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Dual Amplitudes and Scaling in Inelastic Lepton-Hadron Collisions

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Scaling behaviors of the structure functions F_1 and F_2 in high-energy inelastic lepton-hadron collisions are obtained with the help of the off-mass-shell scattering amplitude constructed from five-point Veneziano functions. Our method reveals the drawback of the perturbation approach and suggests a fermionic character of the partons. A quark model is seen to be favored by our results. Unlike the existing theoretical attempts, the over-all results of the present formulation seem to be supported by experimental data.

INTRODUCTION

Recent experimental findings concerning the scaling behavior of structure functions and angular distributions[1,2] in high-energy exclusive reactions have opened a wide field of investigation in elementary-particle physics. Several theoretical explanations of such phenomena have been put forth by different authors.[3-5] Attempts have also been made to obtain compatibility between such theoretical findings, where ambiguities have been seen to be present owing to different basic assumptions. Recently, the sum rule of Callan and Gross[6] (CG) has predicted two relations for the transverse and longitudinal cross sections. As the two results are from different hypotheses regarding the constituents of currents, it is quite natural to hope that experimental verifications of such theoretical predictions will be useful in understanding the basic structure of hadrons.

Until now we have had no criterion to choose between two such alternative characteristics of the basic constituents of matter, although there have been several attempts to select the proper one. Recently Jackiw and Preparata[7] (JP) have shown that the prediction of the CG sum rule, viz.,

$$F_2(\omega) - \omega F_1(\omega) = 0 \quad \text{(quark model)},$$

$$\omega F_1(\omega) = 0 \quad \text{(gauge field algebra)},$$

cannot be tested in a canonical field theory where perturbation is the only tool. In their paper JP have shown that $F_2 - 3\omega F_1$ diverges logarithmically with ν as $\nu \to \infty$, implying a nonscaled behavior of the structure functions. In this note we have demonstrated that such wide deviations are due to the use of perturbation theory, and our results also favor the fermion character of partons. Furthermore our results for F_2 and F_1 suggest a zero for $F(\omega)$ at $\omega = 2$ and the proper behavior as $\omega \to 0$.[8,9]

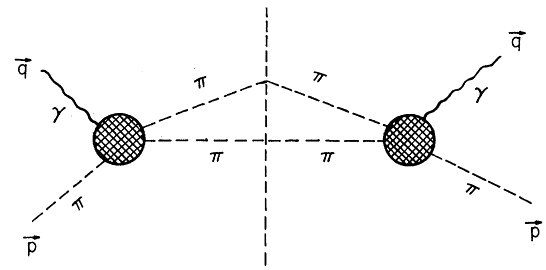

FIG. 1. The diagram contributing to the discontinuity in Eq. (1).

FORMULATION

In order to avoid the complexities of spin we have considered the scattering of an electron and a pion, which in turn implies a study of the imaginary part of the forward virtual Compton amplitude, i.e., the process $\gamma\pi \to \gamma\pi$. Such a discontinuity can easily be computed in the fashion of JP from the diagram of Fig. 1, where the blobs represent all possible strong-interaction effects.

The general structure of the imaginary part of the forward virtual Compton amplitude is written as

$$T_{\mu\nu} = \int dx\, e^{iq\cdot x}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle$$
$$= \nu^{-1}\left(p_\mu + \frac{q_\mu}{\omega}\right)\left(p_\nu + \frac{q_\nu}{\omega}\right)\tilde{F}_2(\omega, \nu)$$
$$- \left(g_{\mu\nu} + \frac{q_\mu q_\nu}{\omega\nu}\right)\tilde{F}_1(\omega, \nu), \tag{1}$$

which defines the structure functions $\tilde{F}_1$ and $\tilde{F}_2$. It is easy to note that $\tilde{F}_1$ ($\tilde{F}_2$) is even (odd) under $\omega \to -\omega$ and $\nu \to -\nu$ and that they are dimensionless. We denote the Bjorken limits of $\tilde{F}_1$ and $\tilde{F}_2$ by

$$\lim_{\nu\to\infty} \tilde{F}_1(\omega, \nu) = F_1(\omega),$$
$$\lim_{\nu\to\infty} \tilde{F}_2(\omega, \nu) = F_2(\omega). \tag{2}$$

An alternative expression for $T_{\mu\nu}$ obtained from Fig. 1 is

$$T_{\mu\nu} = \int\int \delta(p_3^2 - \mu^2)\theta(p_3^0)\delta(p_4^2 - \mu^2)\theta(p_4^0) \times A(s, t)\bar{A}(s, t)d^4p_3 d^4p_4,$$

where A is the amplitude for the process

$$\gamma(\text{virtual})(q) + \pi(p) \to \pi(p_3) + \pi(p_4), \tag{3}$$

with

$$s = (q+p)^2,$$
$$t = (q-p_3)^2, \tag{4}$$
$$u = (q-p_4)^2.$$

The only plausible expression for the scattering amplitude of $\gamma\pi \to \pi\pi$ with γ being off the mass shell can be obtained by the technique of Rashid,[10] from the five-point Veneziano amplitude. Such a procedure for obtaining dual amplitudes for current-hadron scattering has already been tested. In its simple form the amplitude reads

$$A = \epsilon_{\mu\nu\lambda\sigma}\epsilon_\mu p_\nu p_{3\lambda} p_{4\sigma} B(s, t, q^2) \tag{5}$$

and

$$B = \int\int_0^1 du_1 du_4 u_1^{-\alpha_\omega(q^2)}(1-u_1)^{-\gamma_1} u_4^{-\alpha_\rho(s)} \times (1-u_4)^{-\alpha_\rho(t)}(1-u_1u_4)^{\alpha_\rho(t)-1}$$
$$+ (\text{permutation of } s,\ t,\ p_1^2). \tag{6}$$

The constant γ_1 has been fixed to be equal to zero by extrapolating Eq. (3) to the ω-meson pole. (We have neglected isospin totally.) The fourfold integrations in Eq. (3) are done with the help of a known identity,

$$\int d^4p_4\, \delta(p_4^2 - \mu^2)\theta(p_4^0) = \frac{d^3p_4}{p_4^0},$$

when we are left only with an angular integration, with the integrand being a function of the scaling variable ω. As we are interested only in the asymptotic limits, we obtain

$$\lim_{\nu\to\infty} \omega^2 \frac{p_\mu p_\nu}{\nu} T_{\mu\nu} = F_2(\omega) - \omega F_1(\omega) = 0,$$
$$\lim_{\nu\to\infty} \omega T_{\mu\mu} = F_2(\omega) - 3\omega F_1(\omega)$$
$$= (2-\omega)^2 f(\omega), \tag{7}$$

with

$$f(\omega) = \int dz\left(1 - \frac{2}{\omega}\right)^{-2g_+(\omega, z)-1} \frac{\pi g_+^2(\omega, z)}{\sin^2 \pi g_+(\omega, z)}$$
$$\times\left[m - \frac{z(2-\omega)}{z+1}\right]^2 \left(\frac{(2-\omega)zm^2}{z+1} + \frac{2m(2-\omega)^2}{(z+1)^2} - m\left\{m - \left[m^2 + \left(\frac{2-\omega}{z+1}\right)^2\right]^{1/2}\right\}\right)^2$$
$$+\text{a similar integral with } (z+1) \text{ replaced by } (z-1), \tag{8}$$

where

$$g_{\pm}(\omega, z) \equiv \left(\frac{2-\omega}{z \pm 1}\right)^2 + \left\{ m - \left[m^2 + \left(\frac{2-\omega}{z \pm 1}\right)^2 \right]^{1/2} \right\}.$$

DISCUSSIONS

Equations (7) are enough to suggest the[6] proper scaling behavior of F_2 and F_1. Again, as $f(\omega)$ in Eq. (8) is finite, (7) clearly suggest a zero at $\omega = 2$ in $F(\omega)$, which is the case found experimentally. Improvement over the perturbation-theoretic approach is quite clear from (7), which is here finite and scaled while the result of JP diverges as $\ln(\nu^2/\mu^2)$.

In this connection it is interesting to compare our result with some previous attempts to construct the off-mass-shell amplitude for $\gamma\pi \to \gamma\pi$ to study the behaviors of F_1 and F_2. Such an investigation worth mentioning is that of Sakurai.[11] Using a vector-dominance model for the Pomeranchon he obtained a scaling behavior for F_2, but F_1 is not found to be a function of ω. Until now the experimental information about F_1 is very meager, so that the exact behavior of F_1 is still in doubt. But the behavior of fundamental constituents depends on the combination $F_2(\omega) - \omega F_1(\omega)$ which demands a scaling behavior of F_1. We note also that our result suggests

$$4\pi^2 \alpha M (F_2 - \omega F_1) = \sigma_L(\omega) = 0, \quad \sigma_T \neq 0,$$

which yields

$$R = \frac{\sigma_L(\omega)}{\sigma_T(\omega)} = 0,$$

implying a fermionic character for partons, which is also a result of quark-model commutation relations. In light of the above discussions it seems quite appropriate to comment that our purely dynamic approach to the problem of scaling behavior is useful in the search for the basic commutation rules in the theory of fundamental particles.

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