \qquad (2)$$

Its behavior near the symmetry point is a sensitive indicator of the shape of the multiplicity cross sections $\sigma(n_L)$ within each hemisphere.

In a fragmentation picture, with the possibility of large multiplicity fluctuations within each hemisphere, it is usual to assume $\sigma(n_L)\propto(n_L)^{-2}$ for large n_L. This leads to a distribution

$$P_{\text{fragmentation}}(n;n_L)=(\tfrac{1}{2}n)^4(n_L)^{-2}(n-n_L)^{-2} \qquad (3)$$

which is minimal for $n_L=\frac{1}{2}n$, as shown in Fig. 1. Thus *in a fragmentation picture, asymmetric events,* with unequal numbers of particles produced in the right and left hemispheres, are the rule.

In a simple multiperipheral (or independent-emission) model, the multiplicity cross sections follow a Poisson distribution in the variable $(\frac{1}{2}n_L)$, i.e.,

$$\sigma(n_L)=(\tfrac{1}{2}\langle n\rangle)^{(n_L/2-1)}/(\tfrac{1}{2}n_L-1)!\,.$$

This in turn leads to a distribution