$\bar{p}p$ Annihilation in the Multiperipheral Regge Model*

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Using the simple multi-Regge model of Chew and Pignotti, we obtain an acceptable fit to the bulk of $\bar{p}p$ annihilation data. The parameters of the fit are consistent with $\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}} \propto s^{\alpha_\omega(0)-1}$.

I. INTRODUCTION

Let $\sigma_{\bar{p}p}^{\text{tot}}$ and σ_{pp}^{tot} denote the total $\bar{p}p$ and pp cross sections. The difference $\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}}$ is well known to be positive at nonasymptotic energies.[1,2] A Regge analysis of imaginary part of the forward elastic amplitude, including the usual neglect of isospin dependence and assumptions concerning exchange degeneracy, leads one to the conclusion[3]

$$\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}} \propto s^{\alpha_\omega(0)-1}. \tag{1}$$

The data[1,2] also indicate that $\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}} \approx \sigma_{\bar{p}p}^{\text{annih}}$, where $\sigma_{\bar{p}p}^{\text{annih}} = \sum_{n=2}^{\infty} \sigma_{\bar{p}p \to n\pi}$. Ting[4] has argued on the basis of the generalized multi-Regge model (MRM) developed by Chew, Goldberger, and Low[5] (CGL) that the annihilation channels constructed from baryon-exchange diagrams (Fig. 1) can generate the difference (1) via unitarity. However, it remains to show this explicitly. That is, can a set of amplitudes such as shown in Fig. 1 both fit the annihilation data *and* sum to give the result (1)? This is our concern in this paper.

II. THE MODEL

For simplicity, we adopt the Chew-Pignotti[6] approximations to the MRM, and write down the cross-section formulas for the annihilation processes $\bar{p}p \to m\pi^- m\pi^+ k\pi^0$. To start with, we assume that only one $B=1$, $I=\frac{1}{2}$ trajectory with intercept α_B is present. We also assume duality, so that only pions are considered present in the final state. For $k=0$ (no π^0's), only a single graph contributes: the one in which the charged and neutral Reggeons form alternate links in the multiperipheral chain.

For $k \neq 0$, one must add all the permutations resulting from a distribution of the π^0 vertices along the chain. The result is that there are $(2m+k)!/(2m)!k!$ graphs. The "strong ordering" approximation[6,7,8] says that these amplitudes do not interfere.

Next introduce the SU(2)-invariant internal couplings, $(\frac{2}{3})^{1/2}g$ (for $\pi^\pm$) and $(\frac{1}{3})^{1/2}g$ (for π^0), and the end couplings $(\frac{2}{3})^{1/2}G$ and $(\frac{1}{3})^{1/2}G$. The kinematic approximations of Ref. 6 lead to the cross section formula

$$\sigma_{m,k} = \left[G^4 e^{(2\alpha_B - 2)Y} \frac{(g^2Y)^{2m+k-2}}{(2m+k-2)!} \left(\tfrac{1}{3}\right)^k \left(\tfrac{2}{3}\right)^{2m} \right] \times \left[\frac{(2m+k)!}{k!(2m)!} \right], \tag{2}$$

where $Y = \ln(s/M^2)$. In Eq. (2) the first bracket represents the cross section due to each graph, the second bracket the number of graphs.

We can also sum over k to find an expression for the topological cross section

$$\sigma_m \equiv \sum_{k=0}^{\infty} \sigma_{\bar{p}p \to m(\pi^-\pi^+)k\pi^0}.$$

The result is

$$\sigma_m = G^4 e^{(2\alpha_B - 2 + g^2/3)Y} \left[\frac{4}{9} \frac{(\frac{2}{3}g^2Y)^{2m-2}}{(2m-2)!} + \frac{4}{9} \frac{(\frac{2}{3}g^2Y)^{2m-1}}{(2m-1)!} + \frac{1}{9} \frac{(\frac{2}{3}g^2Y)^{2m}}{(2m)!} \right]. \tag{3}$$

The total annihilation cross section is

$$\sigma_{\text{annih}} = \sum_{m=1}^{\infty} \sigma_m \approx \tfrac{1}{2} G^4 e^{(2\alpha_B - 2 + g^2)Y}, \tag{4}$$

ignoring terms of $O(e^{-(2/3)g^2Y})$.

Finally, the charged and neutral average multiplicities may be obtained from Eqs. (2) and (3):

$$\langle n_{\text{ch}} \rangle = \sum_{m=1}^{\infty} (2m)\sigma_m / \sigma_{\text{annih}} = \tfrac{2}{3} g^2 Y + \tfrac{4}{3}, \tag{5a}$$

$$\langle n_0 \rangle = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} k\sigma_{m,k} / \sigma_{\text{annih}} = \tfrac{1}{3} g^2 Y + \tfrac{2}{3}. \tag{5b}$$

III. FITTING OF THE TOPOLOGICAL DATA

The extreme relativistic approximations inherent in the formulas (2)–(5) suggest that comparison with experiment be done at the highest energies available. If we also require that there exist topological data with good statistics at the energies considered, we are inevitably led to confront our model with the data at 3.28 GeV/c,[9] 5.70 GeV/c, and 6.94 GeV/c.[11,12] We deal with two types of

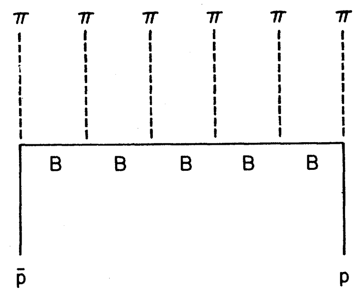

FIG. 1. The process $\bar{p}p \rightarrow$ pions with multi-baryon exchange.

data in this work: (1) topological cross sections as defined above, and (2) constrained cross sections [$\bar{p}p \rightarrow m(\pi^-\pi^+)$, which is quadruply constrained ("4c"), or $p\bar{p} \rightarrow m(\pi^-\pi^+)+\pi^0$, which is singly constrained ("1c")]. Since we are interested in seeing whether the annihilation process can indeed generate the $s^{\alpha_\omega(0)-1}$ behavior of $\sigma_{\bar{p}p}^{tot} - \sigma_{pp}^{tot}$, we would like to compare our formulas with the data representing the bulk of the annihilation cross section. Our procedure has been to obtain the parameters which provide a best fit to the *topological* cross sections, and then to compare the resulting predictions for the *constrained* cross sections with experiment.

Given three energies and four topological cross sections ($m = 1, 2, 3, 4$), there are *a priori* twelve data points. However, the 8-prong data at 3.28 and 5.70 GeV/c were not included for want of sufficient statistics, and the 2-prong data at 6.94 GeV/c were eliminated because of possible contamination by the presence of an $n\bar{n}$ pair in the final state.[11] Thus our fitting is done to nine data points.

In an initial attempt, α_B was fixed by identifying the baryon trajectory with the N_α, which is dominant in the two-body reactions $\pi p \rightarrow p\pi$ and $\bar{p}p \rightarrow \pi^-\pi^+$. A linear extrapolation (from the nucleon) of the N_α trajectory gives[13]

$$\alpha_B = \alpha_{N_\alpha}(0) = -0.38\,. \qquad (6)$$

If we then also impose $\sigma_{\text{annih}} \propto s^{\alpha_\omega(0)-1}$, the constraint

$$2\alpha_B - 1 + g^2 = \alpha_\omega(0) \simeq 0.4 \qquad (7)$$

follows from Eq. (4). The constraints (6) and (7) fix $g^2 = 2.16$. This attempt ended when the "best" fit obtained through varying G^4 gave a $\chi^2 \simeq 81$ for 8 degrees of freedom (d.f.).

Solution 1. As a next step, the "bootstrap" constraint [Eq. (7)] was relaxed, but Eq. (6) was retained. A best fit was obtained for $g^2 = 1.76$ (χ^2/d.f. = 10.9/7). This leads to $\sigma_{\text{annih}} \propto s^{-1}$, badly violating the bootstrap.

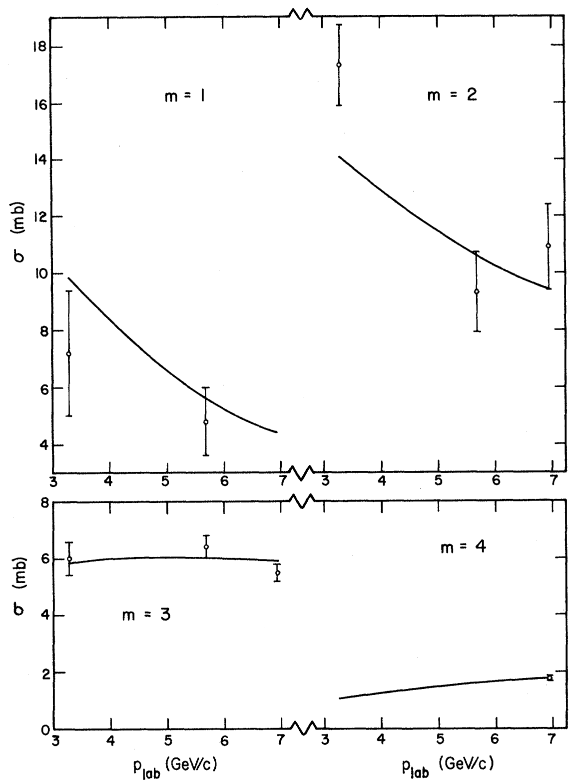

FIG. 2. Fit to topological cross sections $\sigma_m = \sum_{k=0}^{\infty} \sigma_{\bar{p}p\rightarrow m(\pi^-\pi^+)k\pi^0}$. The solid curve is solution 2.

Solution 2. We then relaxed the "intercept" condition (6) but retained the "bootstrap" constraint (7). The resulting best fit, with parameters $g^2 = 1.68$, $\alpha_B = -0.14$, and $G^4 = 240$, is also acceptable (χ^2/d.f. = 13.0/7).

A graphic comparison of solution 2 with the experimental topological cross sections[14] is presented in Fig. 2. The agreement is pleasing, especially when one considers the order-of-magnitude variation in the cross section, and that the MRM curves follow pretty well the very different qualititative behavior of the different topologies.

IV. CONSTRAINED REACTIONS

Substituting the parameters for solutions 1 and 2 into Eq. (2) with $k = 0, 1$ leads to predictions for the constrained reactions.[15] These are shown in Fig. 3 for solution 2. There is good quantitative agreement with three of the larger cross sections ($\sigma_{2,1}$ and $\sigma_{3,0}$ at 3.29 GeV/c, and $\sigma_{3,1}$ at 6.94 GeV/c). There is also qualitative agreement with the consistent dominance of $\sigma_{m,1}$ over $\sigma_{m,0}$. [From Eq. (2), $\sigma_{m,1}/\sigma_{m,0} \simeq \frac{1}{3}g^2 Y$ for $m \gtrsim 3$.]

Since the present work is concerned with the gross properties of the annihilation cross section,

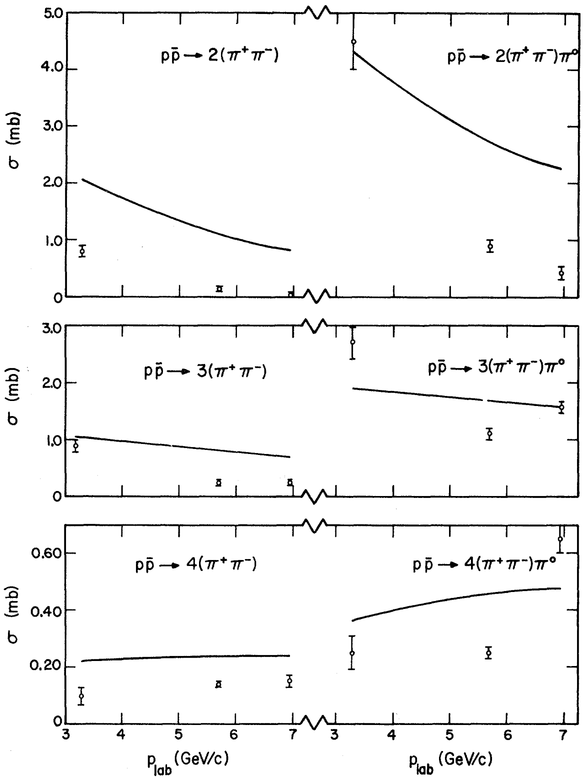

FIG. 3. Constrained cross sections resulting from solution 2.

and the constrained cross sections account for less than $\frac{1}{3}$ of σ_{annih}, we do not at this time concern ourselves with the lack of over-all quantitative agreement for $\sigma_{m,0}$ and $\sigma_{m,1}$.

V. MULTIPLICITY

The average charged multiplicity may be calculated from Eq. (5a). For solution 2, the calculated multiplicities are 3.83 at 3.28 GeV/c, 4.28 at 5.70 GeV/c, and 4.50 at 6.94 GeV/c, in very good agreement with the experimental values 4.0 ± 0.3,[9] 4.3 ± 0.2,[10] and 4.5 ± 0.2[11,12] respectively.

VI. DISCUSSION OF RESULTS

We consider here the physical interpretation of solutions 1 and 2.

The parameters of solution 1 would have $\sigma_{\text{annih}} \propto s^{-1}$, contradicting the bootstrap hypothesis. A behavior $\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}} \propto s^{-1}$ is very difficult to understand theoretically. The possibility of a daughter (presumably of the Pomeranchukon) does not arise, because the Pomeranchukon has an O(4) quantum number $M = 0$, and its daughters lie at $-1, -3, \ldots$. Thus the leading exchange with intercept 0 is the $I = 1$ π-A_1 trajectory. An $I = 1$ exchange is very difficult to reconcile with the annihilation being responsible for $\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}}$, because then we would have $\sigma_{\bar{n}p}^{\text{annih}} \approx \sigma_{\bar{n}p}^{\text{tot}} - \sigma_{np}^{\text{tot}} \approx -(\sigma_{\bar{p}p}^{\text{tot}} - \sigma_{pp}^{\text{tot}}) \approx -\sigma_{\text{annih}}$, a meaningless result. It is possible that the kinematic and other approximations which are made in the Chew-Pignotti approach are too severe in the energy range considered, and that a best fit to future data at higher energies would yield a value of g^2 closer to 2.16, thus satisfying the bootstrap constraint. But for the present, we reject solution 1.

In accepting solution 2, we must find some way of accommodating our intercept $\alpha_B = -0.14$. What is the experimental situation? A best fit to the energy dependence of the backward $\pi^+ p$ data $(d\sigma/du)_{u=0}$ between 6 and 17 GeV/c (Ref. 16) yields an effective intercept $\alpha(0) = -0.20 \pm 0.05$. The same experiment established the dominance of N over Δ exchange, and hence we can identify this value as the intercept of the N (Ref. 17) trajectory, and also consistent with our result. Recent higher-energy data[18] seem to confirm this departure of the intercept from the value -0.38 obtained[13] by linear extrapolation from the Chew-Frautschi plot.

Since $\alpha_B = -0.14$ lies about halfway between -0.38 and $\alpha_\Delta(0) \simeq 0.14$, one may also consider the possibility that solution 2 mocks up the situation where both N and Δ occur in the multiperipheral chain. We know[13] that Δ is suppressed at the end links, but there is very little evidence that the two-Reggeon–pion couplings $N\Delta\pi$ and $\Delta\Delta\pi$ are small[19]; indeed, we know that when *both* N and Δ are on shell at least the $N\Delta\pi$ coupling is comparable to the $NN\pi$ coupling (the "reciprocal bootstrap"[20]).

The cross-section formulas (2)–(4) are incomplete if Δ is present; not only are there two additional g parameters, but the complex combinatorics for $I = \frac{3}{2}$ preclude the possibility of simple formulas. Nonetheless, we may examine whether our results represent some average over the contributions of the two trajectories.

As a simple example of a model which would yield our result, we may consider the bootstrap condition of the CGL multiperipheral equation[5] generalized to the case where two trajectories are present.[21] Neglecting isospin coefficients, one obtains for α_A, the $t = 0$ intercept of the leading trajectory dominating the absorptive part due to annihilation,

$$\alpha_A = \alpha_N + \alpha_\Delta + g_{NN}{}^2 + g_{\Delta\Delta}{}^2 + \tfrac{1}{2}[(\alpha_N - \alpha_\Delta)^2 + 4 g_{N\Delta}{}^4]^{1/2} - 1 . \tag{8a}$$

In the approximation $g_{N\Delta}{}^4 \gg \frac{1}{4}(\alpha_N - \alpha_\Delta)^2 \simeq 0.06$, we obtain

$$\alpha_A \simeq \alpha_N + \alpha_\Delta + (g_{NN}{}^2 + g_{\Delta\Delta}{}^2 + g_{N\Delta}{}^2) - 1 . \tag{8b}$$

We can then identify $g_{NN}{}^2 + g_{\Delta\Delta}{}^2 + g_{N\Delta}{}^2$ with our value g^2, α_A with $\alpha_\omega(0)$, and $\frac{1}{2}(\alpha_N + \alpha_\Delta)$ with our effective baryon intercept $\alpha_B = -0.14$. In a linear approximation to the trajectories $\frac{1}{2}(\alpha_N + \alpha_\Delta) \simeq -0.13$ (Ref. 13), consistent with our value of α_B.

However, in view of all the uncertainties surrounding the trajectory intercepts, it is our distinct preference at this time to simply conclude that the result $\alpha_B = -0.14$ is consistent with -0.20 ± 0.05, the N_α intercept found in fitting the energy dependence of the backward $\pi^+ p$ scattering.

Our main goal has been accomplished: We have shown that the MRM provides a fair fit to the annihilation data *with parameters which would sustain the difference* $\sigma_{\bar{p}p}^{tot} - \sigma_{pp}^{tot} \propto s^{\alpha_\omega(0)-1}$ *observed at very high energy.*

ACKNOWLEDGMENT

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[14]These were obtained by adding the constrained and unconstrained cross sections in Refs. 9–12.

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