Determination of f^0NN and gNN Coupling Constants*

Ronald Aaron and Hyman Goldberg
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 1 March 1972)

Using backward πN dispersion relations (BDR) and sum rules derived from BDR we obtain the spin-flip f^0NN and a combination of the flip and nonflip gNN coupling constants, and fit certain backward πN data. Our results have implications in the one-boson-exchange theory of the nucleon-nucleon force and in theories of elementary particles which assume universal coupling to the stress tensor.

I. INTRODUCTION

We have used sum rules [1] derived from backward dispersion relations (BDR)[1] in conjunction with some recent πN phase-shift analyses to determine the spin-flip f^0NN and a combination of the flip and nonflip gNN coupling constants. With these coupling constants we then use BDR to fit the B^+ and A^- πN amplitudes. After completion of this research we became aware of earlier work by Engels,[2] who, using similar methods, obtained results very much like ours for the f^0 meson and the B^+ amplitude. Nevertheless, we display our results for completeness, in addition to our new results on the g meson and the A^- amplitude. The strength with which the $f^0(1260)$ meson couples to the nucleon is of current interest to a variety of physicists. For example, it is relevant both to those involved with one-boson-exchange models of the nucleon-nucleon force and those interested in single-particle saturation of the stress tensor. We comment upon these topics later.

II. FORMALISM

Assuming Mandelstam analyticity, one of us [1] has derived the following set of unsubtracted dispersion relations describing elastic πN scattering at backward angle ($\cos\theta = -1$):

$$\mathrm{Re}F(s) = (\text{pole terms}) + \frac{1}{\pi}\int_{(M+\mu)^2}^{\infty} ds'\,\mathrm{Im}F(s')\left[P\frac{1}{s'-s} + \frac{r^2}{s'(r^2-ss')}\right]$$

$$+\frac{1}{\pi}\int_{4M^2}^{\infty} dt'\,\frac{\mathrm{Im}F_{N\bar{N}\to\pi\pi}(t')}{(t'-4M^2)^{1/2}(t'-4\mu^2)^{1/2}}\,\frac{2r^2+s(t'-\Sigma)}{r^2+s(t'-\Sigma)+s^2} + \frac{2ir}{\pi}\int_0^{\pi} d\varphi\,\frac{(r-s\cos\varphi)}{r^2+s^2-2rs\cos\varphi}\,F_t(\varphi)\,, \tag{1}$$

where $r = M^2-\mu^2$ and $\Sigma = 2M^2+2\mu^2$. $F(s)$ represents the invariant amplitudes $A^-(s,\ \cos\theta=-1)$ or $B^+(s,\ \cos\theta=-1)$. The pole terms occur only for B^+ and are given by

$$B^+_{\text{pole}} = \frac{G^2}{M^2-s} + \frac{G^2r^2}{M^2(r^2-M^2s)}\ . \tag{2}$$

On the right-hand side of Eq. (1), the first and second integrals are the contributions from direct- and crossed-channel πN scattering, the third integral is the contribution of the $N\bar{N}\to\pi\pi$ channel above threshold, and the last integral is the contribution to $\mathrm{Re}F(s)$ of the absorptive part of the $N\bar{N}\to\pi\pi$ process below $N\bar{N}$ threshold. (F_t can be shown to be pure imaginary.[1]) The value of t at which F_t is evaluated is given in terms of the angle φ by the relation $t=\Sigma-2r\cos\varphi$.[1] Noting that $(s,\ \cos\theta_s=-1)$ corresponds to $(u=r^2/s,\ \cos\theta_u=-1)$ and using the usual crossing relations for the amplitudes A^-, B^+ leads to the conditions [1]

$$A^-(\pm r,\ -1)=0\,,\qquad B^+(\pm r,\ -1)=0\,, \tag{3}$$

which in turn give the nontrivial sum rules [1] for the backward amplitudes

$$0=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^-(s')ds'}{s'} + \frac{1}{\pi}\int_{4M^2}^{\infty}dt'\,\frac{\mathrm{Im}A^-_{N\bar{N}\to\pi\pi}(t')}{(t'-4M^2)^{1/2}(t'-4\mu^2)^{1/2}} + \frac{i}{\pi}\int_0^{\pi}d\varphi A_t^-(\varphi)\,, \tag{4}$$

$$0=\frac{G^2}{M^2} + \int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}B^+(s')ds'}{s'} + \frac{1}{\pi}\int_{4M^2}^{\infty}dt'\,\frac{\mathrm{Im}B^+_{N\bar{N}\to\pi\pi}(t')}{(t'-4M^2)^{1/2}(t'-4\mu^2)^{1/2}} + \frac{i}{\pi}\int_0^{\pi}d\varphi B_t^+(\varphi)\,. \tag{5}$$

III. B^+ SUM RULE AND DISPERSION RELATION

We have evaluated the B^+ sum rule, Eq. (5), taking $G^2/4\pi = 14.8$ and obtaining $\text{Im}B^+(s', -1)$ from either the CERN Theoretical[3] or Glasgow solution A[3,4] phase shifts for $\sqrt{s'} \lesssim 2.0$ GeV. For $\sqrt{s'} \gtrsim 2$ GeV we use $\text{Im}B^+(s', -1)$ obtained from a Regge-pole fit to backward $\pi^\pm p$ scattering.[5]

The annihilation contribution [second integral in Eq. (5)] is very small. There seems to be no resonant $I=0$ activity in $\bar{p}p \to \pi^+\pi^-$ scattering for c.m. energies between 1.9 and 2.6 GeV.[6] We have included a Regge contribution for $N\bar{N} \to \pi\pi$ obtained by line reversal from the $\pi N \to \pi N$ Regge fit,[7] but this contribution is very small due to the much more prominent role played by the nonsense zero at $\alpha = -\frac{1}{2}$ in backward $N\bar{N} \to \pi\pi$ than in $\pi N \to \pi N$. (This can be seen by comparing the Regge s and Regge t contributions to $\text{Re}B^+$ in Table I.) Since the σ and other spin-zero resonances do not contribute to the B^+ amplitude, and there are no other $I^G = 0^+$, $J \geq 2$ resonances below $N\bar{N}$ threshold with sizable coupling to $\pi\pi$,[8] we believe it to be a fairly good approximation to saturate the last integral in Eq. (5) with the $f^0(1260)$. Using a narrow-resonance approximation for the f^0, we finally arrive at the result[9]

$$(\gamma^{(1)}_{f^0NN})^2/4\pi = 61.0 \pm 14$$

or (6)

$$\gamma^{(1)}_{f^0NN} = 27.8 \pm 3.0 .$$

The coupling constant $\gamma^{(1)}_{f^0NN}$ is defined through the covariant vertex function

$$\mathcal{L}_{f^0NN} = \bar{u}(p_2)\left(\frac{\gamma^{(1)}_{f^0NN}}{M} P_\mu \gamma_\nu + \frac{\gamma^{(2)}_{f^0NN}}{M^2} P_\mu P_\nu\right) \times u(p_1)\epsilon^{\mu\nu}(p_2 - p_1), \quad (7)$$

where $\epsilon^{\mu\nu}$ is the symmetric f^0 wave function, $P \equiv \frac{1}{2}(p_1 + p_2)$, and M = nucleon mass. Engels[2] obtains $\gamma^{(1)}_{f^0NN} = 25.8 \pm 3.0$.

The uncertainty in our answer arises mainly from uncertainty in the width of the f^0. The integral in the B^+ sum rule is dominated by the (3, 3) resonance whose contribution to $\text{Im}B^+$ is very well determined by the phase-shift analyses. It is extremely unlikely that uncertainties in the other phase shifts and in the high-energy Regge contribution can significantly alter the value. We base this opinion on the fact that we have used two sets

TABLE I. For several values of W we display the various contributions to $\text{Re}B^+$(th) and compare with $\text{Re}B^+$(exp) obtained from CERN Th. phase shifts. Below 1.2 GeV the results become very sensitive to minor effects like small changes in the P_{33} phase shift (<3%) and $\pi^\pm\pi^0$ and n-p mass differences. "Direct" is the contribution from the first integral in Eq. (1) and "Crossed" is that of the second integral, in both cases for $\sqrt{s'} < 2.2$ GeV. "Regge-s" and "Regge-t" are the high-energy Regge contributions in the s and t channels, respectively. "Pole" is the nucleon pole contribution, and "f pole" is the t-channel contribution assuming saturation by the f meson.

W (GeV)	Pole	Direct	Regge-s	Crossed	f pole	Regge-t	B^+ (th)	B^+ (exp)
1.099	−1057.7	−563.5	−5.7	61.5	−78.2	0.45	−1644.0	−1687.1
1.110	−987.7	−638.4	−5.7	59.4	−75.7	0.45	−1648.6	−1607.5
1.127	−895.1	−727.5	−5.7	56.5	−72.1	0.45	−1644.4	−1686.5
1.160	−754.7	−883.4	−5.7	51.4	−65.4	0.45	−1658.4	−1702.2
1.186	−669.8	−797.1	−5.8	47.9	−60.6	0.45	−1485.9	−1513.9
1.202	−625.7	−528.1	−5.9	45.9	−57.8	0.45	−1172.0	−1186.9
1.221	−579.8	11.2	−5.9	43.8	−54.6	0.45	−585.8	−585.2
1.240	−539.5	503.5	−6.0	41.8	−51.5	0.45	−52.2	−75.9
1.253	−514.8	678.1	−6.0	40.5	−49.6	0.45	+147.8	+129.9
1.275	−477.2	759.0	−6.1	38.5	−46.4	0.45	+267.5	+267.4
1.320	−413.8	638.1	−6.2	34.8	−40.4	0.45	+212.1	+205.4
1.362	−366.8	490.2	−6.3	31.9	−35.6	0.45	+113.0	+115.3
1.416	−318.6	307.8	−6.5	28.7	−30.2	0.45	−19.1	−44.7
1.470	−280.3	220.0	−6.7	26.0	−25.6	0.45	−67.0	−87.1
1.501	−261.7	234.8	−6.8	24.7	−23.3	0.45	−32.8	−30.1
1.524	−249.2	265.8	−6.9	23.7	−21.7	0.45	+11.3	−8.0
1.572	−226.2	304.3	−7.1	21.9	−18.7	0.45	+73.8	+56.5
1.617	−207.7	261.6	−7.3	20.4	−16.3	0.45	+50.3	+35.3
1.644	−197.7	201.4	−7.5	19.6	−15.0	0.45	+0.4	−9.5
1.672	−188.2	130.6	−7.6	18.8	−13.8	0.45	−60.6	−66.9
1.697	−180.4	65.9	−7.8	18.1	−12.8	0.45	−117.3	−118.8
1.716	−174.8	25.9	−7.9	17.7	−12.0	0.45	−151.6	−153.4
1.769	−160.5	−50.2	−8.3	16.4	−10.2	0.44	−213.3	−205.2

TABLE II. Relative contributions of different partial waves to the s' integral in the B^+ and A^- sum rules for CERN Th. and Glasgow A phase-shift analyses. The upper limit of integration is 2200 MeV for CERN and 1960 MeV for Glasgow.

Partial wave	B^+ (CERN) (GeV^{-2})	B^+ (Glasgow) (GeV^{-2})	A^- (CERN) (GeV^{-1})	A^- (Glasgow) (GeV^{-1})
S_{31}	0.095	0.081	−0.125	−0.104
S_{11}	0.044	0.038	0.116	0.098
P_{33}	−7.643	−7.643	−0.708	−0.713
P_{31}	0.580	0.420	0.210	0.127
P_{13}	−0.126	−0.095	0.002	0.007
P_{11}	0.848	0.819	−0.488	−0.460
D_{35}	0.345	0.195	0.043	0.027
D_{33}	−0.578	−0.471	−0.199	−0.151
D_{15}	0.532	0.462	−0.147	−0.133
D_{13}	−0.942	−0.889	0.549	0.481
F_{37}	−0.912	−0.871	−0.163	−0.156
F_{35}	0.532	0.345	0.199	0.121
F_{17}	−0.154	−0.018	0.054	0.006
F_{15}	1.126	1.070	−0.715	−0.659
G_{39}	0.056	0.003	0.012	0.001
G_{37}	−0.111	−0.000	−0.046	−0.000
G_{19}	0.077	0.005	−0.032	−0.002
G_{17}	−0.213	−0.277	0.167	0.182
High-energy Regge ($\sqrt{s'}$ >2200 MeV)	−0.496	−0.496	−0.177	−0.177
Total	−6.940	−7.322	−1.448	−1.505

of phase shifts (CERN Th. and Glasgow A) which differ considerably above the (3, 3) region but still give essentially the same result for $\gamma_{f^0NN}^{(1)}$ [as seen by the error bars in Eq. (9)]. The contributions from individual partial waves other than the (3, 3) are not negligible – they are, however, of roughly the same magnitude ($\lesssim \frac{1}{7}$ of the P_{33} contribution) and alternate in sign, effectively canceling (see Table II).

Knowing the f^0 pole strength, we can now use the BDR [Eq. (1)] to determine ReB^+ in the physical region, and have compared the output result ReB^+ (th) with ReB^+(exp) (i.e., with ReB^+ as determined by phase-shift analysis). The results of using the CERN Th. phase shifts are shown in Fig. 1. The agreement between input and output is quite remarkable in view of the large cancellations which take place in order to give the final values of ReB^+ (see Table I). The agreement between ReB^+ (th) and ReB^+ (exp) does depend rather sensitively on the input πN phase shifts, and leads to a poor determination of $\gamma_{f^0NN}^{(1)}$.[10] The fit with the Glasgow A phase (not shown) is not as good.

To sum up, the f^0NN coupling constant is rather rigidly determined from the B^+ sum rule. From an examination of Table I it is seen that although the f^0 contributes non-negligibly in providing a close fit to the data, it is improbable that such a fit without the sum rule would provide as reliable a determination of $\gamma_{f^0NN}^{(1)}$.

IV. APPLICATIONS

(a) In the one-boson-exchange model of the nucleon-nucleon interaction, the f^0-meson exchange (which is attractive in the S states) is normally

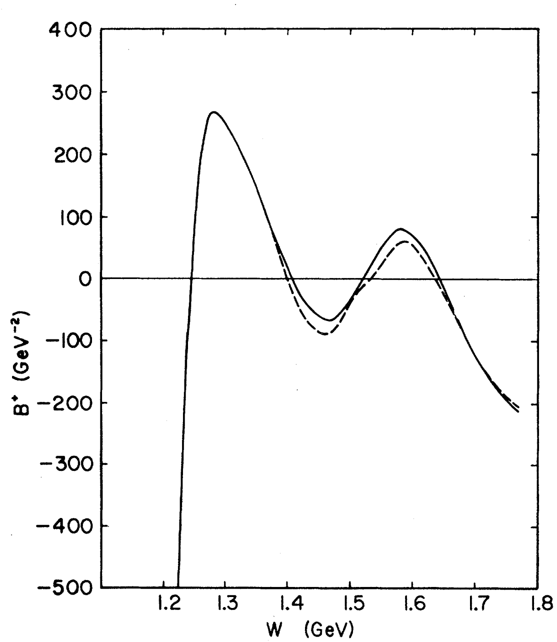

FIG. 1. ReB^+ (th) calculated via dispersion relations (solid curve) compared with ReB^+ (exp) determined by the CERN Th. phase-shift analysis (dashed curve).

omitted, while a relatively large ω-exchange contribution provides the required hard core in the S waves (the $^1S_0$ phase shift is particularly sensitive to this effect). Ueda[11] has recently found that a value of $(\gamma^{(1)}_{f^0NN})^2/4\pi$ as large as 10 would not appreciably affect the ω-produced hard core.[12] With our value of $\gamma^{(1)}_{f^0NN}$, f^0 exchange in the Born approximation is opposite in sign to and roughly twice the magnitude of ω exchange in the S states. If not cut off rather drastically, it could considerably soften the ω-produced hard core.

(b) There have been several determinations[13] of $\gamma^{(1)}_{f^0NN}$ through the use of meson dominance of the stress tensor $\Theta_{\mu\nu}(x)$. These invariably yield values of $\gamma^{(1)}_{f^0NN}\sim 5-10$, a factor of 3 to 6 smaller than our value of ~28 [Eq. (6)]. Another determination involving a theory of scale breaking[14] yields a value of $\gamma^{(1)}_{f^0NN}+\gamma^{(1)}_{f'NN}\simeq 9$, again small compared to ours (unless $\gamma^{(1)}_{f'NN}$ is sizable and negative).

V. A^- SUM RULE AND DISPERSION RELATION

The physics of the A^- sum rule [Eq. (4)] differs in several ways from that of the B^+ sum rule.
(i) There is neither a nucleon pole term nor does the (3, 3) dominate to anywhere near the extent that it did in the B^+ sum rule. Thus the higher phase shifts play a more significant role in the evaluation of the first integral in Eq. (4). (ii) There are at least two resonances with $I^G=1^+$ present in the $\pi\pi\to N\bar{N}$ channel below $N\bar{N}$ threshold: the $\rho(765)$ $(J^P=1^-)$ and the $g(1660)$ $(J^P=3^-$ preferred).
(iii) There is some evidence for weak resonant activity in the $I=1$ component of $p\bar{p}\to\pi^+\pi^-$ scattering.[6] However, the Regge parametrization for this process obtained by line reversal from $\pi N\to\pi N$ seems to provide an adequate interpolation to the data.[7] In the spirit of duality, we have included a Regge contribution in our calculations, but have omitted any higher resonances to prevent double counting. We define covariant vertices for the g:

$$\mathcal{L}_{gNN}=\bar{u}(p_2)\tfrac{1}{2}\tau_a\left(\frac{\gamma^{(1)}_{gNN}}{M^2}\gamma_\mu P_\nu P_\lambda+\frac{\gamma^{(2)}_{gNN}}{M^3}P_\mu P_\nu P_\lambda\right)\times u(p_1)\epsilon^{\mu\nu\lambda}(p_2-p_1),\quad(8)$$

$$\mathcal{L}_{g\pi\pi}=i\epsilon_{abc}\gamma_{g\pi\pi}Q_\mu Q_\nu Q_\lambda\epsilon^{\mu\nu\lambda}(q_2-q_1),\quad(9)$$

where

$$P=\tfrac{1}{2}(p_1+p_2),\qquad Q=\tfrac{1}{2}(q_1+q_2).$$

Then if we adopt ρ universality[15] $[\gamma_{\rho NN}{}^2=\gamma_{\rho\pi\pi}{}^2\simeq(2.3)(4\pi)]$ the sum rule gives a value for the linear combination[16] $\tilde{\gamma}_{gNN}=\gamma^{(1)}_{gNN}-(1-m_g{}^2/4M^2)\gamma^{(2)}_{gNN}$. We obtain the following sum rule:

$$\frac{\gamma_{g\pi\pi}\tilde{\gamma}_{gNN}}{4\pi M}\frac{(m_g{}^2-4\mu^2)}{40}-\frac{\gamma_{\rho NN}\gamma_{\rho\pi\pi}}{4\pi}\left(\frac{\lambda_V}{4M}\right)=\frac{1}{4\pi^2}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^-(s',-1)ds'}{s'}.\quad(10)$$

In Eq. (10) the ρ-meson contribution is 2.265.

The breakdown of contributions to the s' integral is given in Table II. From Eq. (10) we thus obtain

$$\frac{\gamma_{g\pi\pi}\tilde{\gamma}_{gNN}}{4\pi M}\frac{(m_g{}^2-4\mu^2)}{40}=0.80\pm0.05\ \mathrm{GeV}^{-1}.\quad(11)$$

The coupling $\gamma_{g\pi\pi}$ can be obtained from the $g\to\pi\pi$ decay rate through the formula[16]

$$\Gamma_{g\to\pi^+\pi^-}=(\gamma_{g\pi\pi}{}^2/4\pi m_g{}^2)\tfrac{1}{35}\tfrac{1}{128}(m_g{}^2-4\mu^2)^{7/2},\quad(12)$$

with $\Gamma_{g\to\pi^+\pi^-}=150\pm50$ MeV and $m_g=1660\pm20$ MeV. Combining Eqs. (11) and (12) we finally have the result

$$\tilde{\gamma}_{gNN}{}^2/4\pi=2.3\pm0.8.\quad(13)$$

Most of the probable error is due to the uncertainty in $\gamma_{g\pi\pi}$. The main observation to be made at this point is that the ρ definitely does not saturate the A^- sum rule.[17]

Proceeding as in the B^+ case, we use the BDR [Eq. (2)] and our coupling constant [Eq. (13)] to calculate ReA^-(th) at various energies and compare with ReA^-(exp). The various contributions (using the CERN Th. phase shifts) are given in

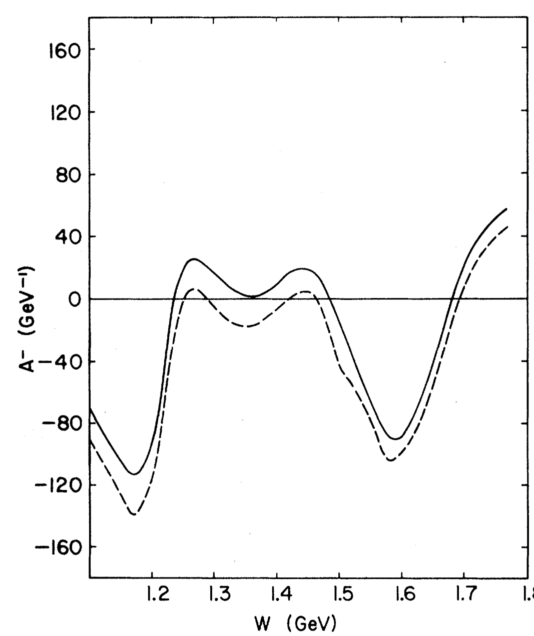

FIG. 2. ReA^-(th) calculated via dispersion relations (solid curve) compared with ReA^-(exp) determined by the CERN Th. phase-shift analysis (dashed curve).

TABLE III. For several values of W we display the various contributions to ReA^-(th), and compare with ReA^-(exp) obtained from CERN Th. phase shifts. The columns are labeled as in Table I, except that here we saturate the t channel with a ρ meson and a g meson. These contributions are labeled ρ and g.

W (GeV)	Direct	Regge-s	Crossed	ρ	g	Regge-t	A^-(th)	A^-(exp)
1.099	−68.9	−2.0	8.7	+0.6	−8.1	−0.15	−69.7	−91.9
1.110	−76.3	−2.0	8.4	−0.5	−8.0	−0.15	−78.4	−89.5
1.127	−85.8	−2.0	8.1	−2.1	−7.8	−0.16	−89.8	−108.4
1.160	−104.1	−2.0	7.4	−4.7	−7.4	−0.16	−111.0	−136.5
1.186	−100.1	−2.0	6.9	−6.4	−7.2	−0.16	−108.9	−132.0
1.202	−77.3	−2.0	6.6	−7.2	−7.0	−0.16	−87.0	−109.9
1.221	−28.0	−2.0	6.4	−8.1	−6.8	−0.16	−38.7	−65.2
1.240	+18.0	−2.1	6.1	−8.9	−6.6	−0.16	+6.4	−17.1
1.253	+33.9	−2.1	5.9	−9.3	−6.5	−0.16	+21.7	−0.8
1.275	+37.6	−2.1	5.6	−10.0	−6.3	−0.16	+24.7	4.6
1.320	+23.2	−2.1	5.1	−10.9	−5.9	−0.16	+9.2	−13.9
1.362	+15.3	−2.2	4.7	−11.4	−5.6	−0.16	+0.7	−17.3
1.416	+30.5	−2.2	4.3	−11.7	−5.2	−0.16	+15.5	−3.5
1.470	+28.3	−2.3	3.9	−11.7	−4.9	−0.16	+13.2	−4.2
1.501	−0.6	−2.3	3.7	−11.6	−4.7	−0.16	−15.7	−42.4
1.524	−28.1	−2.4	3.6	−11.5	−4.6	−0.16	−43.2	−57.4
1.572	−69.2	−2.4	3.3	−11.3	−4.3	−0.15	−84.0	−99.6
1.617	−62.7	−2.5	3.1	−11.0	−4.1	−0.15	−77.3	−89.2
1.644	−34.6	−2.6	3.0	−10.8	−3.9	−0.15	−49.1	−63.0
1.672	−0.2	−2.6	2.8	−10.6	−3.8	−0.15	−14.5	−26.6
1.697	32.1	−2.7	2.7	−10.4	−3.7	−0.15	+17.9	+2.6
1.716	48.9	−2.7	2.7	−10.2	−3.6	−0.15	+34.8	+21.4
1.769	71.1	−2.9	2.5	−9.8	−3.4	−0.15	+57.3	+46.4

Table III, and a graph of the result is shown in Fig. 2. The result is a bit perplexing. The fit would be excellent if the curves were not displaced by a constant amount along the ordinate (Glasgow A does give better results in the higher-energy region). The displacement is quite large, and all explanations of this result that we can imagine are nontrivial. For example, one possibility is that the phase-shift analyses are incorrect at higher energies. The discrepancy *cannot* be explained by the presence in the t channel of a few isolated high-energy resonances or by a constant background; if the strength of these contributions to A_t^- were large enough to provide a fit to ReA^-(exp), the A^- sum rule would be grossly violated. We see that the sum rule provides an important constraint on theoretical ideas.

*Work supported in part by the National Science Foundation under Grant No. GP-29649.

[1]Hyman Goldberg, Phys. Rev. 171, 1485 (1968). BDR had previously been written in the two-sheeted k^2 plane by D. Atkinson [Phys. Rev. 128, 1908 (1962)].

[2]J. Engels, Nucl. Phys. B25, 141 (1971).

[3]D. J. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, LRL Report No. UCRL-20030 πN, 1970 (unpublished).

[4]Since Glasgow A begins at 1320 MeV, we use CERN theoretical phase shifts from threshold to 1320. We do not significantly bias our results in this way because all phase-shift analyses give roughly the same low-energy phase shifts.

[5]V. Barger and D. Cline, Phys. Rev. Letters 21, 392 (1968).

[6]H. Nicholson *et al.*, Phys. Rev. Letters 23, 603 (1969).

[7]V. Barger and D. Cline, Phys. Letters 25B, 415 (1967); B. C. Barish *et al.*, Phys. Rev. Letters 23, 607 (1969).

[8]The f' contribution is suppressed by a factor of at least 5 at the pion vertex; in addition, the quark model predicts $\gamma_{f'NN}^{(1)} = 0$.

[9]For a detailed description of the calculation procedures see Ref. 1. The value of $\gamma_{f^0NN}^{(1)}$ obtained there was much different from that which we give in Eq. (6). The discrepancy arises mainly because a Breit-Wigner form was used to approximate the various πN resonances which were assumed to saturate the s' integral in Eq. (5). This approximation is particularly poor for the dominant P_{33} contribution, where the integral over the imaginary part is overestimated by 50%.

[10]C. Lovelace, in *Pion Nucleon Scattering,* edited by G. L. Shaw and D. Y. Wong (Wiley, New York, 1969).

[11]Tamotsu Ueda, Phys. Rev. Letters 26, 588 (1971).

[12]However, this conclusion is strongly cutoff-dependent (T. Ueda, private communication).

[13]B. Renner, Phys. Letters 33B, 599 (1970); H. F.

Jones and A. Salam, *ibid.* 34B, 149 (1971); K. Raman, Phys. Rev. D 3, 2900 (1971).

[14]R. Arnowitt, M. H. Friedman, and P. Nath, Northeastern Univ. Report No. NUB-2098 (unpublished).

[15]J. J. Sakurai, Phys. Rev. Letters 17, 1021 (1966).

[16]Standard techniques are sued for the evaluation of the g-exchange diagram and the g decay rate. See, for instance, M. D. Scadron, Phys. Rev. 165, 1640 (1968).

[17]J. Engels, G. Höhler, and B. Petersson [Nucl. Phys. B15, 365 (1970)] have recently attempted saturating the A^- sum rule but have made no mention of the g contribution.

PHYSICAL REVIEW D VOLUME 6, NUMBER 9 1 NOVEMBER 1972

Muon Pair Production in Electron-Positron Annihilation and the Bjorken-Johnson-Low Limit

M. A. Ahmed
Physics Department, University of Khartoum, Khartoum, Sudan

(Received 6 March 1972)

Electron-positron annihilation leading to the production of a muon pair and a system of hadrons is investigated in the Bjorken-Johnson-Low asymptotic limit. The equal-time commutators are calculated from light-cone expansions and expressions for differential cross sections obtained. Comparison with the work of Gross and Treiman is made.

In a recent article Gross and Treiman[1] proposed new experiments that probe further the properties of products and commutators of electromagnetic (e.m.) currents near the light cone. Specifically they consider the process

$$e^+ + e^- \to \mu^+ + \mu^- + X_1 + X_2 + \cdots , \tag{1.1}$$

where $\{X\}$ is any system of hadrons, and the process

$$e + p \to e + \mu^+ + \mu^- + X , \tag{1.2}$$

where the hadron system is denoted by X for short. The process (1.1) involves two timelike photons while in (1.2) the incident photon is spacelike and the outgoing one is timelike. We shall limit ourselves here to a discussion of processes (1.1).

To lowest order in electromagnetism, two types of Feynman diagrams are relevant and these are shown in Figs. 1 and 2. Figure 1 describes hadron production in states that are even under charge conjugation while Fig. 2 corresponds to the production of states that are odd under charge conjugation. There is no interference between the contributions arising from these two sets to the inclusive cross section and following Gross and Treiman we restrict our attention to processes of the type shown in Fig. 1.

The Bjorken-Johnson-Low[2] (BJL) asymptotic limit for the process (1.1) is accessible physically and in fact one is probing a new kinematical region for these types of processes. For the equal-time commutators (ETC) that arise in the BJL expansion Gross and Treiman[1] use the quark-gluon model. On the other hand, operator-product expansions for short[3] or lightlike[4] distances have been offered on rather general grounds. These expansions of course determine the ETC's.[3] In this note we treat the process (1.1) with the ETC's calculated from the general framework provided by light-cone expansions. The latter have proved valuable in understanding scaling behavior, and since the ETC's that involve time derivatives of current components or that involve space components are necessarily model-dependent, it seemed to us desirable to extract the properties of these objects, in particular the Schwinger terms, from the light-cone expansions. In this way direct contact between ETC's and the bilocal operators that characterize light-cone expansions is made.

Let l_+, l_- be the momenta of the incident electron and positron pair, k_+, k_- be those of the outgoing μ^+ and μ^-, respectively, and let P be the momentum of the hadron system. Define

$$l = l_+ + l_- , \quad k = k_+ + k_- , \quad l^2 = s , \qquad Q = \tfrac{1}{2}(l+k) , \tag{1.3}$$

and note that $P = l - k$. The graph of Fig. 1 involves the amplitude

$$M_{\mu\nu} = i \int d^4x \, e^{iQ\cdot x} \langle X | T^*(J_\mu(\tfrac{1}{2}x) J_\nu(-\tfrac{1}{2}x)) | 0 \rangle , \tag{1.4}$$

with J_μ being the e.m. current. The BJL limit is $Q_0 \to \infty$ with $\vec{Q}$ and all hadron momenta held fixed. In the c.m. frame of the incident electron-posi-