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Reciprocal Bootstrap of N and N^* with a Nonpolynomial Lagrangian

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A calculation of the reciprocal bootstrap of N and N^* is carried out, using a nonpolynomial Lagrangian given by Weinberg. We include the multipion intermediate states in the static Bethe-Salpeter equation and the N/D equations, and obtain results free from cutoff parameters. It is found that the asymptotic behavior of the πN scattering amplitude depends only on the minor coupling constant and decreases exponentially. The dynamical parameters calculated are $\gamma_{NN\pi} = 3f^2 \simeq 0.23$, $\gamma_{N^*N\pi} \approx 0.12$, and $m_{N^*} - m_N \approx 1.5 m_\pi$, as compared to the corresponding experimental numbers of 0.24, 0.12, and $2.1 m_\pi$.

I. INTRODUCTION

We begin with the following observations:

(1) Nonpolynomial Lagrangians seem to provide[1-5] a source of finite field theories without ghosts.[3] So far, however, the interest has been primarily of a formal nature, dealing with the finiteness of the calculations for various orders in the major coupling constant, the nature of singularities, and unitarity.

(2) One disturbing feature in connection with nonpolynomial Lagrangians is [5,3] that the self-energy terms in different orders of the major coupling constant diverge exponentially as the energy increases. A cautious hope has been expressed in this regard that the summing up of the ladder terms of different orders in the major coupling constant can overcome[2,3] this difficulty.

(3) In a different context,[4] a nonpolynomial Lagrangian was used in the tree approximation as a chiral realization of the soft-pion results. Weinberg[4] has written down a Lagrangian for the πN interaction which gives the correct πN scattering lengths in the soft-pion limit. However, no serious attempt has been made to calculate any other parameters of πN scattering.

(4) It is well known[6] that N and N^* can be shown to evolve together in a self-consistent reciprocal bootstrap framework of a static theory. This has been demonstrated in the N/D method,[6] as well as in the static Bethe-Salpeter equation.[7] But these calculations have the drawback that they need a cutoff so that one can calculate only the ratio of N and N^* coupling to the πN system but not the individual couplings or the masses. It may be that the need for a cutoff is a consequence of confining the intermediate states in these calculations to the elastic channel or introducing only one inelastic channel.[8] Many-particle intermediate states have been avoided due to technical difficulties.

Our contention is that the Weinberg Lagrangian[4] should be taken seriously for a more detailed description of at least the low-energy πN interaction, such as N and N^* masses and their coupling constants. We have performed a static-model calculation of πN scattering in the $I = \frac{1}{2}, \frac{3}{2}$ and $J = \frac{1}{2}, \frac{3}{2}$ channels, including the multipion intermediate states within the framework of the nonpolynomial Lagrangian of Weinberg. We find that the summing up of ladder terms in the major coupling, by the use of the static N/D method or the Bethe-Salpeter equation, converts an exponentially diverging amplitude into an exponentially decreasing amplitude. As a consequence of including the multipion intermediate states, our theory does not require any cutoff, and hence allows us to calculate all the low-energy parameters of πN scattering. Specifically we obtain

$$\gamma_{NN\pi} = 3f^2 \approx 0.23$$

and

$$m_{N^*} - m_N \approx 1.5 m_\pi$$

in addition to the well-known result[6]

$$\gamma_{NN\pi}/\gamma_{N^*N\pi} \approx 2\,.$$

These results compare very satisfactorily with the experimental numbers $\gamma_{NN\pi} \approx 0.24$, $m_{N^*} - m_N \approx 2.1 m_\pi$, and $\gamma_{N^*N\pi} \approx 0.12$. These calculations are applicable to other systems such as $\pi\omega$ and $\pi\pi$, as well as baryons with higher symmetries and spin, but will be reported elsewhere.

II. BETHE-SALPETER EQUATION FOR $(n\pi)N \to (m\pi)N$

The πN interaction Hamiltonian suggested by Weinberg[4] is

$$H_{\text{int}} = \sqrt{4\pi}\, \frac{f}{m_\pi} \bar{N} \gamma_\mu \gamma_5 \vec{\tau} N \cdot \frac{\partial_\mu \vec{\phi}}{1 + a^2 \phi^2}, \tag{1}$$

where $f^2 \approx 0.08$ and $a = 1/F_\pi \approx 0.8/m_\pi$. This differs from the usual interaction by the factor $(1+a^2\phi^2)^{-1}$. The term $\partial_\mu \vec{\phi}$ will create or annihilate a p-wave pion, while the $a^2\phi^2$ terms create or annihilate s-wave pions. The static Bethe-Salpeter equation for

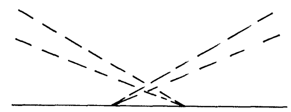

FIG. 1. The particle-exchange potential for Eq. (3).

$$N+\pi_0+(2r')\pi_1+(2p')\pi_2+(2n')\pi_3 \to N+\pi_0+(2r)\pi_1+(2p)\pi_2+(2n)\pi_3\,, \tag{2}$$

i.e., one p-wave pion and $2(r'+p'+n')$ pions in the s wave going to one p-wave pion and $2(r+p+n)$ in the s wave, is given by

$$\begin{aligned} &T(\omega_0', \omega_1', \ldots, \omega'_{2(r'+p'+n')}; \omega_0, \omega_1, \ldots, \omega_{2(r+p+n)}; E) \\ &\quad = \frac{\gamma^x F(r',p',n')F(r,p,n)}{m_x+\sum\omega_i'+\sum\omega_i - E} - \frac{\gamma^x}{\pi}\sum_{R,P,N}\int d\omega_0'' d\omega_1'' \cdots d\omega''_{2(R+P+N)}\rho_0(\omega_0'')\rho_1(\omega_1'')\cdots\rho_1(\omega''_{2(R+P+N)}) \\ &\qquad \times F(r',p',n')F(R,P,N)\,\frac{T(\omega_0'', \omega_1'', \ldots, \omega''_{2(R+P+N)}; \omega_0, \omega_1, \ldots, \omega_{2(r+p+n)}; E)}{(m_x+\sum\omega_i'+\sum\omega_i''-E)(E-\sum\omega_i''-m)}\,, \end{aligned} \tag{3}$$

where m_x is the mass of the exchanged particle,

$$\begin{aligned} &F(r,p,n) = \frac{(r+p+n)!}{r!\,p!\,n!}\,[(2r)!\,(2p)!\,(2n)!]^{1/2}a^{2(r+p+n)}, \\ &\rho_0(\omega_0'') = q_0''^3, \\ &\rho_1(\omega_i'') = \frac{q_i''}{(2\pi)^2}, \\ &q_i'' = (\omega_i''^2 - m_\pi^2)^{1/2}, \\ &\gamma^x = \alpha_{II'}\beta_{JJ'}\gamma_{I'J'}, \end{aligned} \tag{4}$$

α and β being the conventional πN static-model crossing matrices for isotopic spin and spin, respectively. The summation convention is not used for the indices X, I, or J, and I and J take values $\frac{1}{2}$ or $\frac{3}{2}$ depending on the exchanged particle. For the potential, we have taken terms corresponding to Fig. 1, the choice of which was motivated by the corresponding term for the elastic static Bethe-Salpeter equation.[7] For simplifying Eq. (3), we take $E = \omega + m$, and define

$$\sum\omega_i' \equiv \omega'\,, \qquad \sum\omega_i \equiv \omega\,, \qquad \Delta \equiv m_x - m,$$

$$T(\omega_0', \omega_1', \ldots, \omega'_{2(r',p',n')}; \omega_0, \omega_1, \ldots, \omega_{2(r+p+n)}; E) \equiv S(\omega', r', p', n'; \omega, r, p, n)F(r',p',n')F(r,p,n) \tag{5}$$

so that

$$\begin{aligned} S(\omega', r', p', n'; \omega, r, p, n) &= \frac{\gamma^x}{\Delta+\omega'} + \frac{\gamma^x}{\pi}\sum_{R,P,N}\int \rho_0(\omega_0'')\rho_1(\omega_1'')\rho_1(\omega''_{2(R+P+N)})d\omega_0''d\omega_1''\cdots d\omega''_{2(R+P+N)} \\ &\quad \times \frac{F(R,P,N)F(R,P,N)}{(\sum\omega_i''-\omega)(\Delta+\omega'-\omega+\sum\omega_i'')}\,S(\omega'', R, P, N; \omega, r, p, n,)\,. \end{aligned} \tag{6}$$

We use the representation

$$1 = \frac{1}{2\pi}\int_\mu^\infty d\omega''\int_{-\infty}^\infty dt\,\exp[it(\textstyle\sum\omega_i''-\omega'')], \tag{7}$$

in terms of which (6) can be rewritten as

$$S(\omega', r', p', n'; \omega, r, p, n) = \frac{\gamma^x}{\Delta + \omega'} + \frac{\gamma^x}{2\pi^2} \sum_{R,P,N} \int_\mu^\infty d\omega'' S(\omega'', R, P, N; \omega, r, p, n) \frac{1}{(\Delta + \omega' + \omega'' - \omega)(\omega'' - \omega)}$$
$$\times \int_{-\infty}^\infty dt \int \exp[it(\textstyle\sum \omega_i'' - \omega'')] d\omega_0'' \, d\omega_1'' \cdots d\omega''_{2(R+P+N)}$$
$$\times F^2(R, P, N) \rho_0(\omega_0'')\rho_1(\omega_1'') \cdots \rho_1(\omega''_{2(R+P+N)}) . \tag{8}$$

We notice that S is independent of the indices r, p, n, etc., so that we can write

$$S(\omega', \omega) = \frac{\gamma^x}{\Delta + \omega'} + \frac{\gamma^x}{\pi} \int_\mu^\infty \frac{d\omega'' S(\omega'', \omega)\rho(\omega'')}{(\Delta + \omega' + \omega'' - \omega)(\omega'' - \omega)}, \tag{9}$$

$$\rho(\omega'') = \frac{1}{2\pi} \int_{-\infty}^\infty dt \, e^{-it\omega''} g_0(t) \sum_{R,P,N} \frac{[(R+P+N)!]^2 (2R)!(2P)!(2N)!}{[R!\,P!\,N!\,]^2} [a^2 g_1(t)]^{2(R+P+N)}, \tag{10}$$

where

$$g_0(t) = \int_\mu^\infty q_0^3 e^{it\omega_0} d\omega_0 ,$$
$$g_1(t) = \frac{1}{(2\pi)^2} \int_\mu^\infty q_1 e^{it\omega_1} d\omega_1 . \tag{11}$$

This equation is very similar to the elastic static Bethe-Salpeter equation, except that the phase space $\rho(\omega'')$ instead of being q''^3 is given by (10).

Now, though $g_1(t)$ is not well defined, we can formally sum up the series by noting that

$$\sum_{R,P,N=0} \frac{[(R+P+N)!]^2 (2R)!(2P)!(2N)!}{[R!\,P!\,N!\,]^2} x^{(R+P+N)} = \sum_{n=0}^\infty (2n+1)!\, x^n$$
$$= \int_0^\infty \frac{u\, e^{-u}}{1 - xu^2} du . \tag{12}$$

Hence

$$\rho(\omega'') = \frac{1}{2\pi} \int_{-\infty}^\infty dt \, e^{-it\omega''} g_0(t) \int_0^\infty \frac{u\, e^{-u}}{1 - u^2 a^4 [g_1(t)]^2} du . \tag{13}$$

III. ANALYTIC CONTINUATION

In order to give meaning to (13), let us redefine

$$g_0(t) = \lim_{t_0 \to 0} \int_\mu^\infty q_0^3 e^{it\omega_0} e^{-u^{1/2} t_0 \omega_0} d\omega_0 ,$$
$$g_1(t) = \lim_{t_0 \to 0} \frac{1}{4\pi^2} \int_\mu^\infty q_1 e^{it\omega_1} e^{-u^{1/2} t_0 \omega_1} d\omega_1 , \tag{14}$$

where t_0 is a constant. Furthermore we take $\mu = 0$, for simplification, which leads to

$$g_0(t) \approx \lim_{t_0 \to 0} \frac{6}{(t + iu^{1/2} t_0)^4} ,$$
$$g_1(t) \approx \lim_{t_0 \to 0} \frac{-1}{4\pi^2 (t + iu^{1/2} t_0)^2} . \tag{15}$$

We interchange the order of integration, and define

$$\rho(\omega'') = \lim_{t_0 \to 0} \rho(\omega'', t_0) ,$$
$$\rho(\omega'', t_0) = \frac{3}{\pi} \int_0^\infty u\, e^{-u} du \int_{-\infty}^\infty \frac{e^{-it\omega''} dt}{(t + iu^{1/2} t_0)^4 - u^2 (a/2\pi)^4} . \tag{16}$$

It should be noted that the summation in (12) and (13) is legitimate, provided $t_0 > a/2\pi$. This is reflected by the amplitude having a branch point at $t_0 = a/2\pi$ with the branch cut along the real axis for $t_0 < a/2\pi$. Therefore, there is an ambiguity in continuation to $t_0 = 0$. For the choice of the proper branch, we are guided by the knowledge that $S(\omega, \omega)$ is real for $\omega \leq \mu$.

The t integration in (16) yields, for $\omega'' > 0$,

$$\rho(\omega'', t_0) = \tfrac{3}{2}\int_0^\infty \frac{u\, e^{-u}\, e^{-\omega'' t_0 u^{1/2}}}{\delta^3} du (e^{\omega''\delta} - e^{-\omega''\delta} - 2\sin\omega''\delta), \tag{17}$$

where $\delta = (a/2\pi)u^{1/2}$. With this expression for $\rho(\omega'', t_0)$, we analytically continue Eq. (9) to obtain

$$S(\omega', \omega) = \frac{\gamma^x}{\Delta + \omega'} + i\frac{\gamma^x \rho(\omega)}{\Delta + \omega'} S(\omega, \omega)\theta(\omega) + \frac{\gamma^x}{\pi}\text{p.v.}\int_0^\infty d\omega'' \left(\frac{S(\omega'', \omega)\rho_1(\omega'')}{(\Delta + \omega' + \omega'' - \omega)(\omega'' - \omega)} + \frac{S(-\omega'', \omega)\rho_2(\omega'')}{(\Delta + \omega' - \omega'' - \omega)(\omega'' + \omega)} \right), \tag{18}$$

where p.v. stands for principal-value integral and

$$\rho_1(\omega'') = -\tfrac{3}{2}\int_0^\infty \frac{u\, e^{-u}}{\delta^3}(2\sin\omega''\delta + e^{-\omega''\delta}) du,$$
$$\rho_2(\omega'') = \tfrac{3}{2}\int_0^\infty \frac{u\, e^{-u}}{\delta^3} e^{-\omega''\delta} du. \tag{19}$$

Asymptotically, the phase-space functions $\rho_i(\omega'')$ go as $1/\omega''$, so that our integral equation (18) does not need any cutoff. In a general sort of way, $(2\pi/a)$ acts as an in-built cutoff parameter. It may be noted that the lowest-order iterations of (18) agree with the direct calculation of the diagrams.

IV. SOLUTION TO THE EQUATION

For solving the integral equation (18), we use the Noyes[9] method. Let us write

$$S(\omega', \omega) = f(\omega', \omega) S(\omega), \tag{20}$$

which leads to

$$S(\omega) = \frac{\gamma^x}{\Delta + \omega} d^{-1}(\omega), \tag{21}$$

$$d(\omega) = 1 - \frac{i\gamma^x \rho(\omega)}{\Delta + \omega} - \frac{\gamma^x}{\pi}\text{p.v.}\int_0^\infty d\omega'' \left(\frac{f(\omega'', \omega)\rho_1(\omega'')}{(\Delta + \omega'')(\omega'' - \omega)} + \frac{f(-\omega'', \omega)\rho_2(\omega'')}{(\Delta - \omega'')(\omega'' + \omega)} \right), \tag{22}$$

$$f(\omega', \omega) = \frac{\Delta + \omega}{\Delta + \omega'} + \frac{\gamma^x}{\pi}\text{p.v.}\int_0^\infty d\omega'' \left[\frac{f(\omega'', \omega)\rho_1(\omega'')}{\omega'' - \omega} \left(\frac{1}{\Delta + \omega'' + \omega' - \omega} - \frac{\Delta + \omega}{(\Delta + \omega')(\Delta + \omega'')} \right) + \frac{f(-\omega'', \omega)\rho_2(\omega'')}{\omega'' + \omega} \left(\frac{1}{\Delta - \omega'' + \omega' - \omega} - \frac{\Delta + \omega}{(\Delta + \omega')(\Delta - \omega'')} \right) \right]. \tag{23}$$

It is easy to see that, as $\omega \to \infty$,

$$S(\omega) \underset{\omega\to\infty}{\sim} \frac{2i(a/2\pi)^3 e^{-\omega^2 (a/2\pi)^2/4}}{3\pi^{1/2}}, \tag{24}$$

which is independent of the major coupling γ^x and goes to zero rapidly. The amplitude is well behaved for $\omega \to \infty$, unlike the exponentially divergent terms of perturbation diagrams.

As a first approximation for solving (23), let us use the determinantal approximation,

$$f(\omega', \omega) \approx \frac{\Delta + \omega}{\Delta + \omega'}. \tag{25}$$

With this approximation, $d(\omega)$ is evaluated and we look for the zeros of the $d(\omega)$. This allows one to locate the poles of $S(\omega)$ and analyze the residues.

Let us first consider the πN scattering in the $(\frac{3}{2}, \frac{3}{2})$ channel. The forces in this channel are provided primarily by the N exchange. From the crossing matrix one has

$$\gamma^x = \tfrac{4}{9}\gamma, \tag{26}$$

and $\Delta = m_x - m = 0$, where γ is related to πNN coupling. The study of the zeros of $d(\omega)$ then gives the N^* mass and coupling constant as a function of γ. The relations, for the linearized $d(\omega) \approx 1 + c\omega$, are particularly simple:

$$\gamma^* \approx \tfrac{4}{9}\gamma,$$
$$\gamma(m_{N^*} - m_N) \approx 2.7(a/2\pi), \tag{27}$$

where γ^* is the $N^*N\pi$ coupling constant m_{N^*} is the mass of N^*.

For a complete bootstrap of the N, N^* system, we repeat the calculations for πN scattering in the $(\frac{1}{2}, \frac{1}{2})$ channel. The forces in this channel are provided by the N^* exchange. From the crossing matrix, one has

$$\gamma^x = \tfrac{16}{9}\gamma^*, \tag{28}$$

and $\Delta = m_{N^*} - m_N$. The zero in the linearized $d(\omega) \approx 1 + b(\omega + \Delta)$ gives

$$\begin{aligned} &\gamma \approx \tfrac{16}{9}\gamma^*, \\ &\tfrac{16}{9}\gamma^*(m_{N^*} - m_N)\left(\frac{5\pi}{3a} - 2(m_{N^*} - m_N)\right) \approx 1 . \end{aligned} \tag{29}$$

We solve (27) and (29) to obtain

$$\begin{aligned} &\gamma \approx 2\gamma^*, \\ &m_{N^*} - m_N \approx 1.5 m_\pi, \\ &\gamma \approx 0.23 . \end{aligned} \tag{30}$$

These results should be compared with the experimental numbers, $\gamma \approx 0.24$, $\gamma^* \approx 0.12$, and $m_{N^*} - m_N \approx 2.1 m_\pi$. The first of the relations (30) is the well-known Chew result.[6] The other two results are new and are obtained since we do not require a cutoff in our theory. We feel that the agreement with the experimental numbers is encouraging and leads us to believe that the nonpolynomial Lagrangian approach allows us to perform meaningful calculations in the domain of strong interaction.

One may improve the approximation (25) by taking the next iteration or using one of the more complicated approximations such as the Balázs[10] method. However, this introduces unnecessary complications without changing the qualitative nature of the results, and we do not consider them here.

V. THE N/D METHOD

The analysis of the reciprocal bootstrap can be carried out by using the N/D method. Here, we start with the unitarity relation

$$\begin{aligned} &\operatorname{Im} T(\omega_0', \omega_1', \ldots, \omega_{2(r'+p'+n')}'; \omega_0, \omega_1, \ldots, \omega_{2(r+p+n)}) \\ &= \sum_{R,P,N} \int T^*(\omega_0', \omega_1', \ldots, \omega_{2(r'+p'+n')}'; \omega_0'', \omega_1'', \ldots, \omega_{2(R+P+N)}'') \\ &\quad \times T(\omega_0'', \omega_1'', \ldots, \omega_{2(R+P+N)}''; \omega_0, \omega_1, \ldots, \omega_{2(r+p+n)}) \frac{d^3q_0''\, d^3q_1'' \cdots d^3q_{2(R+P+N)}'' \omega_0''^2 \delta(\omega_0'' + \omega_1'' + \cdots + \omega_{2(R+P+N)}'' - \omega)}{(2\omega_0'')(2\omega_1'') \cdots (2\omega_{2(R+P+N)}'')(2\pi)^{3(2R+2P+2N)}} . \end{aligned} \tag{31}$$

As before in (15), we define

$$T(\omega_0', \omega_1', \ldots, \omega_{2(r'+p'+n')}'; \omega_0, \omega_1, \ldots, \omega_{2(r+p=n)}) \equiv S(\omega) F(r', p', n') F(r, p, n), \tag{32}$$

where $F(r, p, n)$ is defined in (4) and

$$\omega = \sum_i \omega_i' = \sum_i \omega_i . \tag{33}$$

Then the phase space can be written in a factorizable form by using the representation

$$\delta(\omega_0 + \omega_1 + \cdots + \omega_l - \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp[-it(\omega - \omega_0 - \omega_1 - \cdots - \omega_l)]dt . \tag{34}$$

The "l-particle" phase space is given by

$$\begin{aligned} \rho_l(\omega) &\equiv \int \frac{d^3q_0 d^3q_1 \cdots d^3q_l \omega_0^2 \delta(\omega_0 + \omega_1 + \cdots + \omega_l - \omega)}{(2\omega_0)(2\omega_1)\cdots(2\omega_l)(2\pi)^{3l}} \\ &= \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{2\pi} g_0(t)[g_1(t)]^l dt . \end{aligned} \tag{35}$$

The index-free amplitude $S(\omega)$ then satisfies the unitarity relation

$$\operatorname{Im} S(\omega) = |S(\omega)|^2 \rho(\omega),$$

where $\rho(\omega)$ is defined in (10). We now define

$$S(\omega) = N(\omega)/D(\omega), \tag{36}$$

where $N(\omega)$ has only the left-hand singularities while $D(\omega)$ has the right-hand singularities. If $S_l(\omega)$ describes the left-hand singularities of $S(\omega)$, we can write

$$N(\omega) = \frac{1}{\pi}\int_l \frac{\operatorname{Im} S_l(\omega') D(\omega')}{\omega' - \omega} d\omega' , \tag{37}$$

$$D(\omega)=1-\frac{\omega-\bar{\omega}}{\pi}\int_0^\infty \frac{\rho(\omega')N(\omega')d\omega'}{(\omega'-\bar{\omega})(\omega'-\omega-i\epsilon)}, \tag{38}$$

where $\bar{\omega}$ is the subtraction point. As for the Bethe-Salpeter equation, the $\rho(\omega)$ as defined in (10) is not well defined. We, therefore, redefine $g_0(t)$ as in (14) which leads us to the definition (16) of $\rho(\omega'')$. Finally one analytically continues Eq. (38) from $t_0 > a/2\pi$ to $t_0=0$. The proper choice of the branch leads to

$$D(\omega)=1-i\rho(\omega)N(\omega)\theta(\omega) - \frac{\omega-\bar{\omega}}{\pi}\text{p.v.}\int_0^\infty d\omega'\left(\frac{N(\omega')\rho_1(\omega')}{(\omega'-\bar{\omega})(\omega'-\omega)} - \frac{N(-\omega')\rho_2(\omega')}{(\omega'+\bar{\omega})(\omega'+\omega)}\right). \tag{39}$$

The dispersion relations (37) and (39) allow us to calculate the scattering amplitude, once $S_l(\omega')$ is known.

For πN scattering in the $(\frac{3}{2},\frac{3}{2})$ channel, the dominant contribution to $S_l(\omega)$ comes from the nucleon exchange, which can be approximated as

$$\text{Im}S_l(\omega')\approx -\pi(\tfrac{4}{9}\gamma)\delta(\omega'). \tag{40}$$

If we further take $\bar{\omega}=0$, we get

$$N(\omega)\approx\frac{\frac{4}{9}\gamma}{\omega}, \tag{41}$$

and the $D(\omega)$ is identical to the $d(\omega)$ in (22) with the determinantal approximation (25). The linearization of $D(\omega)$ therefore leads to the relations (27). Similarly for the πN scattering in the $(\frac{1}{2},\frac{1}{2})$ channel, the dominant contribution from the N^* exchange gives

$$\text{Im}S_l(\omega')\approx -\pi(\tfrac{16}{9}\gamma^*)\delta(\omega'+m_{N^*}-m_N). \tag{42}$$

For this, we take the subtraction point at $\bar{\omega} = -(m_{N^*}-m_N)$, which leads to

$$N(\omega)\approx\frac{\frac{16}{9}\gamma^*}{\omega+m_{N^*}-m_N}. \tag{43}$$

The $D(\omega)$ is identical to the $d(\omega)$ of the Bethe-Salpeter equation with the determinantal approximation. The linearized $D(\omega)$ leads to the relations (29). The combined reciprocal bootstrap leads to the results (30). One may therefore regard the results of N/D method as a particular case of the Bethe-Salpeter equation in this case. The asymptotic behavior of $S(\omega)$ is once again given by (24), which is quite acceptable.

VI. DISCUSSION

We have carried out a dynamical reciprocal bootstrap calculation of N and N^*, using a nonpolynomial πN interaction given by Weinberg. In summing up the ladder terms using the Bethe-Salpeter equation or the N/D method, the inclusion of the multipion intermediate state allows us to do a parameter-free calculation. We find that the asymptotic behavior of the scattering amplitude is given by

$$S(\omega)\sim\frac{i2(a/2\pi)^3 e^{-\omega^2(a/2\pi)^2/4}}{3\pi^{1/2}},$$

an exponentially decreasing behavior in contrast to the exponentially increasing behavior for the individual perturbation terms. The asymptotic behavior is governed by only the minor coupling constant a, but independent of the major coupling constant. This opens up a possibility of the low-energy and the high-energy dynamics being dictated by differing strengths of interactions.

The results of our bootstrap calculations are $\gamma_{NN\pi}=3f^2\approx 0.23$, $\gamma_{N^*N\pi}\approx 0.12$, and $m_{N^*}-m_N \approx 1.5m_\pi$, which should be compared with the corresponding experimental numbers of 0.24, 0.12, and $2.1m_\pi$. We consider the agreement as encouraging. One can extend these results to SU(3) and higher-spin baryon resonances.

The above calculations were made within the framework of the static model, taking the pion mass to be zero. The effect of taking nonzero pion mass may be estimated from the conventional static theory with a cutoff. With a fixed cutoff of about $8m_\pi$, we find that increasing the mass of pion from zero to m_π increases $m_{N^*}-m_N$ by about 30% and decreases f^2 by about 10%, which further improves the agreement. It is hoped that our calculations are only a prelude to a relativistic calculation and that the nice features would survive such an extension.

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