[7]G. Neuhofer *et al.*, Phys. Letters 37B, 438 (1971); G. Barbiellini *et al.*, *ibid.* 39B, 294 (1972); M. Breidenbach *et al.*, *ibid.* 39B, 654 (1972).

[8]This assumption means that $N_L(\vec{p}, s) \to N_L(x, \vec{p}_\perp)$ as $s \to \infty$. In what follows we assume that the scaling limit of N is obtained by inserting the scaling limit of N_L into Eq. (4). See, however, footnote 14.

[9]Equations (6) are valid if the Lorentz transformation $\Lambda_{p'}$ is a pure boost. If, however, it includes a small rotation (for example, if the longitudinal direction for the fireball decay is defined by the beam direction as seen in the fireball system rather than as seen in the c.m. system), then the function $\vec{k}_\perp(x, y, \vec{p}'_\perp)$ is slightly changed; this change is completely irrelevant when one integrates over transverse momentum.

[10]T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970); C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971); E. Predazzi and G. Veneziano, Lett. Nuovo Cimento 2, 749 (1971); L. S. Brown, Phys. Rev. D 5, 748 (1972).

[11]Strictly speaking, Eq. (18) requires the integral to exist in order that $f(0) \neq 0$. This will certainly happen if $g(x)$ is not so singular at $x = 1$ that $\hat{h}(\lambda)$ is not analytic at $\lambda = 0$. This is true in the cases we consider; see, however, footnote 14.

[12]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[13]R. C. Brower and John Ellis, Phys. Rev. D 5, 2253 (1972).

[14]There are technical reasons why it is difficult to include diffractive processes within the bootstrap; that is, to allow them to affect $g(x)$. For example, a triple-Pomeranchukon term (with a linear zero and $\alpha_P = 1$) would imply $g(x) \sim [(1-x)\ln^2(1-x)]^{-1}$ as $x \to 1$, which would make the integral in Eq. (18) diverge; hence $f(0)$ would vanish. Also a triple-Regge term consisting of two Pomeranchukons and an f trajectory would contribute nothing to the scaling limit of $g(x)$ for $|x| < 1$, and yet could affect integrals over g, and so affect $f(x)$ through Eq. (11).

[15]M. Le Bellac, Phys. Letters 37B, 413 (1971); John Ellis, J. Finkelstein, and R. D. Peccei, Nuovo Cimento (to be published).

[16]A. Pignotti and P. Ripa, Phys. Rev. Letters 27, 1538 (1971).

[17]H. D. I. Abarbanel, Phys. Rev. D 3, 2227 (1971).

[18]One way to show this is to express this coefficient in terms of integrals over g, by using Eq. (39) for the ratio of B to A, and by recognizing the integral multiplying B as $\hat{f}(1-\alpha_0) = \hat{g}(1-\alpha_0)/[1-\hat{h}(1-\alpha_0)]$.

[19]J. V. Allaby *et al.*, CERN Report No. CERN 70-12, 1970 (unpublished).

[20]L. W. Jones *et al.*, Phys. Rev. Letters 25, 1679 (1970).

[21]For example, H. Bøggild *et al.*, Nucl. Phys. B27, 285 (1971).

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Spectral-Function Sum Rules and ω-ϕ Mixing

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In this paper we perform a calculation of the ω-ϕ mixing problem, using the spectral-function sum rules of Weinberg, which is more comprehensive than previous similar calculations. In conjunction with the first sum rule we assume asymptotic nonet symmetry. We use the second sum rule modified by corrections estimated using the Gell-Mann, Oakes, and Renner model of the SU(3)⊗SU(3)-breaking strong interactions. The predictions of the model are in good agreement with experiment where accurate experimental data are available. We indicate how it is possible to reconcile previous competing theories within the more comprehensive framework of our model.

I. INTRODUCTION

Various authors[1] have used the spectral-function sum rules of Weinberg[2] to investigate the problem of ω-ϕ mixing. As data accumulate from electron-positron storage-ring experiments[3] and photoproduction of neutral vector mesons, these theories will be put to a more severe test.

In this paper, we extend the earlier work of Das, Mathur, and Okubo[1] and of Oakes and Sakurai[1] by systematically exploiting the first Weinberg sum rule and the *modified*[4] second Weinberg sum rule. We find that the sum rules alone specify the ω-ϕ mixing angle and, in addition, make several predictions which seem to be in reasonable accord with experiment.

II. FIRST SUM RULE

We take the first sum rule to be of the form

$$\int dm^2[m^{-2}\rho^{(1)}_{\alpha\beta}(m^2) + \rho^{(0)}_{\alpha\beta}(m^2)] = S\delta_{\alpha\beta}$$

$$(\alpha, \beta = 0, \ldots, 8), \quad (1)$$

which is to say we assume asymptotic nonet symmetry. This requires U(3) symmetry of the relevant Schwinger terms for which Okubo has provided some theoretical justification.[5] The reasonableness of the results obtained from this assumption (which we shall give below) lends further support to the nonet hypothesis.

The spectral functions for the vector currents are defined through

$$-i\int d^4x\, e^{-iq\cdot x}\langle 0|T\{V_\mu^\alpha(x)V_\nu^\beta(0)\}|0\rangle$$
$$=\int dm^2\,\rho_{\alpha\beta}^{(1)}(m^2)\frac{\delta_{\mu\nu}+m^{-2}q_\mu q_\nu}{q^2+m^2-i\epsilon}$$
$$+\int dm^2\rho_{\alpha\beta}^{(0)}(m^2)\frac{q_\mu q_\nu}{q^2+m^2-i\epsilon}$$
$$+\delta_{\mu 0}\delta_{\nu 0}\int dm^2[m^{-2}\rho_{\alpha\beta}^{(1)}(m^2)+\rho_{\alpha\beta}^{(0)}(m^2)]\ . \tag{2}$$

We define

$$(2q_0V)^{1/2}\langle 0|V_\mu^{(3)}(0)|\rho^0(q)\rangle=\epsilon_\mu(q)F_\rho\ . \tag{3}$$

F_{K^*}, F_ϕ, and F_ω are defined similarly. In addition we define

$$(2q_0V)^{1/2}\langle 0|V_\mu^{(0)}(0)|\phi(q)\rangle=\epsilon_\mu(q)\sigma_\phi\ ,$$
$$(2q_0V)^{1/2}\langle 0|V_\mu^{(0)}(0)|\omega(q)\rangle=\epsilon_\mu(q)\sigma_\omega\ . \tag{4}$$

We assume the existence of a strange scalar (κ) meson satisfying

$$(2q_0V)^{1/2}\langle 0|V_\mu^{(4+i5)}(0)|\kappa^+(q)\rangle=-f_\kappa q_\mu\ . \tag{5}$$

Assuming that the vector spectral functions are dominated by the vector-meson nonet, the first Weinberg sum rules may be written

$$\frac{F_\rho^2}{m_\rho^2}=\frac{F_{K^*}^2}{m_{K^*}^2}+\tfrac{1}{2}f_\kappa^2\ , \tag{6a}$$

$$\frac{F_\rho^2}{m_\rho^2}=\frac{F_\omega^2}{m_\omega^2}+\frac{F_\phi^2}{m_\phi^2}\ , \tag{6b}$$

$$\frac{F_\rho^2}{m_\rho^2}=\frac{\sigma_\omega^2}{m_\omega^2}+\frac{\sigma_\phi^2}{m_\phi^2}\ , \tag{6c}$$

$$\frac{F_\omega\sigma_\omega}{m_\omega^2}+\frac{F_\phi\sigma_\phi}{m_\phi^2}=0\ . \tag{6d}$$

The last sum rule, Eq. (6d), leads[1] to the well-known current-mixing condition

$$\frac{m_\omega}{m_\phi}\tan\theta_Y=\tan\theta=\frac{m_\phi}{m_\omega}\tan\theta_N\ , \tag{7}$$

where θ_Y and θ_N are the mixing angles introduced by Kroll, Lee, and Zumino.[6] As usual, from the sum rule, Eq. (6b), we can derive the Das-Mathur-Okubo[1] relation

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to l^+l^-)=m_\omega\Gamma(\omega\to l^+l^-)+m_\phi\Gamma(\phi\to l^+l^-). \tag{8}$$

III. MODIFIED SECOND SUM RULE

To obtain more information, we need to know the angle θ. The authors in Ref. 1 used different approaches to estimate θ and derived quite different values of θ. In this paper we shall show that the approaches of Das, Mathur, and Okubo[1] and of Oakes and Sakurai[1] may be reconciled. We shall use the modified second Weinberg sum rule together with the first sum rules, Eq. (6), to estimate θ.

The original second sum rule is well known to be incorrect[4] and must be modified. To estimate corrections to it we use the Hamiltonian density proposed by Gell-Mann and co-workers[7] as our model of the symmetry breaking:

$$\mathcal{H}=\mathcal{H}_0-\omega_0u_0-\omega_8u_8\ , \tag{9}$$

where $\mathcal{H}_0$ is SU(3)⊗SU(3)-invariant, and $(\omega_0u_0+\omega_8u_8)$ belongs to the $(3,\bar{3})+(\bar{3},3)$ representation of SU(3)⊗SU(3). At the present time, this model appears to be on a fairly sound footing.

Following the work[8] of Nieh, Kamal, and Lai and Lo, who use the techniques of Bjorken, we write the modified second sum rule in the form

$$\int\rho_{\alpha\beta}^{(1)}(m^2)\,dm^2=d_{\alpha\beta\gamma}\langle Z_\gamma\rangle+\omega_0d_{0\alpha\gamma}d_{\gamma\beta\delta}\langle u_\delta\rangle$$
$$+\omega_8d_{8\alpha\gamma}d_{\gamma\beta\delta}\langle u_\delta\rangle\ , \tag{10}$$

where $\alpha,\beta=0,\ldots,8$ and we consider $\alpha\neq\beta$ only in the case $\alpha=0$, $\beta=8$.

The first term on the right-hand side of Eq. (10) is defined by

$$\int d^3x\langle 0|[[V_i^\alpha(x),H_0]\ ,V_i^\alpha(0)]|0\rangle=d_{\alpha\alpha\gamma}\langle Z_\gamma\rangle\ .$$

The quantities $\langle Z_0\rangle$ and $\langle Z_8\rangle$ are model-dependent (Lai and Lo[8] and Bjorken[8]). Using a gluon model, Bjorken[8] has evaluated the above double commutator and concluded that, in general, $\langle Z_0\rangle$ and $\langle Z_8\rangle$ are quadrically divergent. In this model, when the bare quark masses are the same, we have the interesting result that $\langle Z_8\rangle$ vanishes. It is then possible to obtain an additional relation from Eq. (10) other than the three we obtain below [Eqs. (11a)–(11c)]. We defer discussion of this additional, less general, relation to Sec. IV C. For the moment, we eliminate both $\langle Z_0\rangle$ and $\langle Z_8\rangle$ as follows.

From Eq. (10), we obtain five independent equations by setting $\alpha=\beta=3,4,8,0$ and $\alpha=0$, $\beta=8$. These reduce to three independent equations when $\langle Z_0\rangle$ and $\langle Z_8\rangle$ are eliminated, namely

$$2(\sigma_\phi^2+\sigma_\omega^2)=F_\rho^2+F_\phi^2+F_\omega^2\ , \tag{11a}$$

$$\sqrt{2}\,(\sigma_\phi F_\phi+\sigma_\omega F_\omega)=F_\rho^2-F_\phi^2-F_\omega^2\ , \tag{11b}$$

$$4F_{K^*}^2 - F_\rho^2 - 3(F_\phi^2 + F_\omega^2) = -3\omega_8\langle u_8\rangle = -2f_K^2 m_K^2 . \quad (11c)$$

From the current mixing condition, Eq. (6d), and the first Weinberg sum rule, Eqs. (6b) and (6c), we have

$$\frac{\sigma_\omega}{m_\omega} = \frac{-F_\rho}{m_\rho}\cos\theta = \frac{-F_\phi}{m_\phi} ,$$
$$\frac{\sigma_\phi}{m_\phi} = \frac{-F_\rho}{m_\rho}\sin\theta = \frac{F_\omega}{m_\omega} . \quad (12)$$

Using Eq. (12) we may rewrite Eqs. (11a) and (11b) as

$$2 + \delta = 3\cos^2\theta , \quad (13a)$$

$$\sqrt{2}\sin\theta\cos\theta = \cos^2\theta + \delta , \quad (13b)$$

where

$$\delta = \frac{m_\omega^2 - m_\rho^2}{m_\phi^2 - m_\omega^2} . \quad (14)$$

The solution of Eqs. (13a) and (13b) is

$$\delta = 0 \quad (\text{i.e., } m_\omega = m_\rho) \quad (15)$$

and

$$\theta = \arctan(1/\sqrt{2}) , \quad (16)$$

which are results typical of nonet symmetry.[9]

Using the current mixing condition and first Weinberg sum rule, Eq. (6a), we may rewrite Eq. (11c) as

$$4m_{K^*}^2 - m_\rho^2 - 3(m_\phi^2\cos^2\theta + m_\omega^2\sin^2\theta) = 2(f_K/f_\pi)^2(m_{K^*}^2 - m_K^2) , \quad (17)$$

where we have made use of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[10]

$$F_\rho^2/m_\rho^2 = f_\pi^2 .$$

Making use of Eqs. (15) and (16), we may rewrite Eq. (17) in the simpler form

$$2m_{K^*}^2 - (m_\phi^2 + m_\omega^2) = (f_K/f_\pi)^2(m_{K^*}^2 - m_K^2) . \quad (17')$$

If in addition we make the common assumption that $f_K = 0$, we recover the third standard result of the nonet hypothesis,[9]

$$2m_{K^*}^2 = (m_\phi^2 + m_\omega^2) . \quad (17'')$$

Before proceeding, let us compare the explicit nonet prediction $m_\rho = m_\omega$ with experiment. [The prediction $\theta = \arctan(1/\sqrt{2})$ may be only tested indirectly. Equation (17) is independent of the assumption of asymptotic nonet symmetry.]

The latest data[11] give $m_\omega = 783.9 \pm 0.3$ MeV and from Orsay[3] we have $m_\rho = 780.2 \pm 5.9$ MeV (with the 990-MeV point) or $m_\rho = 775.4 \pm 7.3$ MeV (without the 990-MeV point). This means that the difference $m_\omega - m_\rho$ is only of the order of typical electromagnetic mass differences. Since we are neglecting electromagnetic effects in this analysis, it would seem to be quite consistent to neglect an effect of equivalent order. In addition since it is clearly quite difficult to pin down an "exact" ρ mass because of its broad width, this would seem to be a good way to "fix" the value of m_ρ. Henceforth, we shall assume $m_\rho = m_\omega = 784$ MeV. [Small deviations from this equality could be taken account of by dropping the first Weinberg sum rule, Eq. (6c). In this case it is still possible to calculate θ from the current mixing condition and Eqs. (11a) and (11b). Deviations of θ from the nonet prediction would depend on $\delta = (m_\omega^2 - m_\rho^2)/(m_\omega^2 - m_\phi^2)$ and be quite small.]

IV. COMPARISONS WITH EXPERIMENT

A. Predictions

We now list some of our predictions and their comparison with experiment. In this section we list only those results which depend on $m_\omega = m_\rho$ and $\theta = \arctan(1/\sqrt{2})$.

(1) Introducing the standard dimensionless photon-vector-meson coupling constants

$$f_\rho = m_\rho^2/F_\rho , \; f_\phi = \sqrt{3}\, m_\phi^2/F_\phi , \; f_\omega = \sqrt{3} m_\omega^2/F_\omega ,$$

this model predicts

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9 : 1 : 2\left(\frac{m_\omega}{m_\phi}\right)^2 = 9 : 1 : 1.18 . \quad (18)$$

The theoretical predictions of Das, Mathur, and Okubo[1] are

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9 : 1.2 : 1.0 ,$$

and those of Oakes and Sakurai[1] are

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9 : 0.65 : 1.33 .$$

The experimental results from the Orsay storage ring yield

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9 : 1.25 \pm 0.1 : 2.1 \pm 0.2 .$$

The latest photoproduction experiments as summarized by Söding[12] give somewhat different results:

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9 : 0.9 \pm 0.3 : 1.5 \pm 0.5 .$$

At present, therefore, it does not seem possible to discriminate between the various models because of the substantial experimental errors present in the measurements of the various $f_V^2/4\pi$. In view of the present agreement between measurements of $f_{\rho\pi\pi}^2/4\pi$ from Orsay[3] 2.56 ± 0.4 and photoproduction experiments[12] (2.60, no error quoted), one would hope for better agreement in the future between the different experimental determinations of $f_V^2/4\pi$.

As a final comment we note that, assuming $f_{\phi\pi^0\gamma}=0$ (see below), saturation of the isoscalar $\pi^0\gamma$ form factor with the ω yields[13]

$$\frac{f_{\omega\pi^0\gamma}}{f_\omega}=(2.28\pm0.12)\times10^{-2} .$$

The dimensionless coupling constant $f_{\omega\pi^0\gamma}$ can be deduced from the rate $\Gamma(\omega\to\pi^0\gamma)$ (see below) to be $\pm(0.39\pm0.04)$, from which we conclude

$$f_\omega^2/4\pi=23\pm5 ,$$

which may be compared with the Orsay measurement[3]

$$f_\omega^2/4\pi=18.4\pm2 ,$$

and the prediction of this model, using the Orsay result $f_\rho^2/4\pi=2.56\pm0.27$ together with Eq. (18),

$$f_\omega^2/4\pi=23.0\pm2.4 ,$$

in good agreement with the deduction of $f_\omega^2/4\pi$ from $\Gamma(\omega\to\pi^0\gamma)$ above.

(2) In the narrow-width approximation, saturation of the K-meson isoscalar form factor with the ω and ϕ mesons yields

$$\frac{f_{\omega K\bar{K}}}{f_\omega}+\frac{f_{\phi K\bar{K}}}{f_\phi}=\frac{1}{2} . \tag{19}$$

Combining this with the result

$$\frac{\sigma_\omega f_{\omega K\bar{K}}}{m_\omega^2}+\frac{\sigma_\phi f_{\phi K\bar{K}}}{m_\phi^2}=0 ,$$

since there is no unitary singlet electromagnetic current, yields

$$\frac{f_{\phi K\bar{K}}}{f_\phi}=\tfrac{1}{2}\cos^2\theta=\frac{1}{3} ,\qquad \frac{f_{\omega K\bar{K}}}{f_\omega}=\tfrac{1}{2}\sin^2\theta=\frac{1}{6} . \tag{20}$$

We have used the standard definitions[6]

$$f_\phi/f_\omega=-\tan\theta_Y ,$$

$$\frac{\sigma_\phi/m_\phi^2}{\sigma_\omega/m_\omega^2}=\tan\theta_N .$$

We may also deduce

$$f_{\omega K\bar{K}}/f_{\phi K\bar{K}}=-\tan\theta_N . \tag{21}$$

We shall make use of this later.

Gourdin[13] has noted that by measuring the cross section for

$$e^+ + e^- \to \phi\to K\bar{K} ,$$

it is possible to deduce from the Orsay data[3]

$$|f_{\phi K\bar{K}}/f_\phi|=0.349\pm0.025 \tag{22}$$

in good agreement with our prediction, Eq. (20).

(3) Assuming SU(3) invariance of the vector–pseudoscalar–vector-current vertex (i.e., d-type coupling) we have

$$\sqrt{2}\langle\pi^0|J_\mu^8(0)|\rho^0\rangle=\langle\pi^0|J_\mu^0(0)|\rho^0\rangle .$$

Using the vector-dominance model[6] (VDM) for the (isoscalar) electromagnetic and baryonic currents, one obtains from this the relation

$$\sqrt{2}\left(\frac{F_\phi}{m_\phi^2}g_{\rho\phi\pi}+\frac{F_\omega}{m_\omega^2}g_{\rho\omega\pi}\right)=\frac{\sigma_\phi}{m_\phi^2}g_{\rho\phi\pi}+\frac{\sigma_\omega}{m_\omega^2}g_{\rho\omega\pi} , \tag{23}$$

in a straightforward fashion. Using the results of Eq. (12), Eq. (23) reduces to

$$\sqrt{2}\left(\frac{\cos\theta}{m_\phi}g_{\rho\phi\pi}-\frac{\sin\theta}{m_\omega}g_{\rho\omega\pi}\right)=-\frac{\sin\theta}{m_\phi}g_{\rho\phi\pi}-\frac{\cos\theta}{m_\omega}g_{\rho\omega\pi} . \tag{23$'$}$$

Insertion of the nonet value for θ [Eq. (16)] yields the result

$$g_{\rho\phi\pi}=0 . \tag{23$''$}$$

This means that the strong decay $\phi\to\rho\pi\to3\pi$ is forbidden in our nonet scheme, provided that one has SU(3) invariance of the vector–pseudoscalar–vector-current vertex as we have assumed. The fact that the decay $\phi\to3\pi$ is very inhibited compared with what one might expect from the decay rate of $\omega\to3\pi$ supports both of these assumptions. (According to Pilkuhn[14] the ratio of the coupling constants squared $g_{\rho\omega\pi}^2:g_{\rho\phi\pi}^2$ is approximately 400 : 1.)

Using the usual vector-dominance model[15] for electromagnetic decays of vector mesons $V\to P\gamma$ (which is simply $\omega\to\rho\pi^0\to\gamma\pi^0$, $\rho\to\omega\pi^0\to\gamma\pi^0$ because the ρ-ϕ-π vertex cannot contribute) we predict

$$g_{\omega\pi^0\gamma}:g_{\rho\pi^0\gamma}:g_{\phi\pi^0\gamma}=\frac{1}{f_\rho}:\frac{1}{f_\omega}:0=3:-1:0 \tag{24}$$

for the ratios of the various dimensionless coupling constants. (See Gourdin[13] for their definitions.)

From the radiative decay width[11]

$$\Gamma(\omega\to\pi^0\gamma)=1.1\pm0.2\text{ MeV}$$

we may deduce[13]

$$|g_{\omega\pi^0\gamma}|=0.39\pm0.04 . \tag{25}$$

Although the decay $\rho \to \pi^0\gamma$ has not yet been experimentally observed, it is possible to estimate $g_{\rho\pi^0\gamma}$ from the $\pi^0\gamma$ electromagnetic form factor using ρ dominance of the isovector part.[13]

Using[3] $f_{\rho\pi\pi}{}^2/4\pi = 2.56 \pm 0.4$, we estimate

$$g_{\rho\pi^0\gamma} = 0.13 \pm 0.02 \ . \tag{26}$$

From the branching ratio (Lefrancois[3])

$$\Gamma(\phi \to \pi^0\gamma)/\Gamma(\phi \to \text{all}) = (0.25 \pm 0.09)\% ,$$

we estimate

$$|g_{\phi\pi^0\gamma}| = 0.03 \pm 0.01 \ . \tag{27}$$

Finally, from ω-ϕ dominance of the isoscalar part of the $\pi^0\gamma$ electromagnetic form factor,[13] we find that $g_{\omega\pi^0\gamma}$ and $g_{\rho\pi^0\gamma}$ are relatively negative. Therefore from experiment and VDM we conclude that

$$g_{\omega\pi^0\gamma} : g_{\rho\pi^0\gamma} : g_{\phi\pi^0\gamma} = -(0.39 \pm 0.04) : (0.13 \pm 0.02) : -(0.03 \pm 0.01) , \tag{28}$$

in good agreement with the prediction, Eq. (24).

(4) From the Orsay result[3] $f_\rho{}^2/4\pi = 2.56 \pm 0.27$ and our predictions for f_ω/f_ρ and f_ϕ/f_ρ [Eq. (19)], we estimate

$$\begin{aligned} f_\omega{}^2/4\pi &= 23.0 \pm 2 \ , \\ f_\phi{}^2/4\pi &= 19.5 \pm 2 \ . \end{aligned} \tag{29}$$

Using these estimates together with Eq. (20), we predict

$$\begin{aligned} \frac{f_{\phi K\bar K}{}^2}{4\pi} &= \frac{1}{9}\left(\frac{f_\phi{}^2}{4\pi}\right) = 2.17 \pm 0.22 \ , \\ \frac{f_{\omega K\bar K}{}^2}{4\pi} &= \frac{1}{36}\left(\frac{f_\omega{}^2}{4\pi}\right) = 0.64 \pm 0.06 \ , \end{aligned} \tag{30}$$

Using the notation of Gourdin[13] to relate coupling constants in broken SU(3), we have

$$\frac{f_{K^*K\pi}{}^2}{4\pi} \Big/ \frac{f_{\rho\pi\pi}{}^2}{4\pi} = \frac{3}{8}\left(\frac{1-\frac{1}{2}\alpha}{1+\alpha}\right)^2 . \tag{31}$$

From the decay width[11]

$$\Gamma(K^* \to K\pi) = 50 \pm 1 \text{ MeV}$$

we estimate

$$f_{K^*K\pi}{}^2/4\pi = 1.28 \pm 0.03 \ . \tag{32}$$

Combining this with the Orsay result[3] $f_{\rho\pi\pi}{}^2/4\pi = 2.56 \pm 0.4$ we find

$$\alpha = -0.09 \ , \tag{33}$$

where we have dropped the error in α which arises principally from the error in the $\rho\pi\pi$ coupling constant.

Now using the earlier result

$$f_{\omega K\bar K}/f_{\phi K\bar K} = -\tan\theta_N \ , \tag{21}$$

we have in the above notation

$$\begin{aligned} \frac{f_{\phi K\bar K}{}^2}{4\pi} &= \frac{3}{4}\frac{f_{\rho\pi\pi}{}^2}{4\pi}\left(\frac{1-\alpha}{1+\alpha}\right)^2 \cos^2\theta_N \ , \\ \frac{f_{\omega K\bar K}{}^2}{4\pi} &= \frac{3}{4}\frac{f_{\rho\pi\pi}{}^2}{4\pi}\left(\frac{1-\alpha}{1+\alpha}\right)^2 \sin^2\theta_N \ . \end{aligned} \tag{34}$$

Note that our expressions for $f_{\phi K\bar K}$ and $f_{\omega K\bar K}$ in broken SU(3) differ from those of Gourdin,[13] who uses the incorrect mixing angle.

With $\theta = \arctan(1/\sqrt{2}) = 35.2°$, $\theta_N = 28.6°$, we find

$$\begin{aligned} \frac{f_{\phi K\bar K}{}^2}{4\pi} &= 2.16 \pm 0.3 \ , \\ \frac{f_{\omega K\bar K}{}^2}{4\pi} &= 0.65 \pm 0.1 \ , \end{aligned} \tag{35}$$

in excellent agreement with the predictions in Eq. (30). [Note that the error estimates given in Eq. (35) are fairly crude and take into account only the experimental error in $f_{\rho\pi\pi}$ and not any error associated with our estimate of α.]

In summary, we have estimated $f_{\phi K\bar K}$ and $f_{\omega K\bar K}$ from $f_{\rho\pi\pi}$ using broken SU(3) symmetry for the *VPP* coupling constants and found these estimates to be in very close agreement with the values predicted by vector-meson dominance together with the nonet mixing angle. This gives us further confidence in the internal consistency of our treatment and additional support for the asymptotic nonet symmetry hypothesis.

B. Further Predictions

In Sec. III we found the relation

$$2m_{K^*}{}^2 - (m_\phi{}^2 + m_\omega{}^2) = (f_K/f_\pi)^2(m_{K^*}{}^2 - m_K{}^2). \tag{16$'$}$$

If $f_K \neq 0$ it is not possible to deduce information from this equation about f_K or m_K without some further assumption. We recall that if one has a field theory containing a scalar field and a vector field which interact, it is necessary to introduce a new vector field which will correspond to the physical spin-1 particle. This must be done in order to prevent mixing of the physical spin-0 and spin-1 particles.

In certain theories[16] the mass of the vector particle is not shifted by the presence of the scalar particle. If this were the case, m_{K^*} would be independent of f_K. (m_K does, of course, depend on f_K.) We would then deduce from Eq. (16$'$) the standard nonet result

$$2m_{K^*}{}^2 = m_\phi{}^2 + m_\omega{}^2 \ , \tag{16$''$}$$

together with the interesting equality

$$m_\kappa = m_{K^*} \ , \tag{36}$$

i.e., the κ and K^* are degenerate in mass. We are unable, however, to predict f_κ.

Using[11] $m_\phi = 1020$ MeV and $m_\omega = 784$ MeV we estimate

$$m_{K^*} = 910 \text{ MeV} \ , \tag{37}$$

which seems a little high even allowing for the fact that $m^0_{K^*}$ is heavier than $m^\pm_{K^*}$. (From the data of Ref. 11, we estimate an average K^* mass of 897 MeV.) Therefore we reject the simple hypothesis that the K^* mass is not shifted by the existence of the κ.[17]

In order to proceed further, we must adopt a specific model of the K^*-κ mixing. It is interesting, therefore, to consider the work of Gasiorowicz and Geffen,[18] which not only provides us with a model, but also indicates a way in which the theories of Das, Mathur, and Okubo[1] and of Oakes and Sakurai[1] may be reconciled by taking account of the kappa meson. Using effective-Lagrangian techniques, they construct a so-called "super-Lagrangian" which enables one to estimate the K^* mass shift in terms of f_κ. They obtain a generalization of the current-mixing mass formula of Oakes and Sakurai, namely

$$\frac{4\Gamma_\kappa}{m_{K^*}^2} - \frac{1}{m_\rho^2} = 3\left(\frac{\cos^2\theta}{m_\phi^2} + \frac{\sin^2\theta}{m_\omega^2}\right) \ , \tag{38}$$

where Γ_κ is defined by

$$\frac{F_\rho^2}{m_\rho^2} = \Gamma_\kappa \frac{F_{K^*}^2}{m_{K^*}^2} \ . \tag{39}$$

From the first Weinberg sum rule [Eq. (6a)] and the KSRF relation, we have

$$\Gamma_\kappa \approx 1 + \tfrac{1}{2}(f_\kappa/f_\pi)^2 \ . \tag{40}$$

[Note that Eq. (38) reduces exactly to the mass formula of Oakes and Sakurai when $f_\kappa = 0$.]

Since we have assumed current mixing for ω-ϕ (by virtue of assuming the validity of the first Weinberg sum rule), it is clearly an attractive proposition to extend this concept[19] to the whole nonet of vector mesons. Using the results $m_\rho = m_\omega$, $\theta = \arctan(1/\sqrt{2})$ of asymptotic nonet symmetry, the mass formula Eq. (38) of Gasiorowicz and Geffen simplifies to

$$\frac{2\Gamma_\kappa}{m_{K^*}^2} = \frac{1}{m_\phi^2} + \frac{1}{m_\omega^2} \ . \tag{38$'$}$$

Assuming $m_\phi = 1020$ MeV, $m_\omega = 784$ MeV, and $\bar{m}_{K^*} = 897$ MeV, we find

$$(f_\kappa/f_\pi)^2 = 0.08 \ . \tag{41}$$

Combining this with Eq. (16′), we estimate that m_κ lies in the range 1000–1100 MeV. (This is within the currently accepted experimental range for m_κ.[11]) It does not seem worthwhile to attempt to pin down the mass m_κ more accurately than this because of the errors involved in the other quantities which enter into the equation defining m_κ [Eq. (16′)].

We see then that it is possible within our model (which has the mixing angle θ fixed through the assumption of asymptotic nonet symmetry) to reconcile the superficially different approaches of Das, Mathur, and Okubo[1] and of Oakes and Sakurai[1] by allowing $f_\kappa \neq 0$. This was in fact hinted at by Oakes and Sakurai, who pointed out that the difference between the two methods of determining the mixing angle was only of second order in the SU(3) breaking. We have explicitly exhibited the difference here by including $(f_\kappa/f_\pi)^2$, which is of second order in the SU(3) breaking.

As a check on the above speculation, we can obtain independent information on f_κ by saturating the weak K-π vertex with the K^*. We obtain

$$f_+(0) = \frac{2}{\sqrt{3}} \frac{F_{K^*} f_{K^*K\pi}}{m_{K^*}^2} \ , \tag{42}$$

which may be rewritten as

$$\sqrt{2} f_+(0) \Gamma_\kappa^{\ 1/2} = \left(\frac{8}{3}\right)^{1/2} \frac{m_\rho}{f_\rho} \frac{1}{m_{K^*}} f_{K^*K\pi} \ , \tag{43}$$

using Eq. (39). From the experimental result[3] $f_\rho^2/4\pi = 2.56 \pm 0.27$ and our previous estimate $f_{K^*K\pi}^2/4\pi = 1.28 \pm 0.03$, we may deduce

$$[\sqrt{2}\, f_+(0)]^2 \Gamma_\kappa = 1.027 \ , \tag{44}$$

where we have dropped the errors arising from f_ρ and $f_{K^*K\pi}$.

If we combine this with the Glashow-Weinberg[20] formula

$$\sqrt{2}\, f_+(0) = \frac{f_K^2 + f_\pi^2 - f_\kappa^2}{2 f_K f_\pi} \ , \tag{45}$$

and the experimental result[21]

$$f_K/\sqrt{2}\, f_+(0) f_\pi = 1.27 \pm 0.03 \ , \tag{46}$$

we are able to deduce

$$(f_\kappa/f_\pi)^2 = 0.09 \ , \tag{47}$$

and

$$\begin{aligned} f_K/f_\pi &= 1.26 \ , \\ \sqrt{2}\, f_+(0) &= 0.99 \ , \end{aligned} \tag{48}$$

where we have consistently dropped the errors on the experimental quantities. The value for $(f_\kappa/f_\pi)^2$ is in surprisingly good agreement with that deduced above, lending support to our adoption of the model of Gasiorowicz and Geffen for κ-K^* mixing as an additional hypothesis to our own hypothesis of asymptotic nonet symmetry. In addition, the value obtained for $\sqrt{2}\, f_+(0)$ is about what one would expect.

C. A Further Sum Rule

In Sec. III we mentioned that, in a certain model, the quantity $\langle Z_8 \rangle$ vanished, and consequently it was possible to derive another modified second sum rule in addition to Eqs. (11a), (11b), (11c). From the current-mixing condition and the first Weinberg sum rules, Eqs. (6), we may write this additional relation in the form

$$(m_\rho/f_\rho)^2(m_{K^*}{}^2 - m_\rho{}^2) = \tfrac{1}{2}(f_K{}^2 m_K{}^2 - f_\pi{}^2 m_\pi{}^2 + f_\kappa{}^2 m_{K^*}{}^2) \; . \tag{49}$$

It would have been possible to use this relation in the previous section to deduce f_κ rather than go to the trouble of invoking a particular model[18] of K^*-κ mixing. However, we deferred discussion of this relation until now because, as indicated in Sec. III, it appears to be model-dependent.

It is of interest to check the consistency of this sum rule, Eq. (49), with the sum rule Eq. (16′) (which is model-independent) together with the specific model of K^*-κ mixing discussed in the previous section. Using the value

$$(f_\kappa/f_\pi)^2 = 0.08 \; ,$$

deduced from the mixing model of Gasiorowicz and Geffen together with the result $f_K/f_\pi = 1.26$, which we may deduce from Eqs. (41), (45) and (46), we rewrite Eq. (49) as

$$\left(\frac{m_\rho}{f_\rho}\right)^2 \frac{m_{K^*}{}^2 - m_\rho{}^2}{f_\pi{}^2} = 0.219 \; . \tag{49′}$$

The left-hand side of Eq. (49′) may be evaluated numerically using $\overline{m}_{K^*} = 897$ MeV, $m_\rho = 784$ MeV, $f_\pi = 130$ MeV, together with the experimental result[3] $f_\rho{}^2/4\pi = 2.56 \pm 0.27$. We then arrive at the "equation"

$$0.215 = 0.219 \; , \tag{49″}$$

(The 10% error on the left-hand side arising from the error in $f_\rho{}^2/4\pi$ has been dropped.)

It is clear that the model-dependent sum rule Eq. (49) is extremely well satisfied numerically and lends support to our earlier estimate of f_κ/f_π based on the mixing model of Gasiorowicz and Geffen. Because this sum rule is satisfied much better than one might have expected, we must conclude that the vanishing of $\langle Z_8 \rangle$ is not as model-dependent as we previously have suggested. In the absence of a realistic model of the strong interactions, it is not clear what the full implications of $\langle Z_8 \rangle = 0$ might be. However, if we assume a specific model such as the gluon model of Bjorken, the implication is that the bare quark masses are equal so that any inequalities in the physical quark masses arise from SU(3) breaking in the interaction Hamiltonian.

V. SUMMARY AND CONCLUSION

In this paper we have performed a calculation of the ω-ϕ mixing problem, using the spectral-function sum rules of Weinberg, which is more comprehensive than previous similar calculations. In conjunction with the first sum rule we have assumed asymptotic nonet symmetry. This is a useful simplifying assumption and seems to be in keeping with other approximations that are made; for example, deviations from exact nonet symmetry seem to be of about the order of electromagnetic effects (which are consistently neglected here). In addition we have worked with the second sum rule, modified by corrections estimated using the Gell-Mann, Oakes, and Renner model of the SU(3)⊗SU(3)-breaking strong interactions.

The results for f_ρ, f_ω, f_ϕ presented in Sec. IV A are in no better or worse agreement with experiment than previous theoretical calculations. In order to distinguish between the various theories on the basis of these quantities, more accurate experimental data are needed. However, the additional results presented in this section seem to be in quite good agreement with experiment.

In Sec. IV B, we introduced an additional assumption by using the "super-Lagrangian" of Gasiorowicz and Geffen to estimate the K^* mass shift from the nonet prediction due to the presence of the kappa meson. With this additional assumption, we were able to link the previous works of Das, Mathur, and Okubo and of Oakes and Sakurai in a logical fashion, rather than regard them as competing theories, as is usually assumed. A value for the kappa mass was deduced which lay within the present experimentally accepted region.

In Sec. IV C a further sum rule, thought to be model-dependent, was discussed. It was found that, in fact, it was completely consistent with the model-independent results of the previous sections to an accuracy well within experimental error. This leads us to have further confidence in our attempt to unite the competing theories of Das, Mathur, and Okubo and of Oakes and Sakurai by explicitly including the κ meson within the framework of the mixing model of Gasiorowicz and Geffen. In this sense it is hoped that our current-mixing model for ω-ϕ is not simply a third alternative to the two standard theories, but rather an intermediate, more exact, model which is hopefully closer to the truth than the previous theories.

We note finally that the accuracy with which the sum rule discussed in Sec. IV C holds indicates strongly that it is *not* in fact model-dependent (as we suggested it was in Sec. III). The reasons for this are not clear to the author at the present time.

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[1]T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967); R. J. Oakes and J. J. Sakurai, *ibid.* 19, 1266 (1967).

[2]S. Weinberg, Phys. Rev. Letters 18, 507 (1967); S. Glashow, H. Schnitzer, and S. Weinberg, *ibid.* 19, 139 (1967).

[3]J. E. Augustin *et al.*, Phys. Letters 28B, 503 (1969); J. C. Bizot *et al.*, *ibid.* 32B, 420 (1970); J. Lefrancois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

[4]We abandon the original form of the second sum rule. For specific objections to the original form see, for example, J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967).

[5]S. Okubo, Lectures given at the University of Islamabad, 1967 (unpublished).

[6]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[7]M. Gell-Mann, Physics 1, 63 (1964); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968). See also S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[8]H. T. Nieh, Phys. Rev. 163, 1769 (1967); A. N. Kamal, Nucl. Phys. B12, 123 (1969); C. S. Lai and S. Y. Lo Proc. Phys. Soc. London (Gen.) A 4, 859 (1971); J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[9]S. Okubo, Phys. Letters 5, 165 (1963) and also in *Proceedings of the 1967 International Conference on Particles and Fields,* edited by C. Hagen, G. Guralnik, and V. Mathur (Interscience, New York, 1967).

[10]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 144, 1071 (1966).

[11]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[12]P. Söding, Trieste Report No. IC/71/134, 1971 (unpublished).

[13]M. Gourdin, in *Hadronic Interactions of Electrons and Photons,* edited by J. Cummings and H. Osborne (Academic, New York, 1971). See also M. Gourdin, in *Springer Tracts in Modern Physics,* edited by G. Höhler (Springer, New York, 1970), Vol. 55, p. 191; M. Gourdin, *Unitary Symmetries* (North-Holland, Amsterdam, 1967).

[14]H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967).

[15]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[16]H. Munczek and L. M. Brown, *Symmetry Principles at High Energy* (Benjamin, New York, 1968).

[17]We may note here that V. E. Barnes, Purdue Report No. C00-1428-293 (unpublished), has suggested that the existence of a κ resonance practically degenerate with the K^* would require experimental estimates of the K^* mass to shift upwards by about 10 MeV. If this were the case, the prediction Eq. (37) for the K^* mass, associated with the theoretical K^*-κ mass degeneracy, Eq. (36), would be quite reasonable.

[18]S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[19]S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964).

[20]S. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[21]L. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Reports (to be published).