Impact Picture and Scattering of Deuterons. I*

Hung Cheng†
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Tai Tsun Wu
Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 7 February 1972)

In this paper we apply the theory of the impact picture to the process of scattering of deuterons. We show that the impact-factor representation plays the same role in a complete relativistic scattering process as the Glauber form does in the scattering of a particle on a static deuteron. Therefore, as we use the impact-factor representation for the deuteron scattering amplitude, we include automatically the recoil effects of the deuteron, which must be taken into account when the recoil momentum of the deuteron is not much smaller than the deuteron mass. The main difference between the impact-factor representation and the Glauber form is that in the impact-factor representation, the internal wave function of the outgoing particle is Lorentz-contracted. This also accounts for the difference we found earlier between our results in quantum electrodynamics and the corresponding ones in the droplet model. To obtain the Lorentz-contracted wave function explicitly, we give the Lorentz transformation equations relating the laboratory system and the recoil system, defined to be the system in which the outgoing deuteron is at rest.

I. INTRODUCTION

Much of our knowledge of the neutron is derived from deuteron scattering. Among others, the scattering from deuteron targets provides us with information on (i) $\pi+n\rightarrow\pi+n$, (ii) $K+n\rightarrow K+n$, and (iii) $e+n\rightarrow e+n$.

Since the deuteron is a bound state of a proton-neutron pair, it is necessary to have a theoretical tool to extract the amplitude of neutron scattering from that of deuteron scattering. In the past, this role was adequately filled by the Glauber form, to which a vast amount of work has been devoted.[1]

The Glauber form is based on the assumption that the deuteron can be treated as a static target. When the recoil momentum Δ of the deuteron is comparable to the deuteron mass M_d, however, this assumption is no longer valid. Thus a new method is required to deal with deuteron scattering when the recoil momentum Δ is of the order of 1 BeV – a region now accessible to experiments.

One of the treatments of nonstatic deuterons has been given by Casper and Gross.[2] They applied it to $e+d\rightarrow e+d$, and showed that the effect of Lorentz contraction due to the recoil is responsible for the disagreement of $dG_{En}(-\Delta^2)/d\Delta^2$ at $\Delta^2=0$ with the electron-neutron interaction experiment. (Their treatment emphasizes only the region of small recoil $\Delta\ll M_d$, where the recoil effects are in general quite small.) Other developments along this line were pursued by a number of authors.[3]

Quite independent of such developments, our understanding of a general high-energy scattering process has greatly increased over the past few years. In particular, there has emerged, out of a study of quantum field theories, a unified picture of high-energy scattering. We have now at our disposal many powerful tools to deal with problems of high-energy collisions. In this paper, we shall apply them to a specific problem – the scattering of a deuteron. Our approach will lead to a very simple and natural formulation of the problem. Furthermore, this formulation is applicable not only to a loosely bound system like the deuteron, but also to a tightly bound system such as a hadron. Indeed, for a hadron with a mass under 1 BeV, the recoil effects already become important when the momentum transfer is of the order of a few hundred MeV.

II. PRELIMINARY CONSIDERATIONS

In various examples in quantum electrodynamics[4] and ϕ^3 theory,[5] we have demonstrated that, in the limit $s\rightarrow\infty$ with the momentum transfer fixed, the scattering amplitude is always in the form of the impact-factor representation. Consider, for example, the special case of Compton scattering where the incident particles may exchange an arbitrary number of photons. (The corresponding impact diagram is illustrated in Fig. 1.) This example is chosen here because the incident photon in this process is treated as a bound system of two particles (an electron and a positron), and is therefore similar to the case of the deuteron. Furthermore, photon exchanges always give amplitudes linear in s, in approximate agreement with the behavior of diffractive amplitudes if logarithmic dependence of the energy is neglected. The scattering amplitude for this process is given by

$$\tfrac{1}{2}is\int d^2b\,d^2x_\perp \exp(-i\vec{\Delta}\cdot\vec{b})I^\gamma(\vec{r}_1,\vec{x}_\perp)I^e \times\left\{1-\exp\left[\frac{ie^2}{2\pi}K_0(\vec{b}-\tfrac{1}{2}\vec{x}_\perp)-\frac{ie^2}{2\pi}K_0(\vec{b}+\tfrac{1}{2}\vec{x}_\perp)\right]\right\}. \tag{2.1}$$

In (2.1), $\vec{r}_1=\frac{1}{2}\vec{\Delta}$, I^e is the electron impact factor (in the position space) given by

$$I^e=m_e^{-1}, \tag{2.2}$$

with m_e the mass of the electron, and $I^\gamma(\vec{r}_1,\vec{x}_\perp)$ is the photon impact factor proportional to[6]

$$\int_{-\infty}^{\infty} dx_+\,\mathrm{Tr}[\psi_i(x)\psi_j^\dagger(x)]_{x_-}=0, \tag{2.3}$$

where

$$x_\pm=t\pm z, \tag{2.4}$$

and ψ_i and $\psi_j^\dagger$ are respectively the initial- and the final-state wave functions of the electron-positron pair representing the photon. We have taken the positive z axis to be in the direction of the spatial momentum of the incident electron.

It is now instructive to compare the impact factor representation (2.1) with the Glauber form[1] or the equivalent formula in the droplet model.[7] The Glauber form for the π-d elastic scattering amplitude is

$$\frac{ik}{2\pi}\int d^2b\,\exp(-i\vec{\Delta}\cdot\vec{b})\int d^3x\,\phi_f^*(\vec{x})\phi_i(\vec{x}) \times\{1-\exp[i\chi_n(\vec{b}-\tfrac{1}{2}\vec{x}_\perp)+i\chi_p(\vec{b}+\tfrac{1}{2}\vec{x}_\perp)]\}. \tag{2.5}$$

In (2.5), ϕ_i (ϕ_f) is the wave function for the internal state of the incoming (outgoing) nucleon, $k\vec{e}_z$ is the momentum of the incoming pion, and χ_n and χ_p are the phase shifts associated with π-n and π-p scattering, respectively, and are the counterparts of $(ie^2/2\pi)K_0$ and $(-ie^2/2\pi)K_0$ in (2.1). Let us define the deuteron impact factor as

$$I^d(\vec{x}_\perp)=\int_{-\infty}^{\infty} dz\,\phi_f^*(\vec{x}_\perp,z)\phi_i(\vec{x}_\perp,z), \tag{2.6}$$

where, for clarity, we have written $\phi_i(\vec{x}_\perp,z)$ and $\phi_f(\vec{x}_\perp,z)$ in place of $\phi_i(\vec{x})$ and $\phi_f(\vec{x})$. Then (2.5) is seen to be in the form of the impact factor representation

$$\frac{ik}{2\pi}\int d^2b\,d^2x_\perp\exp(-i\vec{\Delta}\cdot\vec{b})I^d(\vec{x}_\perp) \times\{1-\exp[i\chi_n(\vec{b}-\tfrac{1}{2}\vec{x}_\perp)+i\chi_p(\vec{b}+\tfrac{1}{2}\vec{x}_\perp)]\}. \tag{2.7}$$

There is, however, a major difference between (2.1) and (2.7): The deuteron impact factor I^d as given by (2.6) is a function of $\vec{x}_\perp$ only, while the photon impact factor depends, in addition, on the momentum transfer as well. This is the same disagreement we found several years ago when we compared our results with those of the droplet model.[8]

To understand the reason for this discrepancy, let us study (2.6). Equation (2.6) tells us that the deuteron impact factor is related to the overlapping of the internal wave functions of the deuteron. There are two important points we wish to emphasize: (i) The function $\phi_f(\vec{x})$ describes the internal state of the deuteron *in the Lorentz frame in which the outgoing deuteron is at rest*. When Δ is comparable to the deuteron mass, the wave function of the deuteron to an observer at rest in the laboratory system is not $\phi_f(\vec{x})$ but is related to $\phi_f(\vec{x})$ by a Lorentz transformation. (ii) The wave functions in (2.6) should *not* be interpreted as the wave functions at $t=0$. Rather, they are wave functions on the light cone $t-z=0$ if the momentum of the pion is taken to be in the direction of the positive z axis. (If the momentum of the pion is taken to be in the direction of the negative z axis, then ϕ_i and ϕ_f in (2.6) are wave functions on $t+z=0$.) This is because the pion is traveling with a velocity near that of light and the deuteron is therefore hit at time $t\sim z$.

With the two observations we made above, let us *call* the wave function for the internal state of the outgoing deuteron as seen by an observer in the laboratory system to be $\psi_f^*(\vec{\Delta},\vec{x}_\perp,z)$; then, instead of (2.6), the deuteron impact factor should read

$$I^d(\vec{r}_1,\vec{x}_\perp)=\tfrac{1}{2}\int_{-\infty}^{\infty} dx_+\,\psi_f^*(\vec{\Delta},\vec{x}_\perp,z)\phi_i(\vec{x}_\perp,z) =\int_{-\infty}^{\infty} dz\,\psi_f^*(\vec{\Delta},\vec{x}_\perp,z)\phi_i(\vec{x}_\perp,z), \tag{2.8}$$

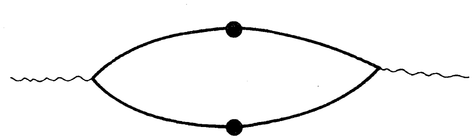

FIG. 1. Lowest-order impact diagram for the scattering of a photon by an external field. The lowest-order impact diagram for Compton scattering is obtained from this figure by replacing the external field with an electron.

where the wave functions are understood to be those on the light cone $t-z=0$. From (2.8), we see that the deuteron impact factor is dependent on Δ. Thus the dependence of the deuteron impact factor on Δ has a very simple physical reason: It comes from the recoil of the target. This explains the lack of such dependence in the droplet model in its most straightforward interpretation. Indeed, this is the underlying reason why, as pointed out three years ago, the impact-factor representation and the droplet model agree for Compton scattering only in the nonrelativistic limit.[8] The explanation of Lee[9] for the disagreement between the results of photon scattering in quantum electrodynamics and the corresponding ones in the droplet model is therefore irrelevant.

In the next section, we shall determine $\psi_f(\vec{\Delta}, \vec{x}_\perp, z)$.

III. THE RECOIL SYSTEM

In the preceding section, we have pointed out that $\phi_f(\vec{x}_\perp, z)$ is the wave function of the outgoing deuteron in the rest system of the *outgoing* deuteron, while the expression (2.8) for the deuteron impact factor involves the wave function of the outgoing deuteron in the rest system of the *incoming* deuteron (the laboratory system). We shall refer to the rest system of the outgoing deuteron as the *recoil system*. In this section, we shall study the Lorentz transformation which connects the recoil system with the laboratory system.

In the laboratory system, the outgoing deuteron has the four-momentum

$$P_f = (P_0, P_3, \vec{P}_\perp) = (M_d + \tfrac{1}{2}\Delta^2 M_d^{-1}, \tfrac{1}{2}\Delta^2 M_d^{-1}, \vec{\Delta}), \tag{3.1}$$

while in the recoil system, the four-momentum of the outgoing deuteron is

$$P_f' = (M_d, 0, \vec{0}).$$

Notice that

$$P_{f-} = P_{f-}', \tag{3.2}$$

where the prime indicates the recoil system. There are, of course, more than one Lorentz transformation which can bring P_f into P_f'. To see this, we observe that P_f' as given by (3.2) is invariant under all rotations and space reflections. As a result, if L is a Lorentz transformation which brings P_f into P_f', (i.e., $P_f' = LP_f$), then RL is another such Lorentz transformation where R is any rotation with or without space reflection. Indeed, the most general transformation is given by this form RL, which forms a family of transformations with three parameters.

In order to remove this undesirable ambiguity due to R, we find it most natural to use the following criteria: (i) The transformation does not change the y axis, i.e., the axis perpendicular to the scattering plane; and (ii) in the recoil system, the spatial momentum of the other incident particle remains in the direction of the positive z axis. These conditions (i) and (ii) specify the Lorentz transformation L uniquely. Call the momentum of the other incident particle to be $k(k')$ in the laboratory (recoil) system. Then in the high-energy limit, we have

$$k \sim (\omega, \omega, 0, 0).$$

Since $kP_f = k'P_f'$, we obtain, by making use of (3.2),

$$k_+' = k_+,$$

which implies that

$$k' \sim k \sim (\omega, \omega, 0, 0). \tag{3.3}$$

Thus the vectors (1, 1, 0, 0) and (0, 0, 0, 1) are both invariant under $L(\Delta)$ for all Δ. Since the subgroup that keeps two four-vectors invariant is one-dimensional, $L(\Delta)$ must be this subgroup. We shall verify this statement explicitly in the present section.

Let A_μ be an arbitrary four-vector. Then, from

$$Ak = A'k',$$

we easily obtain

$$A_- = A_-', \tag{3.4}$$

where

$$A_\pm = A_0 \pm A_3. \tag{3.5}$$

Thus the minus component of a four-vector is the same in the laboratory system as in the recoil system. In fact, the minus component is invariant under any finite Lorentz transformation which leaves the momentum of the other incident particle in the positive z direction. Thus the light cone $t-z=0$ has an invariant meaning in all of these Lorentz frames. Physically, this is because in all of these Lorentz frames, the other incoming particle is always traveling with a velocity near that of light in the positive z direction, and the deuteron is always hit at time $t \sim z$.

Next, we require that the volume of the parallelopiped formed by the vectors A, P_f, k, and $\vec{e}_2$ be invariant, i.e.,

$$\begin{vmatrix} A'_+ & A'_- & A'_1 & A'_2 \\ M_d+\dfrac{\Delta^2}{M_d} & M_d & \Delta & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} A_+ & A_- & A_1 & A_2 \\ M_d & M_d & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}. \tag{3.6}$$

From (3.6), we immediately get

$$A'_1 = A_1 - \frac{\Delta}{M_d}A_- . \tag{3.7}$$

Since the y axis is invariant, we have

$$A'_2 = A_2 . \tag{3.8}$$

Equations (3.7) and (3.8) can be summarized as

$$\vec{A}'_\perp = \vec{A}_\perp - \frac{A_-}{M_d}\vec{\Delta} . \tag{3.9}$$

Finally, from $A^2 = A'^2$, we get

$$A'_+ = A_+ - 2M_d^{-1}\vec{\Delta}\cdot\vec{A}_\perp + \Delta^2 M_d^{-2}A_- . \tag{3.10}$$

Equations (3.4), (3.9), and (3.10) completely define the Lorentz transformation connecting the laboratory system and the recoil system.

Alternatively, (3.4), (3.9), and (3.10) can be written as

$$A'_0 = \left(1+\frac{1}{2}\frac{\Delta^2}{M_d^2}\right)A_0 - \frac{1}{2}\frac{\Delta^2}{M_d^2}A_3 - \frac{\Delta}{M_d}A_1 , \tag{3.11}$$

$$A'_3 = \frac{1}{2}\frac{\Delta^2}{M_d^2}A_0 + \left(1-\frac{1}{2}\frac{\Delta^2}{M_d^2}\right)A_3 - \frac{\Delta}{M_d}A_1 , \tag{3.12}$$

$$A'_1 = -\frac{\Delta}{M_d}A_0 + \frac{\Delta}{M_d}A_3 + A_1 , \tag{3.13}$$

$$A'_2 = A_2 , \tag{3.14}$$

where $\vec{\Delta}$ is taken to be in the x direction. Thus

$$\begin{bmatrix} A'_0 \\ A'_3 \\ A'_1 \\ A'_2 \end{bmatrix} = L(\Delta)\begin{bmatrix} A_0 \\ A_3 \\ A_1 \\ A_2 \end{bmatrix} , \tag{3.15}$$

where

$$L(\Delta) = \begin{bmatrix} 1+\dfrac{1}{2}\dfrac{\Delta^2}{M_d^2} & -\dfrac{1}{2}\dfrac{\Delta^2}{M_d^2} & -\dfrac{\Delta}{M_d} & 0 \\ \dfrac{1}{2}\dfrac{\Delta^2}{M_d^2} & 1-\dfrac{1}{2}\dfrac{\Delta^2}{M_d^2} & -\dfrac{\Delta}{M_d} & 0 \\ -\dfrac{\Delta}{M_d} & \dfrac{\Delta}{M_d} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{3.16}$$

It is remarkable that, unlike the more familiar forms of Lorentz transformation, no square root appears in (3.16) for $L(\Delta)$.

It is easily checked that the elements $L(\Delta)$ form a group. We have

$$L(\Delta_1)L(\Delta_2) = L(\Delta_1+\Delta_2) . \tag{3.17}$$

Thus the inverse of $L(\Delta)$ is $L(-\Delta)$ and the unit element is $L(0)$. This group is actually the little group which leaves the null vector $[1, 1, 0]$ invariant, in the case of $2+1$ space-time dimensions with the y axis omitted. The infinitesimal generator for this group is given by

$$\Gamma = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{3.18}$$

To understand the meaning of Γ, let us rewrite (3.18) as

$$\Gamma = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{3.19}$$

The first term in the right-hand side of (3.19) is the infinitesimal generator for a rotation in the x_0-x_1 plane, and the second term in the right-hand side of (3.19) is the infinitesimal generator for a rotation in the x_1-x_3 plane.

In terms of Γ, we have

$$L(\Delta) = \exp\left(\frac{\Delta}{M_d}\Gamma\right) . \tag{3.20}$$

Since

$$\Gamma^2 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.21}$$

and

$$\Gamma^3 = 0 , \tag{3.22}$$

we have

$$L(\Delta) = 1 + \frac{\Delta}{M_d}\Gamma + \frac{1}{2}\left(\frac{\Delta}{M_d}\right)^2\Gamma^2 . \tag{3.23}$$

If we substitute (3.18) and (3.21) into (3.23), we get precisely (3.16).

IV. CASE OF SCALAR NUCLEONS

In order to simplify the discussion, let us ignore the spin of the nucleon and assume that the deuteron is a bound state of a scalar proton and a scalar neutron. The case of spin-$\frac{1}{2}$ nucleons will be treated in Paper II of this series.

A. Wave Function in Position Space

Let the wave function of the internal state of the outgoing deuteron in the recoil system be called $\Phi_f(x')$, where x' is the (4-dimensional) relative coordinate of the proton-neutron system. Let this wave function on the light cone $t'=z'$ be called

$$\phi_f(\vec{x}'_\perp, z') = \Phi_f(x')\big|_{t'=z'} . \tag{4.1}$$

By (3.13) and (3.12) on the light cone $t'-z'=0$, the relative coordinates x_μ in the laboratory system related to x'_μ by

$$\vec{x}'_\perp = \vec{x}_\perp , \tag{4.2}$$

$$z' = z - \frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}. \tag{4.3}$$

Thus the wave function of the outgoing deuteron in the laboratory system is

$$\phi_f(\vec{x}'_\perp, z') = \phi_f\left(\vec{x}_\perp, z - \frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}\right). \tag{4.4}$$

B. Wave Function in Momentum Space

The recoil effect can also be described in the language of the momentum variables. Let us first recall that, for the wave function at equal time $t=0$, the spatial momentum of a virtual state is conserved, while the energy is not. Similarly, for the wave function at the light cone $t-z=0$, conservation of momentum holds for the minus component and the transverse components, but not for the plus component. From (3.4) and (3.9), this conservation law holds both in the laboratory system and in the recoil system.

Let us first specify the momenta variables in the recoil system. The minus component of the outgoing deuteron is equal to M_d. If we denote the momenta of the proton and the neutron in the outgoing deuteron as k'_p and k'_n, respectively, then, as a result of momentum conservation for the minus and the transverse components, we have

$$k'_{p-} + k'_{n-} = M_d , \tag{4.5}$$

$$\vec{k}'_{p_\perp} + \vec{k}'_{n_\perp} = 0 . \tag{4.6}$$

Let us put

$$k'_{p-} = \beta M_d, \qquad k'_{n-} = (1-\beta)M_d ; \tag{4.7}$$

then

$$0 \leqslant \beta \leqslant 1 , \tag{4.8}$$

as k'_{p-} and k'_{n-} are necessarily positive. We shall call the relative momentum between these two particles to be p, i.e.,

$$p' = \tfrac{1}{2}(k'_p - k'_n) ; \tag{4.9}$$

then

$$p'_- = \alpha M_d , \tag{4.10}$$

where

$$\alpha = \beta - \tfrac{1}{2} \tag{4.11}$$

and, because of (4.8), is restricted to

$$-\tfrac{1}{2} \leqslant \alpha \leqslant \tfrac{1}{2} . \tag{4.12}$$

From (4.6)–(4.11), we get

$$k'_{p-} = (\tfrac{1}{2}+\alpha)M_d , \qquad \vec{k}'_{p_\perp} = \vec{p}'_\perp , \tag{4.13}$$

$$k'_{n-} = (\tfrac{1}{2}-\alpha)M_d , \qquad \vec{k}'_{n_\perp} = -\vec{p}'_\perp . \tag{4.14}$$

From (4.13) and (4.14), we easily obtain k_p and k_n as

$$k_{p-} = (\tfrac{1}{2}+\alpha)M_d , \qquad \vec{k}_{p_\perp} = \vec{p}'_\perp + (\tfrac{1}{2}+\alpha)\vec{\Delta} , \tag{4.15}$$

$$k_{n-} = (\tfrac{1}{2}-\alpha)M_d , \qquad \vec{k}_{n_\perp} = -\vec{p}'_\perp + (\tfrac{1}{2}-\alpha)\vec{\Delta} . \tag{4.16}$$

Furthermore,

$$p_- = \alpha M_d , \qquad \vec{p}_\perp = \vec{p}'_\perp + \alpha\vec{\Delta} . \tag{4.17}$$

In the above, the momenta without the prime denote the quantities in the laboratory system.

The wave function of the internal state of the outgoing deuteron on the light cone is a function of the minus and the transverse components of the relative momentum. Denoting this wave function in the recoil system as $f(\vec{p}'_\perp, \alpha)$, we immediately obtain from (4.17) that this wave function in the laboratory system is

$$f(\vec{p}_\perp - \alpha\vec{\Delta}, \alpha) . \tag{4.18}$$

It is interesting to observe from (4.18) that the wave function in the laboratory system can be obtained from that in the recoil system by a translation of the transverse component of the relative momentum. It is also significant that (4.18) is much simpler than the corresponding formula given by Casper and Gross[2] and is valid for a larger region. (It is valid for arbitrary Δ rather than for small Δ only).

The relationship between f and ϕ will be given in (5.6) below.

V. SUMMARY

We now summarize the results as follows.

A. Position Space

The π-d scattering amplitude, with the recoil of the outgoing deuteron taken into account, is equal to

$$\frac{ik}{2\pi}\int d^2b\,\exp(-i\vec{\Delta}\cdot\vec{b})\int d^3x\,\phi_f^*\left(\vec{x}_\perp, z-\frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}\right)\phi_i(\vec{x}_\perp,z)\,\{1-\exp[i\chi_n(\vec{b}-\tfrac{1}{2}\vec{x}_\perp)+i\chi_p(\vec{b}+\tfrac{1}{2}\vec{x}_\perp)]\}\,, \tag{5.1}$$

where ϕ_i and ϕ_f are the initial and the final internal state wave functions of the deuteron, respectively, and χ_n and χ_p are the phase shifts for π-n and π-p scattering, respectively. Equation (5.1) can be considered to be the relativistic generalization of the Glauber form. This equation is surprisingly simple, with the recoil effects represented by a *translation in the z coordinate*. Needless to say, with a trivial change of notations, (5.1) holds for the scattering amplitude of a particle from a system of two particles.

We may define the deuteron impact factor to be

$$I^d(\vec{r}_1,\vec{x}_\perp)=\int_{-\infty}^{\infty}dz\,\phi_f^*\left(\vec{x}_\perp,z-\frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}\right)\phi_i(\vec{x}_\perp,z)\,; \tag{5.2}$$

then the π-d scattering amplitude is in the form of the impact factor representation

$$\frac{ik}{2\pi}\int d^2b\,d^2x_\perp\exp(-i\vec{\Delta}\cdot\vec{b})I^d(\vec{r}_1,\vec{x}_\perp)\times\{1-\exp[i\chi_n(\vec{b}-\tfrac{1}{2}\vec{x}_\perp)+i\chi_p(\vec{b}+\tfrac{1}{2}\vec{x}_\perp)]\}\,. \tag{5.3}$$

B. Momentum Space

The impact factor and the scattering amplitude can also be expressed by integrals over the momentum space. Let us define the deuteron impact factor in the momentum space to be

$$g^d(\vec{r}_1,\vec{q}_\perp)=\int d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}I^d(\vec{r}_1,\vec{x}_\perp)\,; \tag{5.4}$$

then

$$g^d(\vec{r}_1,\vec{q}_\perp)=(2\pi)^{-3}\int d\vec{p}_\perp\int_{-1/2}^{1/2}d\alpha\times f^*(\vec{p}_\perp+\vec{q}_\perp-\alpha\vec{\Delta},\alpha)f(\vec{p}_\perp,\alpha)\,, \tag{5.5}$$

where

$$\phi(\vec{x}_\perp,z)=(2\pi)^{-3}M_d^{1/2}\int d\vec{p}_\perp\int_{-1/2}^{1/2}d\alpha\times e^{-i\alpha M_d z+i\vec{p}_\perp\cdot\vec{x}_\perp}f(\vec{p}_\perp,\alpha)\,. \tag{5.6}$$

We see that, in the momentum space, the recoil effect is represented by a *translation in the transverse component* of the momentum.

The π-d scattering amplitude can be expressed in the impact factor representation:

$$\frac{ik}{2\pi}\int\frac{d^2q_\perp}{(2\pi)^2}g^d(\vec{r}_1,\vec{q}_\perp)[(2\pi)^4\delta(\vec{r}_1+\vec{q}_\perp)\delta(\vec{r}_1-\vec{q}_\perp)-S_n(\vec{r}_1+\vec{q}_\perp)S_p(\vec{r}_1-\vec{q}_\perp)]\,, \tag{5.7}$$

where

$$S_n(\vec{Q}_\perp)=\int d^2x_\perp e^{-i\vec{Q}_\perp\cdot\vec{x}_\perp}\exp[i\chi_n(\vec{x}_\perp)]\,, \tag{5.8}$$

is proportional to the π-n scattering amplitude of momentum transfer $\vec{Q}_\perp$. A similar expression holds for S_p.

C. Discussion

We make the following remarks:

(i) If we replace the π by an electron, i.e., we consider e-d scattering, then it suffices to retain the lowest-order terms in the electromagnetic interaction. This means that we can replace $[1-\exp(i\chi_n+i\chi_p)]$ in (5.1) by $-i(\chi_n+\chi_p)$. The scattering amplitude for e-d scattering is then proportional to the form factor

$$\int d^3x\,\exp(-i\tfrac{1}{2}\vec{\Delta}\cdot\vec{x}_\perp)\phi_f^*\left(\vec{x}_\perp,z-\frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}\right)\phi_i(\vec{x}_\perp,z)\,. \tag{5.9}$$

We observe from (5.9) that the Fourier transform of the form factor is not the charged density, as commonly supposed. Rather, the quantity that is relevant to the form factor is the overlapping of two displaced wave functions on the light cone. This displacement becomes important as the momentum transfer is comparable to the mass of the particle.

To estimate the magnitude of the recoil effect, let us take

$$\phi_i(\vec{x})=\phi_f(\vec{x})=(a/\pi)^{3/4}e^{-a\vec{x}^2/2}\,; \tag{5.10}$$

then the form factor given by (5.9) is equal to

$$(1+\tfrac{1}{4}\Delta^2M_d^{-2})^{-1/2}\exp\left(-\frac{\Delta^2}{16a(1+\frac{1}{4}\Delta^2M_d^{-2})}\right). \tag{5.11}$$

It is interesting to observe that although the form factor decreases exponentially when $\Delta^2\ll 4M_d^2$, this exponential decrease levels off as Δ^2 becomes comparable to $4M_d^2$ as a result of the recoil effect. Detailed numerical analysis of the recoil effect will be given in a forthcoming paper.

(ii) In the present paper, the spin of the nucleons is ignored. When Δ is comparable to the nucleon mass, the recoil effect due to the spin is significant. This will be discussed in Paper II of this series.

(iii) In the present paper, we restrict ourselves to elastic processes only. This discussion is easily extended to an inelastic process $A+B\to C+D$, as will be given in Paper III of this series.

(iv) It is straightforward to generalize the impact factor representation to the process of deuteron-deuteron scattering. Let the impact distance between the two deuterons be $\vec{b}$ and the relative coordinates be $\vec{x}$ and $\vec{x}'$. Let p and n be the proton and the neutron, respectively, of the first deuteron, and p' and n' be those of the second deuteron. The impact distances between the particles are

$$pp',\quad \vec{b}+\tfrac{1}{2}\vec{x}_\perp-\tfrac{1}{2}\vec{x}'_\perp,$$

$$pn',\quad \vec{b}+\tfrac{1}{2}\vec{x}_\perp+\tfrac{1}{2}\vec{x}'_\perp,$$

$$np',\quad \vec{b}-\tfrac{1}{2}\vec{x}_\perp-\tfrac{1}{2}\vec{x}'_\perp,$$

$$nn',\quad \vec{b}-\tfrac{1}{2}\vec{x}_\perp+\tfrac{1}{2}\vec{x}'_\perp.$$

Thus the d-d scattering amplitude is equal to

$$\frac{ik}{2\pi}\int d^2b\,d^2x_\perp d^2x'_\perp \exp(-i\vec{\Delta}\cdot\vec{b})I^d(\vec{r}_1,\vec{x}_\perp)I^d(\vec{r}_1,\vec{x}'_\perp)$$
$$\times\{1-\exp[i\chi_{pp}(\vec{b}+\tfrac{1}{2}\vec{x}_\perp-\tfrac{1}{2}\vec{x}'_\perp)+i\chi_{pn}(\vec{b}+\tfrac{1}{2}\vec{x}_\perp+\tfrac{1}{2}\vec{x}'_\perp)+i\chi_{np}(\vec{b}-\tfrac{1}{2}\vec{x}_\perp-\tfrac{1}{2}\vec{x}'_\perp)+i\chi_{nn}(\vec{b}-\tfrac{1}{2}\vec{x}_\perp+\tfrac{1}{2}\vec{x}'_\perp)]\},\tag{5.12}$$

where χ_{pp} is the phase shift for pp scattering and the other notations are obvious.

(v) In the discussion on π-d scattering here, the pion is assumed to have a large momentum in the *positive z* direction. In other cases, it may be convenient to choose the momentum of the pion to be in the *negative z* direction. The modification is trivial and we give it explicitly below.

The Lorentz transformation which connects the recoil system and the laboratory system is

$$A'_+=A_+,\tag{5.13}$$

$$\vec{A}'_\perp=\vec{A}_\perp-\frac{A_+}{M_d}\vec{\Delta},\tag{5.14}$$

$$A'_-=A_--2M_d^{-1}\vec{\Delta}\cdot\vec{A}_\perp+\Delta^2M_d^{-2}A_+,\tag{5.15}$$

or, alternatively, we have

$$A'_0=\left(1+\frac{1}{2}\frac{\Delta^2}{M_d^2}\right)A_0+\frac{1}{2}\frac{\Delta^2}{M_d^2}A_3-\frac{\Delta}{M_d}A_1,\tag{5.16}$$

$$A'_3=-\frac{1}{2}\frac{\Delta^2}{M_d^2}A_0+\left(1-\frac{1}{2}\frac{\Delta^2}{M_d^2}\right)A_3+\frac{\Delta}{M_d}A_1,\tag{5.17}$$

$$A'_1=-\frac{\Delta}{M_d}A_0-\frac{\Delta}{M_d}A_3+A_1,\tag{5.18}$$

$$A'_2=A_2.\tag{5.19}$$

Equations (5.16)–(5.19) are the same as (3.11)–(3.14) with $A_3\to -A_3$, $A'_3\to -A'_3$ and can be summarized into

$$\begin{bmatrix}A'_0\\A'_3\\A'_1\\A'_2\end{bmatrix}=L(\Delta)\begin{bmatrix}A_0\\A_3\\A_1\\A_2\end{bmatrix},\tag{5.20}$$

where

$$L(\Delta)=\begin{bmatrix}1+\frac{1}{2}\frac{\Delta^2}{M_d^2} & \frac{1}{2}\frac{\Delta^2}{M_d^2} & -\frac{\Delta}{M_d} & 0\\ -\frac{1}{2}\frac{\Delta^2}{M_d^2} & 1-\frac{1}{2}\frac{\Delta^2}{M_d^2} & \frac{\Delta}{M_d} & 0\\ -\frac{\Delta}{M_d} & -\frac{\Delta}{M_d} & 1 & 0\\ 0&0&0&1\end{bmatrix}.\tag{5.21}$$

The impact factor is equal to the overlapping of the wave functions on the light cone $x_+=0$, and is explicitly given by, instead of (5.2),

$$\int_{-\infty}^{\infty}dz\,\phi_f^*\left(\vec{x}_\perp,z+\frac{\vec{\Delta}\cdot\vec{x}_\perp}{M_d}\right)\phi(\vec{x}_\perp,z).\tag{5.22}$$

Needless to say, conservation of momentum for a virtual state holds for the plus component, but not for the minus component. If we interpret α in (5.5) as the plus component of the relative momentum divided by M_d, then (5.6) becomes

$$\phi(\vec{x}_\perp,z)=(2\pi)^{-3}M_d^{1/2}\int d\vec{p}_\perp\int_{-1/2}^{1/2}d\alpha$$
$$\times c^{i\alpha M_d z+i\vec{p}_\perp\cdot\vec{x}}f(\vec{p}_\perp,\alpha).\tag{5.23}$$

APPENDIX

In this appendix, we give some diagrammatic examples of the wave function on the light cone.

A. Scalar Particle with Pion Loop

As a model for deuteron scattering, consider the elastic scattering of a neutral and spinless particle in an external field. The scattering proceeds through the following steps: (i) The incident particle turns into a π^+-π^- pair; (ii) this pair is scattered by the external field; (iii) the pair recombines to form the outgoing scalar particle. The impact diagram is the same as the one illustrated in Fig. 1. Applying the impact diagram rules,[10] we get

$$\mathcal{I}(\vec{r}_1, \vec{q}_\perp) = g^2(2\pi)^{-3}\int d\vec{p}_\perp \int_0^1 d\beta \frac{\beta(1-\beta)}{[\vec{p}_\perp{}^2 + m^2 - \lambda^2\beta(1-\beta)]\{[\vec{p}_\perp + \vec{q}_\perp - (2\beta-1)\vec{r}_1]^2 + m^2 - \lambda^2\beta(1-\beta)\}}. \quad \text{(A1)}$$

In (A1), m is the mass of the pion and λ is the mass of the scalar particle. We see that the impact factor is indeed in the form (5.5). By comparing (A1) with (5.5), we get

$$f(\vec{p}_\perp, \alpha) = \frac{g(\frac{1}{4} - \alpha^2)^{1/2}}{\vec{p}_\perp{}^2 + m^2 - \lambda^2(\frac{1}{4} - \alpha^2)}. \quad \text{(A2)}$$

Also, from (5.6), we have

$$\begin{aligned}\phi(\vec{x}_\perp, z) &= (2\pi)^{-3}\lambda^{1/2}\int d\vec{p}_\perp \int_{-1/2}^{1/2} d\alpha\, e^{-i\alpha\lambda z + i\vec{p}_\perp\cdot\vec{x}_\perp} \frac{g(\frac{1}{4} - \alpha^2)^{1/2}}{\vec{p}_\perp{}^2 + m^2 - \lambda^2(\frac{1}{4} - \alpha^2)} \\ &= (2\pi)^{-2}\lambda^{1/2} g \int_{-1/2}^{1/2} d\alpha\, e^{-i\alpha\lambda z} K_0([m^2 - \lambda^2(\tfrac{1}{4} - \alpha^2)]^{1/2}|\vec{x}_\perp|)(\tfrac{1}{4} - \alpha^2)^{1/2}. \quad \text{(A3)}\end{aligned}$$

If the pion pair forms a loosely bound "deuteron," i.e., if

$$\lambda = 2m - \epsilon, \quad \text{(A4)}$$

where

$$\epsilon \ll m, \quad \text{(A5)}$$

(A3) can be further simplified. We have

$$\phi(\vec{x}) \sim (2\pi)^{-3}\lambda^{1/2}\int d\vec{p}_\perp \int_{-1/2}^{1/2} d\alpha\, e^{-i\alpha\lambda z + i\vec{p}_\perp\cdot\vec{x}_\perp} \frac{g(\frac{1}{4} - \alpha^2)^{1/2}}{\vec{p}_\perp{}^2 + \lambda^2\alpha^2 + m\epsilon}. \quad \text{(A6)}$$

In the limit $\epsilon \to 0$, the dominant region of integration in (A6) is

$$\lambda\alpha = O(\epsilon^{1/2}), \quad p_\perp = O(\epsilon^{1/2}). \quad \text{(A7)}$$

Thus we may make the approximation $\frac{1}{4} - \alpha^2 \sim \frac{1}{4}$ and then replace the region of integration in α to $(-\infty, \infty)$. Thus (A6) becomes

$$\phi(\vec{x}) \sim \tfrac{1}{2}(2\pi)^{-3}\lambda^{-1/2} g \int d^3p\, e^{i\vec{p}\cdot\vec{x}}(p^2 + m\epsilon)^{-1} = \tfrac{1}{2}(2m)^{-1/2} g \frac{\exp(-\sqrt{m\epsilon}|\vec{x}|)}{4\pi|\vec{x}|}, \quad \text{(A8)}$$

which is just the Yukawa form.

We may also consider higher-order impact diagrams such as the ones in Fig. 2. It is easy to verify that the impact factor from the sum of such diagrams is always in the form of (5.5). Thus, by a proper choice of the wave function $f(\vec{p}_\perp, \alpha)$, we may obtain a nonperturbative approximation to the scattering amplitude.

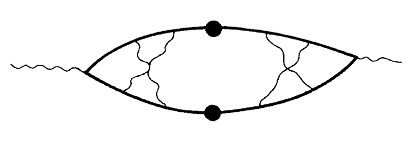

FIG. 2. Higher-order impact diagram for the scattering of a two-body system by an external field.

B. Vector Particle with Pion Loop

Next we shall consider the case of a neutral meson scattered elastically by an external field. The impact diagram is still the one illustrated in Fig. 1, where the incident and the outgoing particles are understood to be of spin 1. Applying the impact diagram rules,[10] we get

$$\mathcal{I}(\vec{r}_1,\vec{q}_\perp)=g^2(2\pi)^{-3}\int d\vec{p}_\perp\int_0^1 d\beta\frac{4\beta(1-\beta)p_i(p+q_\perp)_j}{[\vec{p}_\perp{}^2+m^2-\lambda^2\beta(1-\beta)]\{[\vec{p}_\perp+\vec{q}_\perp-(2\beta-1)\vec{r}_1]^2+m^2-\lambda^2\beta(1-\beta)\}}, \tag{A9}$$

where i and j denote the polarization of the incoming and the outgoing meson, respectively, and where p is half the difference of the momenta of the pions created by the incident vector meson:

$$p=(p_0,p_3,\vec{p}_\perp)=\left(\tfrac{1}{2}\alpha\lambda-\frac{2\alpha(\vec{p}_\perp{}^2+m^2)}{(1-4\alpha^2)\lambda},\tfrac{1}{2}\alpha\lambda+\frac{2\alpha(\vec{p}_\perp{}^2+m^2)}{(1-4\alpha^2)\lambda},\vec{p}_\perp\right). \tag{A10}$$

Thus

$$f(\vec{p}_\perp,\alpha)=\frac{g(\frac{1}{4}-\alpha^2)^{1/2}p_i}{\vec{p}_\perp{}^2+m^2-\lambda^2(\frac{1}{4}-\alpha^2)}, \tag{A11}$$

with

$$p_i=-\left[\tfrac{1}{2}\alpha\lambda+\frac{2\alpha(\vec{p}_\perp{}^2+m^2)}{(1-4\alpha^2)\lambda}\right]\epsilon_3-\vec{p}_\perp\cdot\vec{\epsilon}_\perp, \tag{A12}$$

where $\vec{\epsilon}$ is the polarization vector of the vector meson.

In the position space, we have

$$\phi(\vec{x})=(2\pi)^{-3}\lambda^{1/2}g\int d\vec{p}_\perp\int_{-1/2}^{1/2}d\alpha\, e^{i\alpha\lambda z+i\vec{p}_\perp\cdot\vec{x}_\perp}\frac{(\frac{1}{4}-\alpha^2)^{1/2}p_i}{\vec{p}_\perp{}^2+m^2-\lambda^2(\frac{1}{4}-\alpha^2)}. \tag{A13}$$

In the limit of small binding energy $\epsilon\to0$, we have

$$\phi(\vec{x})\sim-\tfrac{1}{2}(2\pi)^{-3}\lambda^{-1/2}g\int d^3p\, e^{i\vec{p}\cdot\vec{x}}\frac{(\vec{p}\cdot\vec{\epsilon})}{\vec{p}^2+m\epsilon}=-\tfrac{1}{2}(2m)^{-1/2}g(-i\vec{\epsilon}\cdot\vec{\nabla})\frac{\exp(-\sqrt{m\epsilon}\,|\vec{x}|)}{4\pi|\vec{x}|}. \tag{A14}$$

The wave function $\phi(\vec{x})$ is as singular as $|\vec{x}|^2$ at the origin. Thus the impact factor in the momentum space, as given by (A9), is only formal as the integral is divergent. However, for the scattering of a pair of pions in an electrostatic field, the origin does not contribute to the scattering amplitude. This can be seen by observing that the bracket in (2.1) vanishes at $\vec{x}_\perp=0$. Physically, when π^+ and π^- coincide, there is no electric charge anywhere and interaction with the electrostatic field does not occur. We may therefore subtract out the contribution from the origin and replace (A9) by

$$\mathcal{I}(\vec{r}_1,\vec{q}_\perp)=g^2(2\pi)^{-3}\int dp_\perp\int_0^1 d\beta \times\left\{\frac{4\beta(1-\beta)p_i(p+q_\perp)_j}{[\vec{p}_\perp{}^2+m^2-\lambda^2\beta(1-\beta)]\{[\vec{p}_\perp+\vec{q}_\perp-(2\beta-1)\vec{r}_1]^2+m^2-\lambda^2\beta(1-\beta)\}}-\text{preceding term with }\vec{q}_\perp=\vec{r}_1\right\}. \tag{A15}$$

The second term in the bracket of (A18), being independent of $\vec{q}_\perp$, corresponds to a delta function $\delta(\vec{x}_\perp)$ in the position space. Hence the inclusion of it in (A15) does not change the scattering amplitude. The expression on the right-hand side of (A15) is now a convergent integral. This definition of the impact factor in the momentum space was indeed adopted in quantum electrodynamics.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3227.

†Work supported in part by the National Science Foundation under Grant No. GP-29 463.

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CP Nonconservation and $K \to l\bar{l}\gamma$ Decays*

S. K. Singh
Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 14 December 1971; revised manuscript received 21 June 1972)

A simple model which explains the observed suppression of the $K_L \to \mu^+\mu^-$ rate has been used to calculate the *CP*-violating contribution to $K \to l\bar{l}\gamma$ decays. For $K_L \to \mu^+\mu^-\gamma$ this rate is comparable to the Dalitz-pair rate while for $K_L \to e^+e^-\gamma$ and $K_S \to l\bar{l}\gamma$ the contribution is small.

Recently there have been many attempts to explain[1] theoretically the observed lower limit on the $K_L \to \mu^+\mu^-$ rate.[2] If *CP* nonconservation plays a dominant role in these decays as suggested by Lee and Christ[1] then its implication in other rare decay modes is worth examining. In this note we discuss the question of *CP* nonconservation in $K \to l\bar{l}\gamma$ decays. The *C* and *CP* properties of the various final states possible in these decays are shown in Table I. The $K \to l\bar{l}\gamma$ decays can take place through electromagnetic interactions which are *CP*-conserving. Since the lepton pair state has to be produced through a photon conversion it must be a *C*-odd *CP*-even state. Thus $K_L \to l\bar{l}(^3S_1)\gamma(M1)$ and $K_S \to l\bar{l}(^3S_1)\gamma(E1)$ are the *CP*-conserving electromagnetic decays. The *CP*-admixed states in K_L and K_S will be neglected because their contribution to the $K_{L(S)} \to l\bar{l}\gamma$ rate is small.

We consider the possibility of the existence of *CP*-odd nonelectromagnetic interactions. Such an interaction has been described by Wolfenstein[3] which explains the suppression of $K_L \to \mu^+\mu^-$ rate without upsetting any other experimental result. In this model the interaction Hamiltonian is given by

$$H' = iG' \sin\theta_C A_\lambda^7 \bar{\psi}_\mu \gamma_\lambda \gamma_5 \psi_\mu , \tag{1}$$

where A_λ^7 is *C*-odd $|\Delta S| = 1$ axial-vector current and transforms as the seventh component of SU(3). G' is the strength of the interaction and is given by

$$3.8\times10^{-2} \geq G'/G \geq 1.1\times10^{-2} . \tag{2}$$

This interaction requires the lepton pair to be in *C*-even *CP*-even state and thus only $K_L \to l\bar{l}(^3P)\gamma(E1)$ is allowed. $K_S \to l\bar{l}\gamma$ decays are not allowed because the matrix element $\langle\gamma|A_\lambda^7|K_S\rangle$ is zero due to *C* invariance. This however can take place through bremsstrahlung in which K_S decays into $l\bar{l}(^1S_0)$ state and then one of the leptons emits a photon, giving $l\bar{l}(^1P_1)\gamma(E1)$ final state. *CP*-conserving decays are the electromagnetic $K_L \to l\bar{l}(^3S_1)\gamma(M1)$ and $K_S \to l\bar{l}(^3S_1)\gamma(E1)$. As a result the decay rate is simply the sum of *CP*-even and *CP*-odd rates and there are no *CP*-violating K_L-K_S interference effects in the photon spectrum. In this model a definite *CP* violation could be manifested through a correlation between muon spins and photon polarization.

It is natural to add to H' an interaction Hamiltonian[4] H'' where the hadronic current A_λ^7 is coupled to a vector leptonic current in the form

$$H'' = iG'' \sin\theta_C A_\lambda^7 \bar{\psi}_\mu \gamma_\lambda \psi_\mu . \tag{3}$$

This interaction allows $K_L \to l\bar{l}(^3S_1)\gamma(E1)$ decay, and K_L-K_S interference effects might be observed in the photon spectrum because K_S is electromagnetically allowed to decay into the same final states. This interaction, however, does not contribute to $K \to l\bar{l}$ decays and the conclusions regarding $K_L \to \mu^+\mu^-$ are unaltered. In the following, we