J. Primack, Ann. Phys. (N.Y.) 52, 315 (1969); R. Krajcik and L. Foldy, Phys. Rev. Letters 24, 545 (1970); A. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150 (1970); R. Blankenbecler and R. Sugar, *ibid.* 2, 3024 (1970); J. F. Gunion and R. Blankenbecler, *ibid.* 3, 2125 (1971).

[4]H. Cheng and T. T. Wu, Phys. Rev. Letters 22, 666 (1971).

[5]H. Cheng and T. T. Wu, Phys. Rev. D 5, 3170 (1972); 6, 1693 (1972).

[6]See Eqs. (A10), (A11), and (A14) in H. Cheng and T. T. Wu, Phys. Rev. D 1, 1083 (1970). $I^{\gamma}(\vec{r}_1, \vec{x})$ is equal to the Fourier transform (with respect to $\vec{q}_\perp$) of $I^{\gamma}(\vec{r}_1, \vec{q}_\perp)$ given by Eq. (2) of Ref. 4.

[7]T. T. Chou and C. N. Yang, Phys. Rev. 175, 1832 (1968).

[8]See in particular Sec. 8 D of H. Cheng and T. T. Wu, Phys. Rev. 182, 1852 (1969).

[9]B. W. Lee, Phys. Rev. D 1, 2361 (1970).

[10]H. Cheng and T. T. Wu, Phys. Rev. D 1, 1069 (1970); 1, 1083 (1970).

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CP Nonconservation and $K \to l\bar{l}\gamma$ Decays*

S. K. Singh

Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

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A simple model which explains the observed suppression of the $K_L \to \mu^+\mu^-$ rate has been used to calculate the *CP*-violating contribution to $K \to l\bar{l}\gamma$ decays. For $K_L \to \mu^+\mu^-\gamma$ this rate is comparable to the Dalitz-pair rate while for $K_L \to e^+e^-\gamma$ and $K_S \to l\bar{l}\gamma$ the contribution is small.

Recently there have been many attempts to explain[1] theoretically the observed lower limit on the $K_L \to \mu^+\mu^-$ rate.[2] If *CP* nonconservation plays a dominant role in these decays as suggested by Lee and Christ[1] then its implication in other rare decay modes is worth examining. In this note we discuss the question of *CP* nonconservation in $K \to l\bar{l}\gamma$ decays. The *C* and *CP* properties of the various final states possible in these decays are shown in Table I. The $K \to l\bar{l}\gamma$ decays can take place through electromagnetic interactions which are *CP*-conserving. Since the lepton pair state has to be produced through a photon conversion it must be a *C*-odd *CP*-even state. Thus $K_L \to l\bar{l}\,(^3S_1)\gamma(M1)$ and $K_S \to l\bar{l}\,(^3S_1)\gamma(E1)$ are the *CP*-conserving electromagnetic decays. The *CP*-admixed states in K_L and K_S will be neglected because their contribution to the $K_{L(S)} \to l\bar{l}\gamma$ rate is small.

We consider the possibility of the existence of *CP*-odd nonelectromagnetic interactions. Such an interaction has been described by Wolfenstein[3] which explains the suppression of $K_L \to \mu^+\mu^-$ rate without upsetting any other experimental result. In this model the interaction Hamiltonian is given by

$$H' = iG' \sin\theta_C A_\lambda^7 \bar{\psi}_\mu \gamma_\lambda \gamma_5 \psi_\mu , \qquad (1)$$

where A_λ^7 is *C*-odd $|\Delta S| = 1$ axial-vector current and transforms as the seventh component of SU(3). G' is the strength of the interaction and is given by

$$3.8\times10^{-2} \geq G'/G \geq 1.1\times10^{-2} . \qquad (2)$$

This interaction requires the lepton pair to be in *C*-even *CP*-even state and thus only $K_L \to l\bar{l}\,(^3P)\gamma(E1)$ is allowed. $K_S \to l\bar{l}\gamma$ decays are not allowed because the matrix element $\langle\gamma|A_\lambda^7|K_S\rangle$ is zero due to *C* invariance. This however can take place through bremsstrahlung in which K_S decays into $l\bar{l}\,(^1S_0)$ state and then one of the leptons emits a photon, giving $l\bar{l}\,(^1P_1)\gamma(E1)$ final state. *CP*-conserving decays are the electromagnetic $K_L \to l\bar{l}\,(^3S_1)\gamma(M1)$ and $K_S \to l\bar{l}\,(^3S_1)\gamma(E1)$. As a result the decay rate is simply the sum of *CP*-even and *CP*-odd rates and there are no *CP*-violating K_L-K_S interference effects in the photon spectrum. In this model a definite *CP* violation could be manifested through a correlation between muon spins and photon polarization.

It is natural to add to H' an interaction Hamiltonian[4] H'' where the hadronic current A_λ^7 is coupled to a vector leptonic current in the form

$$H'' = iG'' \sin\theta_C A_\lambda^7 \bar{\psi}_\mu \gamma_\lambda \psi_\mu . \qquad (3)$$

This interaction allows $K_L \to l\bar{l}\,(^3S_1)\gamma(E1)$ decay, and K_L-K_S interference effects might be observed in the photon spectrum because K_S is electromagnetically allowed to decay into the same final states. This interaction, however, does not contribute to $K \to l\bar{l}$ decays and the conclusions regarding $K_L \to \mu^+\mu^-$ are unaltered. In the following, we

TABLE I. C and CP of various $l\bar{l}\gamma$ states.

$l\bar{l}$	γ	C	CP
$^3P_1$	$E1$	−	+
$^3S_1$	$E1$	+	+
$^1P_1$	$M1$	+	+
$^3P_1$	$M1$	−	−
$^3S_1$	$M1$	+	−
$^1P_1$	$E1$	+	−

limit ourselves to a discussion of the rates and the photon spectrum only.

The electromagnetic decay takes place through $K(p)\to\gamma(k)+\gamma(k')\to\gamma(k)+l(p_-)+\bar{l}(p_+)$ as shown in Fig. 1(a). The matrix element for this diagram is given by

$$m=eH_{PS}(k'^2)\frac{1}{k'^2}\times\epsilon_{\mu\nu\rho\sigma}\epsilon_\nu k_\rho k'_\sigma\bar{u}(p_-)\gamma_\mu v(p_+)\,, \tag{4}$$

where $H_{PS}(k'^2)$ is the form factor defined through the $K_L\to 2\gamma$ matrix element. Assuming a constant form factor, H_{PS} is related to the $K_L\to 2\gamma$ rate R as

$$R(K_L\to 2\gamma)=\frac{H_{PS}{}^2 m_K{}^3}{64\pi}\,. \tag{5}$$

The decay spectrum is then calculated to be given by[5]

$$\frac{d\omega}{dx}=\frac{1}{3}\left(\frac{H_{PS}}{4\pi}\right)^2\alpha m_K{}^3 I\,, \tag{6}$$

where

$$I=\left(1-\frac{x^2}{m_K{}^2}\right)^3\left(1+\frac{2m_l{}^2}{x^2}\right)\left(1-\frac{4m_l{}^2}{x^2}\right)^{1/2}\frac{1}{x}\,, \tag{7}$$

where x is the dilepton mass and α is the fine-structure constant. The photon spectrum is peaked around $x=240$ MeV and is shown in Fig. 2(a). Using $\Gamma(K_L\to 2\gamma)/\Gamma(K_L\to\text{all})=4.5\times10^{-4}$ (Ref. 6) and $\Gamma(K_S\to 2\gamma)/\Gamma(K_S\to\text{all})\le 1.2\times10^{-3}$ (Ref. 7) we calculate the rates for $K\to l\bar{l}\gamma$ decays and the results are shown in Table II.

The CP-violating contribution to the decay $K_L\to l\bar{l}\gamma$ is due to the diagram shown in Fig. 1(b). The matrix element can be written using the standard procedure[8] as

$$m=iG'e\sin\theta_C\,\epsilon_\lambda(k)m_{\lambda\mu}l_\mu\,, \tag{8}$$

where

$$l_\mu=\bar{u}(p_-)\gamma_\mu\gamma_5 v(p_+)\quad\text{for interaction } H', \tag{9}$$

$$l_\mu=\bar{u}(p_-)\gamma_\mu v(p_+)\quad\text{for interaction } H'' \tag{10}$$

and

$$m_{\lambda\mu}=i\int d^4x\, e^{ik\cdot x}\langle 0|T(j_\lambda^{\text{e.m.}}(x)A_\mu^7(0))|K(p)\rangle\,. \tag{11}$$

It is easy to see that this vanishes in the exact SU(3) limit and our results are proportional to the SU(3) breaking. Since there are no kaon poles in $m_{\lambda\mu}$ the most general form of $m_{\lambda\mu}$ can be written on grounds of covariance alone to be

$$m_{\lambda\mu}=A(\nu,k^2)g_{\lambda\mu}+B(\nu,k^2)p_\lambda k_\mu+C(\nu,k^2)p_\lambda p_\mu+D(\nu,k^2)k_\lambda k_\mu+E(\nu,k^2)k_\lambda k_\mu\,, \tag{12}$$

where $\nu=k\cdot p$. For the physical process ($k^2=0$), the gauge invariance condition requires $m_{\lambda\mu}$ to be of the following form:

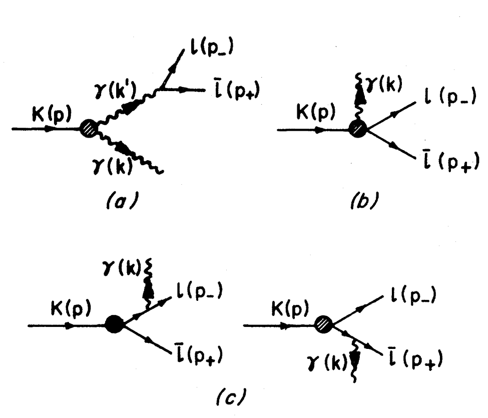

FIG. 1. (a) Electromagnetic contributions to $K\to l\bar{l}\gamma$; (b) direct weak contributions to $K\to l\bar{l}\gamma$; (c) bremsstrahlung contribution to $K\to l\bar{l}\gamma$.

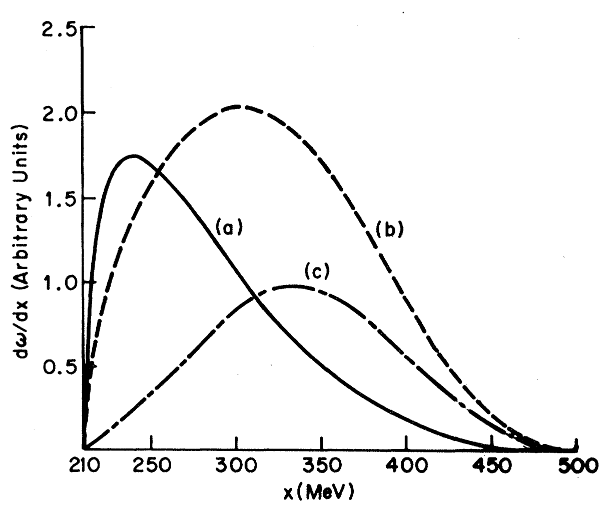

FIG. 2. (a) $dw/dx\sim x$ for electromagnetic contributions to $K\to\mu^+\mu^-\gamma$; (b) $dw/dx\sim x$ for $K_L\to\mu^+\mu^-\gamma$ using interaction H'; (c) $dw/dx\sim x$ for $K_L\to\mu^+\mu^-\gamma$ using interaction H''.

TABLE II. Branching ratio $R(K\to l\bar{l}\gamma)=\Gamma(K\to l\bar{l}\gamma)/\Gamma(K\to \text{all})$ for the various interactions considered.

Process	Dalitz-pair rate	CP-violating rate with $G', G''=2.5\times10^{-2}G$ for H'	H''
$K_L\to\mu^+\mu^-\gamma$	1.8×10^{-7}	3.3×10^{-7}	1.4×10^{-7}
$K_L\to e^+e^-\gamma$	7.0×10^{-6}	5.1×10^{-7}	4.2×10^{-7}
$K_S\to\mu^+\mu^-\gamma$	$\leq4.8\times10^{-7}$	$\sim10^{-9}$	not allowed
$K_S\to e^+e^-\gamma$	$\leq1.9\times10^{-5}$	small	not allowed

$$m_{\lambda\mu}=\nu a(\nu)\left(g_{\lambda\mu}-\frac{p_\lambda k_\mu}{\nu}\right), \tag{13}$$

and $a(\nu)$ is related to the form factor $A(\nu,k^2)$ by the condition

$$A(\nu,0)=\nu a(\nu). \tag{14}$$

The matrix element for the process $K_L\to l\bar{l}\gamma$ is then given by

$$m=iG'\sin\theta_C e\nu a(\nu)\epsilon_\lambda\left(g_{\lambda\mu}-\frac{p_\lambda k_\mu}{\nu}\right)l_\mu. \tag{15}$$

The decay spectrum is calculated from Eq. (15) to be

$$\frac{d\omega}{dx}=\left(\frac{G'\sin\theta_C}{2\pi}\right)^2\frac{\alpha m_K^3}{8}a^2(\nu)\left[I_1+\left(\frac{G''}{G'}\right)^2 I_2\right], \tag{16}$$

where

$$I_1=\left(1-\frac{x^2}{m_K^2}\right)^3\left(1+\frac{2m_l^2}{x^2}\right)\left(1-\frac{4m_l^2}{x^2}\right)^{1/2}x^3,$$

$$I_2=\left(1-\frac{x^2}{m_K^2}\right)^3\left(1-\frac{4m_l^2}{x^2}\right)^{3/2}x^3,$$

$$x^2=(p-k)^2.$$

We have plotted in Figs. 2(b) and 2(c) the decay spectrum given in Eq. (16) for constant $a(\nu)$ $[=a(0)]$ calculated in soft-kaon limit using PCAC (partially conserved axial-vector current) and current algebra. From Eq. (14) $a(0)$ is given by

$$a(0)=\left.\frac{dA(\nu,0)}{d\nu}\right|_{\nu=0}. \tag{17}$$

Following Das *et al.*,[9] we get

$$a(0)=\frac{3\sqrt{2}\,\beta}{m_\rho^2 f_K\alpha},$$

where

$$\beta=\left[\left(1-\frac{m_\rho^2}{m_\varphi^2}\right)m_\varphi\Gamma_\varphi+\left(1-\frac{m_\rho^2}{m_\omega^2}\right)m_\omega\Gamma_\omega\right]. \tag{18}$$

Γ_ϕ and Γ_ω are the decay widths[10] for the decays $\phi\to e^+e^-$ and $\omega\to e^+e^-$ and f_K is the kaon decay constant. We have taken the experimental values of m_V and Γ_V ($V=\rho,\omega,\phi$) to evaluate the decay rates and obtain

$$\frac{\Gamma(K_L\to\mu^+\mu^-\gamma)}{\Gamma(K_L\to\text{all})}=1.07\times10^{-3}\left(\frac{G'\beta}{G}\right)^2\left[1+0.42\left(\frac{G''}{G'}\right)^2\right]. \tag{19}$$

G' is given in Eq. (2) while there is no estimate for G''. Assuming $G''=G'$ as in Okubo's[4] model we calculate the rates for $G'=G''=2.5\times10^{-2}G$. The results are given in Table II for H' and H''. To compare these results with the predictions of this model for $K_S\to\mu^+\mu^{-3}$ we find

$$\frac{\Gamma(K_L\to\mu^+\mu^-\gamma)/\Gamma(K_L\to\text{all})}{\Gamma(K_S\to\mu^+\mu^-)/\Gamma(K_S\to\text{all})}=0.07\left[1+0.42\left(\frac{G''}{G'}\right)^2\right]. \tag{20}$$

For K_S there is no direct term, but the bremsstrahlung terms as shown in Fig. 1(c) contribute through H'. The photon spectrum is typical bremsstrahlung spectrum dominated by the infrared photons. This contribution being proportional to the mass of the radiating particle is negligibly small for the electrons. For muons the rate is quoted in Table II for $E_\gamma\geq30$ MeV.

To conclude, the CP-violating contribution to $K_L\to\mu^+\mu^-\gamma$ rate due to H' is comparable to the Dalitz-pair rate. The uncertainty in the calculations is due to the lack of an adequate model to describe SU(3) breaking. The CP violation is hard to see because there is no interference of any kind in the photon spectrum. The CP-violating and CP-conserving spectra are, however, different and could be experimentally distinguished if enough events were to be observed.

In case of H'', the CP violation shows up in the photon spectrum due to K_L-K_S interference. The interference is limited to the middle part of the photon spectra. Since the CP-conserving and CP-violating spectra are quite different in this case too, the interference effect in the total rate will be small.

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[1]N. Christ and T. D. Lee, Phys. Rev. D 4, 209 (1971); G. R. Farrar and S. B. Treiman, *ibid.* 4, 257 (1971); M. K. Gaillard, Phys. Letters 35B, 431 (1971); 36B, 114 (1971); H. H. Chen and S. Y. Lee, Phys. Rev. D 4, 903 (1971); S. Barshay, Phys. Letters 36B, 571 (1971); unpublished report.

[2]A. R. Clark, T. Ellioff, R. C. Field, H. J. Frisch, R. P. Johnson, L. T. Kerth, and W. A. Wenzel, Phys. Rev. Letters 26, 1667 (1971).

[3]L. Wolfenstein, University of Michigan report (unpublished); in *Particles and Fields-1971*, proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971).

[4]S. Okubo and M. Bace, Nucl. Phys. B40, 541 (1972).

[5]R. H. Dalitz, Proc. Phys. Soc. (London) A64, 667 (1951); N. M. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955); C. Jarsklog and H. Pilkuhn, Nucl. Phys. B1, 264 (1967).

[6]K. Winter, in *Proceedings of the Amsterdam International Conference on Elementary Particles, 1971*, edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1972).

[7]J-P. Repellin, B. Wolff, J. C. Chollet, J-M. Gaillard, M. R. Jane, and K. R. Schubert, Phys. Letters 36B, 603 (1971).

[8]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

[9]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 859 (1967).

[10]Experimental numbers for Γ_ϕ and Γ_ω are taken from J. Lefrancois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

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Dual Amplitudes and Scaling in Inelastic Lepton-Hadron Collisions

T. Roy and A. Roy Chowdhury
Department of Physics, Jadavpur University, Calcutta-32, India
(Received 31 August 1971)

Scaling behaviors of the structure functions F_1 and F_2 in high-energy inelastic lepton-hadron collisions are obtained with the help of the off-mass-shell scattering amplitude constructed from five-point Veneziano functions. Our method reveals the drawback of the perturbation approach and suggests a fermionic character of the partons. A quark model is seen to be favored by our results. Unlike the existing theoretical attempts, the over-all results of the present formulation seem to be supported by experimental data.

INTRODUCTION

Recent experimental findings concerning the scaling behavior of structure functions and angular distributions[1,2] in high-energy exclusive reactions have opened a wide field of investigation in elementary-particle physics. Several theoretical explanations of such phenomena have been put forth by different authors.[3-5] Attempts have also been made to obtain compatibility between such theoretical findings, where ambiguities have been seen to be present owing to different basic assumptions. Recently, the sum rule of Callan and Gross[6] (CG) has predicted two relations for the transverse and longitudinal cross sections. As the two results are from different hypotheses regarding the constituents of currents, it is quite natural to hope that experimental verifications of such theoretical predictions will be useful in understanding the basic structure of hadrons.

Until now we have had no criterion to choose between two such alternative characteristics of the basic constituents of matter, although there have been several attempts to select the proper one. Recently Jackiw and Preparata[7] (JP) have shown that the prediction of the CG sum rule, viz.,

$$F_2(\omega) - \omega F_1(\omega) = 0 \quad \text{(quark model)},$$

$$\omega F_1(\omega) = 0 \quad \text{(gauge field algebra)},$$

cannot be tested in a canonical field theory where perturbation is the only tool. In their paper JP have shown that $F_2 - 3\omega F_1$ diverges logarithmically with ν as $\nu \to \infty$, implying a nonscaled behavior of the structure functions. In this note we have demonstrated that such wide deviations are due to the use of perturbation theory, and our results also favor the fermion character of partons. Furthermore our results for F_2 and F_1 suggest a zero for $F(\omega)$ at $\omega = 2$ and the proper behavior as $\omega \to 0$.[8,9]