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Regge Effects in $V^0\Delta^{++}$ Production

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It is argued that a recent interpretation of unnatural-parity exchange in $\rho^0\Delta^{++}$ and $\omega^0\Delta^{++}$ production becomes more satisfactory if a second exchange-degenerate trajectory is introduced in additon to the π-B: namely, A_1-R. Two advantages are the allowance of a realistic slope for the π-B trajectory and interpretation of the narrow dip in ω^0 production near $t=-0.25$ (GeV/c)^2 as a ghost effect. The broad dip near $t=-0.75$ (GeV/c)^2 for ρ^0 production becomes more flexibly interpreted, and it could vanish under suitable circumstances.

Recent measurements[1] of $\rho^0\Delta^{++}$ and $\omega^0\Delta^{++}$ production by π^+p at 3.7 GeV/c were made for the purpose of displaying a particularly simple Regge exchange. Exchanges contributing to the partial cross section $\sigma_0^-=\rho_{00}d\sigma/d|t|$ must have $I=1$, $P=(-1)^{J+1}$ and signature factors fixed to give real resonances only with $G=-1$ $(+1)$ for ρ^0 (ω^0) production. These were taken, respectively, as the π and B trajectories, assumed to be degenerate. In ρ^0 production σ_0^- showed a broad minimum at $t\approx -0.8$ (GeV/c)^2, which was interpreted as a zero in the π-B trajectory. The corresponding σ_0^- curve for ω^0 production showed instead a narrow dip at $t\simeq -0.25$ (GeV/c)^2, which was noted but not explained.

In the data of Ref. 1 the dip in σ_0^- for $\rho^0\Delta^{++}$ appears definitely significant; what little other information is available tends more to confirm than refute this conclusion. A sample[2] of similar total size over five incident momenta from 2.95 to 4.08 GeV/c had necessarily to be of less resolution in t but showed values of σ_0^- compatible with Ref. 1. An experiment[3] at 5.45 GeV/c shows a just-significant dip at $t\approx -0.6$ (GeV/c)^2, in the sense that the average σ_0^- for $t=-0.8$ to -1.4 (GeV/c)^2 somewhat exceeds the error on the zero cross section observed at the dip. Another measurement[4] at 5 GeV/c shows a similar but quantitatively nonsignificant dip; this reference, however, quotes very small values of σ_0^- for all t, being a factor 2 or 3 below those of Ref. 3. This smallness away from the dip region could be a cause for insignificance of the observed dip. In summary, the dip structure claimed in Ref. 1 for σ_0^- in $\rho^0\Delta^{++}$ production receives mild support and no contradiction from other measurements.

Reference 3 notes, however, that the slope $\alpha'\approx 1.2$ (GeV/c)$^{-2}$ assumed[5] for the linear π-B Regge trajectory in the interpretation of this dip is rather large. Those authors point out that a linear trajectory containing both π and B would have a slope of about one half the cited value: namely, $\alpha'\approx 0.65$ (GeV/c)$^{-2}$.

The present note proposes an alternative fit by introducing a second, independent Regge trajectory based on the observed A_1 with $J^P=1^+$ and a corresponding $J^P=2^-$ somewhere in the R region ($m=1.6$ to 1.8 GeV/c^2). This trajectory is assumed to be exchange-degenerate like the π-B trajectory, but has opposite G parity; only a total of two and not four trajectories is involved in the fit.[6] Since the trajectories are fixed by observed points, there is just one adjustable parameter, the ratio $r=\beta_A/\beta_B$ of the trajectory couplings. At the cost of this parameter, it is possible to accommodate at least the following three features of the situation instead of one, as by assuming[5] $r=0$: (i) linear trajectories for both π-B and A_1-R contain the observed mesons, (ii) the dip in σ_0^- for $\rho^0\Delta^{++}$ results from interference between the trajectories, (iii) the narrow dip[7] in σ_0^- for $\omega^0\Delta^{++}$ at $t\approx -0.25$ (GeV/c)^2 results from a suppressed A-trajectory ghost.

These conditions turn out to specify r fairly uniquely and consistently, $\pm r\approx 1.5\pm 0.5$. The ambiguity of sign is associated with the quadratic dependence of cross section on amplitude.

One would *a priori* except the $\pi\rho$ vertex to couple to the A_1 trajectory about as strongly as to the π trajectory, and empirically the $A_1\to\pi\rho$ and $\rho\to\pi\pi$ widths are comparable. If linearity is assumed, the slope of the π-B trajectory is

$$\alpha_B'=m_B{}^2c^2-m_\pi{}^2c^2$$

$$=0.66\pm 0.06\ (\mathrm{GeV}/c)^{-2}.$$

For the A_1 trajectory a ghost at $t_G\approx -0.25$ (GeV/c)^2 implies a slope of

$$\alpha_A'=(m_A{}^2c^2-t_G)^{-1}$$

$$=0.72\pm 0.06\ (\mathrm{GeV}/c)^{-2}.$$

These values are not significantly different, so we adopt an average

$$\alpha' = 0.7 \pm 0.1\ (\mathrm{GeV}/c)^{-2}. \tag{1}$$

This is, however, significantly different from $\alpha' = 1.2\ (\mathrm{GeV}/c)^{-2}$.

A Veneziano-type amplitude involving the π-B trajectory has a factor $\Gamma(1-\alpha_B)$; since the A_1 trajectory starts its resonances (real mesons) at $\alpha_A = 1$ instead of $\alpha_B = 0$, we expect a corresponding factor $\Gamma(2-\alpha_A)$. Hence with $\zeta^{\pm}$ signature factors, the amplitudes as $s \to \infty$ are

$$A_B \sim \zeta_B^{\pm}\Gamma(1-\alpha_B) = \frac{\pi\beta_B(1 \pm e^{-i\pi\alpha_B})}{\sin\pi\alpha_B\,\Gamma(\alpha_B)}\left(\frac{s-u}{2s_0}\right)^{\alpha_B},$$

$$A_A \sim \zeta_A^{\pm}\Gamma(2-\alpha_A) = -\frac{\pi\beta_A(1 \pm e^{-i\pi\alpha_A})}{\sin\pi\alpha_A\,\Gamma(\alpha_A - 1)}\left(\frac{s-u}{2s_0}\right)^{\alpha_A}. \tag{2}$$

Generalize the one-particle propagator for exchange by[5]

$$G_B(t - m_B{}^2)^{-1} \to \alpha' \frac{A_B}{\alpha_B},$$

$$G_A(t - m_A{}^2)^{-1} \to \alpha' \frac{A_A}{\alpha_A - 1};$$

the amplitude for ρ^0 production is

$$A^{\rho} = \frac{-\pi\alpha'\beta^{\rho}}{\Gamma(\alpha+2)}\left(\frac{s-u}{2s_0}\right)^{\alpha}\{r(1+\alpha_A)\alpha_A[\tan(\tfrac{1}{2}\pi\alpha_A) + i] - (1+\alpha_B)[\cot(\tfrac{1}{2}\pi\alpha_B) - i]\}. \tag{3}$$

In the factor outside the brackets we have neglected any slight difference between α_A and α_B; this can be absorbed in the variation of the coefficients β_A, β_B, which will be neglected anyhow.

Equation (3) for $|A^{\rho}|^2$ yields an interference minimum around $t = -0.8\ (\mathrm{GeV}/c)^2$ for $\pm r = 2 \pm 1$, if we use linear trajectories with slope from Eq. (1), and $(m_\pi c)^2 = 0.02\ (\mathrm{GeV}/c)^2$, $(m_A c)^2 = 1.15\ (\mathrm{GeV}/c)^2$. The positive sign for r gives a somewhat deeper and sharper minimum.

For ω^0 production the trajectory signatures are reversed from Eq. (3), so that a ghost could arise as $\alpha_A \to 0$. This is eliminated by the conventional factor α_A, but the resultant function is perfectly smooth in the neighborhood of the ghost. To obtain a dip, some additional hypothesis is needed, and the most immediate in the current climate is to ascribe some complex part to the trajectory. Thus, the ghost-forming and -suppressing factors may get slightly out of register and yield a narrow oscillation, which is essentially the observed feature.

The most physical way of introducing a complex trajectory[8] is to regard it as a sum over real single-particle exchanges, the single particles comprising an infinite sequence of Regge recurrences. Then a reaction amplitude has the form

$$T(s,t) = \sum{}^{\pm} \frac{b(J,t)}{[M^2(J) - t]} P_J(z). \tag{4}$$

Here the sum is over even ($\sum^{+}$) or odd ($\sum^{-}$) integral, non-negative J, with $z = (u-s)(4m^2 - t)^{-1}$ in the case of a single mass m for the reacting particles. The denominator in Eq. (4) is simply the propagator of the exchanged particle of spin J with angular function $P_J(z)$ and amplitude $b(J,t)$.

The amplitude $T(s,t)$ in Eq. (4) can be expanded about its poles, say $J = J_0$, when $t = t_0$. For the expansion it is convenient to define an inverse function,

$$\alpha^{-1}(x) = M^2(x), \quad x > 0. \tag{5}$$

Take α and x real to begin with; this limitation will be relaxed below. A pole of Eq. (4) is then specifically

$$\alpha^{-1}(J_0) = t_0,$$

$$J_0 = \alpha(t_0). \tag{6}$$

Consider the function

$$J - \alpha(t) = \alpha(\alpha^{-1}(J)) - \alpha(t) \tag{7}$$

and expand to first order about the pole:

$$[J - \alpha(t)] - [J_0 - \alpha(t_0)] = \alpha'(t_0)\{[\alpha^{-1}(J) - t] - [\alpha^{-1}(J_0) - t_0]\}. \tag{8}$$

By Eq. (6) this becomes

$$\frac{\alpha'(t_0)}{J - \alpha(t)} = \frac{1}{\alpha^{-1}(J) - t} = \frac{1}{M^2(J) - t}, \tag{9}$$

which by insertion into Eq. (4) yields a standard Regge form. This argument is just a repetition of that in Ref. 8, perhaps too much expanded, but given in order to facilitate extension to complex trajectories in a "realistic" way.

The extension is trivial: All physical particles are subject to spontaneous decay even while undergoing virtual exchange, so in Eq. (4) we should put $M^2(J) \to M^2(J) - i\Gamma(J)M(J)$. Here $\Gamma(J) \ll M(J)$ in general, so we can treat the imaginary part as a small correction. In the previous treatment α and t_0 now become slightly complex, but J_0 and t remain real. Equation (6) becomes

$$J_0 = \alpha(t_0)$$
$$= \alpha(M_0{}^2 - i\Gamma_0 M_0)$$
$$\simeq \alpha(M_0{}^2) - i\Gamma_0 M_0 \alpha'(M_0{}^2)\,. \qquad (10)$$

Taking $J_0 = \alpha_R(t_0)$ and $\Gamma_0 M_0 \alpha' = \alpha_I(t_0)$, we can write

$$\alpha(t_0) = \alpha_R(t_0) + i\alpha_I(t_0)\,, \qquad (11)$$

where α_I is positive for $t_0 > 0$. The previous arguments now repeat exactly, leading to

$$\frac{1}{M^2(J) - t} = \frac{\alpha'(t_0)}{J - \alpha_R(t) - i\alpha_I(t_0)}\,. \qquad (12)$$

Here $\alpha'(t_0)$ has a slightly complex argument as in Eq. (10); for its imaginary part to be non-negative would require $\alpha''(t_0) \leq 0$. For a strictly linear trajectory α' is a real constant, which is sufficient for our purposes.

A more abstract procedure[9] is to replace a single Regge trajectory by a pair of complex conjugate trajectories. The total amplitude then remains as real as before, although the physical necessity for this is not argued. In any case, this is a more formal procedure; the complex conjugate parts are introduced explicitly in regions of negative t and hence are necessarily devoid of direct interpretation in terms of particle masses or decay widths. With this procedure the usual singularity $\beta(\alpha - \alpha_0)^{-1}$ in the neighborhood of a Regge pole is replaced by $\beta_+(\alpha - \alpha_+)^{-1} + \beta_-(\alpha - \alpha_-)^{-1}$, where $\alpha_\pm = \alpha_0 \pm i\alpha_I$ and $\beta_+^* = \beta_-$. In order to approach the usual trajectory as $\alpha_I \to 0$, let $\beta_+ + \beta_- = \beta$; then the prescription becomes

$$\frac{\beta}{\alpha - \alpha_0} \to \beta\left[\frac{(\alpha - \alpha_0)}{(\alpha - \alpha_0)^2 + \alpha_I{}^2} - i\lambda\,\frac{\alpha_I}{(\alpha - \alpha_0)^2 + \alpha_I{}^2}\right]\,, \qquad (13)$$

where $\lambda = (\beta_- - \beta_+)/(\beta_+ + \beta_-)$. Here λ is pure imaginary; in the realistic approach of Eq. (12) we put $\alpha_R = \alpha$ and recognize J as α_0, so that $\lambda = 1$. The two situations can be combined under Eq. (13) by the restriction

$$\mathrm{Re}\lambda \geq 0\,. \qquad (14)$$

The replacement in Eq. (13) is necessary only when $|\alpha - \alpha_0|$ is of order α_I or less; in the present analysis this situation occurs only near the A-trajectory ghost at $t_G = -0.25$ $(\mathrm{GeV}/c)^2$. This anomaly is taken to have an experimental width of $\Delta t \approx 0.05$ $(\mathrm{GeV}/c)^2$. Its actual shape is of secondary concern here, and depends on the choice of λ. The solid curve in Fig. 1 corresponds to $\lambda = 0$; the better fit of the dashed curve for $\lambda = 1$ corresponds to a single complex trajectory of conventional

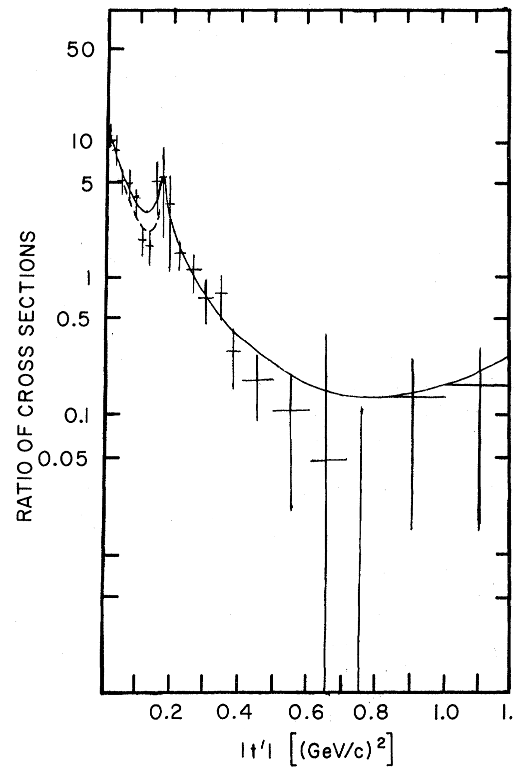

FIG. 1. Fit to $\sigma_0^-(\rho^0)/\sigma_0^-(\omega^0)$ with parameters described in text; experimental points are from Ref. 1.

imaginary sign.[10] The ghost structure is sensitive to the value of r and determines $\pm r \approx 1.5 \pm 0.5$; as before, there is some variation of the fit with sign.

Note that here $|\beta^\rho/\beta^\omega|^2 = A = 0.4 \pm 0.1$, in good agreement with Ref. 5; this ratio depends mainly on the forward cross sections and hence on the B trajectory (i.e., the pion pole near $t = 0$). For β^ρ/β^ω real and positive the relative phase of ω to ρ^0 production at $|t| \lesssim 0.22$ $(\mathrm{GeV}/c)^2$ is $\beta = 0.8$–0.9 rad, as compared with the observation[11] $\beta = 1.5 \pm 0.3$ rad. The present model, especially with no background terms in Eq. (3), may be too simple to fit these refinements.

The ratio $\sigma_0(\rho)/\sigma_0^-(\omega)$ plotted in Fig. 1 has no special significance beyond being the most efficient way to display the points raised here. If the present interpretation is correct, a narrow dip in Fig. 1 should occur near $t = -1.4$ $(\mathrm{GeV}/c)^2$ as a ghost effect in the B trajectory. There are no present data on $\rho_{00}d\sigma/dt$ of sufficient statistics to reveal any narrow fluctuations of this sort; and the specific shape of this "ghost effect" is hardly predictable. Similar effects seem to occur in $\pi^+ p \to \eta^0\Delta^{++}$.

The A trajectory lies slightly higher than the B and should dominate as $s \to \infty$. This emergence will be slow but implies the eventual disappearance of the ρ^0 dip around $t=-0.7$ (GeV/c)^2 with increasing s, while the narrow ω^0 dip at $t\approx -0.25$ (GeV/c)^2 remains. Note that the present interpretation, unlike that of Ref. 5, does not absolutely require the existence of the ρ^0 dip.

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[1]G. S. Abrams, J. W. J. Barnham, W. R. Butler, D. G. Coyne, G. Goldhaber, B. H. Hall, and J. MacNaughton, Phys. Rev. Letters 25, 617 (1970).

[2]D. Brown, G. Gidal, R. W. Birge, S. Y. Fung, W. Jackson, and R. T. Poe, Phys. Rev. D 1, 3053 (1970).

[3]I. J. Bloodworth, W. C. Jackson, J. D. Prentice, and T. S. Yoon, Nucl. Phys. B35, 79 (1971).

[4]C. L. Pols, D. J. Schotanus, D. Z. Toet, R. T. Van de Walle, K. Böckmann, K. Sternberger, B. Wagini, G. Winter, J. V. Major, E. Cirba, R. Vanderhagen, G. Rinaudo, and A. Werbrouck, Nucl. Phys. B25, 109 (1970).

[5]G. S. Abrams and U. Maor, Phys. Rev. Letters 25, 621 (1970).

[6]The use of two different, non-Pomeranchuk trajectories in a Regge fit is a familiar idea: e.g., the ρ and ρ' to explain polarization in pion charge exchange, H. Högaasen and W. Fischer, Phys. Letters 22, 516 (1966) and R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters 18, 259 (1967); the ρ and B trajectories for $\pi\to\omega$, J. Tran Thanh Van, Lett. Nuovo Cimento 2, 678 (1970); the π and A_2 trajectories for $\pi\to\rho$, $K\to K^*$, M. Markytan and P. Schmid, Lett. Nuovo Cimento 3, 51 (1970). These examples are illustrative only and not exhaustive.

[7]This dip is significant in Refs. 1–4, and also in $\pi^- p\to\omega^0 n$: L. E. Holloway, B. Huld, M. Jordan, D. W. Mortara, E. I. Rosenberg, A. D. Russell, S. Bernstein, M. H. Garrell, S. Margulies, and D. W. McLeod, Phys. Rev. Letters 27, 1671 (1971).

[8]L. Van Hove, Phys. Letters 24B, 183 (1967).

[9]J. S. Ball, G. Marchesini, and F. Zachariasen, Phys. Letters 31B, 583 (1970).

[10]We have not pursued the complex conjugate trajectory pair here, but it must also fit qualitatively. Equation (13) will yield a dip-and-bump structure at a ghost, the relative positions of the two being opposite for $\lambda=\pm i|\lambda|$.

[11]G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. N. MacNaughton, and G. H. Trilling, Phys. Rev. Letters 23, 1351 (1969).

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Fermion Loops and the $K_2^0 \to \mu^+\mu^-$ Puzzle

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We show that the fermion-loop model for $K_2^0\to\gamma\gamma$ recently proposed by Rockmore and Wong makes the puzzle of the missing $K_2^0\to\mu^+\mu^-$ rate some four times worse, and that the difficulty cannot be removed by the usual CP-violation hypothesis.

In a recent letter, Rockmore and Wong[1] have shown that the fermion-loop model can be used to provide a quantitative explanation of the $K_2^0\to\gamma\gamma$ rate. It is the purpose of this comment to point out that their explanation only makes the $K_2^0\to\mu^+\mu^-$ puzzle worse.

The branching ratio

$$R=\frac{\Gamma(K_2^0\to\mu^+\mu^-)}{\Gamma(K_2^0\to\gamma\gamma)}$$

is measured to be less than 0.31×10^{-5} with 90% confidence.[2] If one assumes CP invariance and standard electrodynamics, one can bound this ratio by $R\geq 1.17\times10^{-5}$ by using unitarity and only the imaginary part of $K_2^0\to\mu^+\mu^-$.[3] The experiment is outside this bound. It has been suggested by Christ and Lee[4] that a CP violation could produce destructive interference and vitiate the use of the unitarity bound. However, given a model for the $K_2^0\to\gamma\gamma$ process such as Rockmore and Wong's, one need not just bound the $K_2^0\to\mu^+\mu^-$ amplitude, but rather one can calculate the whole thing. As-