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Hyperon-Nucleon Scattering. I. Invariant and Helicity Amplitudes

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We investigate the invariant and helicity amplitudes for $A+B \rightarrow C+D$, where A, B, C, and D are $J^P = \frac{1}{2}^+$ particles. New variables are introduced to simplify the otherwise complicated expressions for the regularized, parity-conserving helicity amplitudes in all three channels. Simple algebraic methods are used to derive the equivalence theorems and the Fierz transformations for four spinors. The reduction of this general formalism to describe hyperon-nucleon scattering (e.g., $\Sigma N \rightarrow \Lambda N$, $\Sigma N \rightarrow \Sigma N$) is immediate.

I. INTRODUCTION

The study of hyperon-nucleon scattering is a natural extension of the old and thoroughly studied nucleon-nucleon problem. For example, one might like to see a strange (virtual) bound state similar to the deuteron in the low-energy region. Also, at high energies, a Regge-pole and/or -cut analysis may be appropriate; hence more experiments with high-energy hyperon beams are welcome.

There exist good review articles on hyperon-nucleon scattering.[1] It is clear that the kinematics is fairly complicated. On the other hand, the problem of anomalous thresholds,[2] which do occur in the s channel of $\Sigma N \rightarrow \Sigma N$ and $\Sigma N \rightarrow \Lambda N$ (but not $\Lambda N \rightarrow \Lambda N$), must be settled, in view of its potentially important effects on the low-energy parameters.

In this first part of two articles on hyperon-nucleon scattering, we try to *standardize* the invariant and helicity amplitudes for a general $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$ (all positive parity) scattering,[3] by examining their relations in all three (s, t, and u) channels. The reduction of the eight independent invariant amplitudes to six for elastic scattering (e.g., $\Sigma N \rightarrow \Sigma N$), and to five for $NN \rightarrow NN$, is well known. It also emerges quite naturally in the helicity formalism.

Invariant amplitudes are most suitable for a dispersion-relation approach. The absence of kinematic singularities in the invariant amplitudes can be directly "proved" by examining the explicit s-, t-, and u-channel helicity amplitudes (h.a.). Helicity amplitudes, in addition, are feasible for a partial-wave decomposition in both the direct and the crossed channels, which is necessary for a full phenomenological analysis in the low- and high-energy regions, respectively.

When evaluating the s-channel h.a., say, $[\bar{u}(p_2)u(p_1)][\bar{u}(k_2)u(k_1)]$, in the center-of-mass (c.m.) frame of p_1 (p_2) and k_1 (k_2), the resulting expressions are very messy. Two reasons are responsible for this complexity: (i) p_1 (k_1) and p_2 (k_2) are not collinear, and (ii) they do not have the same magnitude of momentum.[4] The situation changes completely in the t-channel c.m. frame of k_1 and $-k_2$ (to evaluate $[\bar{u}(p_2)v(p_1)][\bar{v}(k_2)u(k_1)]$), as well as in the u-channel c.m. frame of k_1 and $-p_2$ (to evaluate $[\bar{u}(k_2)v(p_1)][\bar{v}(p_2)u(k_1)]$). These latter circumstances therefore simplify the algebraic manipulation greatly. This fact makes possible an algebraic derivation of the *equivalence theorems*[3] and the Fierz transformations,[3] by means of the t- and u-channel h.a., respectively.

In Sec. II we evaluate the s-channel h.a. in terms of eight invariant amplitudes for a general $A+B \rightarrow C+D$. The specialization to hyperon-nucleon scattering can be easily made. In Sec. III the h.a. in the t channel are calculated. As has been mentioned, the resulting expressions are simpler due to the collinearity in the momenta. This allows for a simple algebraic derivation of the equivalence theorems for four spinors. The same observation leads to another algebraic derivation of Fierz transformations using the u-channel h.a. in Sec. IV. The correctness of these u-channel h.a. (as assured by yielding correct Fierz transformations) serves to check the algebra of the complicated s-channel h.a. (Sec. II) via the "crossing symmetry" between s and u. A summary is given in Sec. V. Finally, the Appendix lists the definition of the eight regularized, parity-conserving h.a. in all three channels.

II. s-CHANNEL AMPLITUDES

Consider the process

$$A(p_1, \lambda_1)+B(k_1, \mu_1)\rightarrow C(p_2, \lambda_2)+D(k_2, \mu_2), \qquad (1)$$

where A, B, C, and D are $J^P = \frac{1}{2}^+$ particles with momenta and helicities (p_1, λ_1), (k_1, μ_1), (p_2, λ_2), and (k_2, μ_2), respectively. The s-channel scattering amplitude T^s can be decomposed into eight invariant amplitudes:

$$\langle p_2, \lambda_2; k_2, \mu_2 | T^s | p_1, \lambda_1; k_1, \mu_1 \rangle$$
$$= \sum_{i=1}^{8} [\bar{u}_C(p_2, \lambda_2) O_i^{(1)} u_A(p_1, \lambda_1)] \times [\bar{u}_D(k_2, \mu_2) O_i^{(2)} u_B(k_1, \mu_1)] F_i(s, t)$$
$$= \sum_{i=1}^{8} [\bar{u}_C \bar{u}_D (O_i^{(1)} \otimes O_i^{(2)}) u_A u_B] F_i(s, t)$$
$$= \sum_{i=1}^{8} [\bar{u}_C \bar{u}_D O_i u_A u_B] F_i(s, t)$$
$$= \sum_{i=1}^{8} [\bar{u}_C \bar{u}_D O_i' u_A u_B] F_i'(s, t), \qquad (2)$$

with

$$O_1 = 1 \otimes 1, \qquad O_2 = \gamma_5 \otimes \gamma_5,$$
$$O_3 = \gamma_\mu \otimes \gamma_\mu, \qquad O_4 = i\gamma_5\gamma_\mu \otimes i\gamma_5\gamma_\mu,$$
$$O_5 = \tfrac{1}{2}\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}, \qquad O_6 = i\gamma \cdot K \otimes 1 - 1 \otimes i\gamma \cdot P, \qquad (3)$$
$$O_7 = \gamma_5 i\gamma \cdot K \otimes \gamma_5 + \gamma_5 \otimes \gamma_5 i\gamma \cdot P,$$
$$O_8 = \gamma_5 i\gamma \cdot K \otimes \gamma_5 - \gamma_5 \otimes \gamma_5 i\gamma \cdot P,$$

where $P = \frac{1}{2}(p_1 + p_2)$ and $K = \frac{1}{2}(k_1 + k_2)$.[5] The second set O_i' of spinor covariants is related to the first set O_i by

$$O_i' = O_i, \quad i = 1 \text{ to } 6,$$
$$O_7' = \tfrac{1}{2}(O_7 + O_8) = \gamma_5 i\gamma \cdot K \otimes \gamma_5, \qquad (4)$$

and

$$O_8' = \tfrac{1}{2}(O_7 - O_8) = \gamma_5 \otimes \gamma_5 i\gamma \cdot P.$$

It will be shown later that F_7' and F_8' are more convenient than F_7 and F_8 from the point of view of $s \leftrightarrow u$ crossing, when $A = C$ (or $B = D$).

To evaluate the helicity amplitudes in the c.m. frame [Fig. 1(a)], we use the phase convention of Jacob and Wick.[6] The typical spinor for $A(p_1, \lambda_1)$ is

$$u_A(p_1, \lambda_1) = [2m_A(E_A + m_A)]^{-1/2} \begin{pmatrix} E_A + m_A \\ 2p_A\lambda_1 \end{pmatrix}, \qquad (5)$$

where m_A is the mass of particle A,

$$E_A = (s + m_A{}^2 - m_B{}^2)/2s^{1/2}, \qquad (6)$$

and

$$p_A{}^2 = [s - (m_A - m_B)^2][s - (m_A + m_B)^2]/4s. \qquad (7)$$

The other three spinors can be obtained from (5) by performing proper rotations and by multiplying with suitable phases.

Because of the unequal-mass kinematics, a straightforward calculation of the h.a. with (2) and (5) is rather awkward. We therefore introduce the following *quasi*variables between a pair of *collinear* particles (A and B, C and D, in the s channel):

$$E_{ij} = \tfrac{1}{2}[s - (m_i - m_j)^2]^{1/2},$$
$$m_{ij} = E_{ij}(m_i + m_j)/s^{1/2}, \qquad (8)$$
$$p_{ij} = \tfrac{1}{2}[s - (m_i + m_j)^2]^{1/2},$$
$$\Delta_{ij} = p_{ij}(m_i - m_j)/s^{1/2},$$

which reduce to E_i, m_i, p_i, and zero, respectively, when $m_i = m_j$. (Note that $\Delta_{ij} = -\Delta_{ji}$.) Although just E_{AB}, p_{AB}, E_{CD}, and p_{CD} should be enough, we add m_{AB}, Δ_{AB}, m_{CD}, and Δ_{CD} to facilitate easy manipulation in the actual calculation. It also eliminates the explicit s dependence of the h.a., and maintains a high symmetry in the resulting expressions.

The eight regularized, parity-conserving h.a. (see the Appendix) thus read

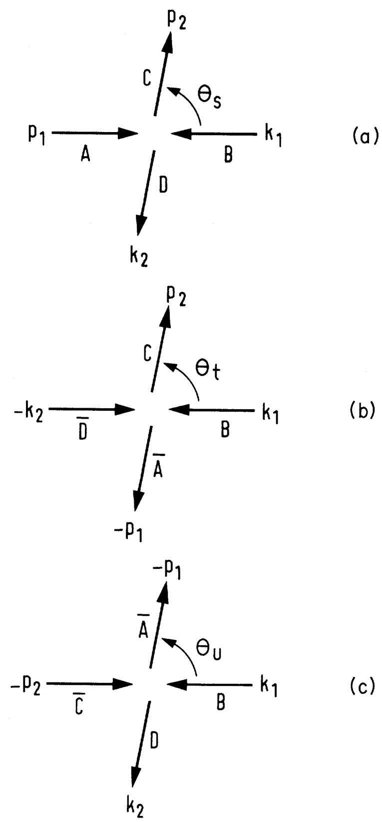

FIG. 1. c.m. frames for the s, t, and u channels.

$$f_1 = -F_1(p_{AB}p_{CD} + \Delta_{AB}\Delta_{CD}) - F_2(p_{AB}p_{CD} - \Delta_{AB}\Delta_{CD}) + 2F_3(2p_{AB}p_{CD} - \Delta_{AB}\Delta_{CD}) - 2F_4(2p_{AB}p_{CD} + \Delta_{AB}\Delta_{CD})$$
$$+ 6F_5 p_{AB}p_{CD} + F_6[p_A E_{AB}\Delta_{CD} + p_C \Delta_{AB} E_{CD} + W(p_{AB}\Delta_{CD} + \Delta_{AB}p_{CD})]$$
$$+ F_7[-p_A m_{AB} p_{CD} + p_C p_{AB} m_{CD} + D(p_{AB}\Delta_{CD} - \Delta_{AB}p_{CD})]$$

$$+F_8[-p_A E_{AB}\Delta_{CD}+p_C\Delta_{AB}E_{CD}-W(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})]$$
$$+\cos\theta_s\{F_1(E_{AB}E_{CD}+m_{AB}m_{CD})+F_2(E_{AB}E_{CD}-m_{AB}m_{CD})+2(F_3+F_4)m_{AB}m_{CD}+2F_5E_{AB}E_{CD}$$
$$+F_6[p_A\Delta_{AB}E_{CD}+p_C E_{AB}\Delta_{CD}+D(E_{AB}m_{CD}+m_{AB}E_{CD})]$$
$$+F_7[p_A p_{AB}m_{CD}-p_C m_{AB}p_{CD}+W(E_{AB}m_{CD}-m_{AB}E_{CD})]$$
$$+F_8[p_A\Delta_{AB}E_{CD}-p_C E_{AB}\Delta_{CD}-D(E_{AB}m_{CD}-m_{AB}E_{CD})]\},$$

$$f_2=F_1(E_{AB}E_{CD}+m_{AB}m_{CD})+F_2(E_{AB}E_{CD}-m_{AB}m_{CD})+2F_3(2E_{AB}E_{CD}-m_{AB}m_{CD})$$
$$-2F_4(2E_{AB}E_{CD}+m_{AB}m_{CD})-6F_5E_{AB}E_{CD}+F_6[p_A\Delta_{AB}E_{CD}+p_C E_{AB}\Delta_{CD}+D(E_{AB}m_{CD}+m_{AB}E_{CD})]$$
$$+F_7[p_A p_{AB}m_{CD}-p_C m_{AB}p_{CD}+W(E_{AB}m_{CD}-m_{AB}E_{CD})]$$
$$+F_8[p_A\Delta_{AB}E_{CD}-p_C E_{AB}\Delta_{CD}-D(E_{AB}m_{CD}-m_{AB}E_{CD})]$$
$$+\cos\theta_s\{-F_1(p_{AB}p_{CD}+\Delta_{AB}\Delta_{CD})-F_2(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})+2(F_3+F_4)\Delta_{AB}\Delta_{CD}-2F_5p_{AB}p_{CD}$$
$$+F_6[p_A E_{AB}\Delta_{CD}+p_C\Delta_{AB}E_{CD}+W(p_{AB}\Delta_{CD}+\Delta_{AB}p_{CD})]$$
$$+F_7[-p_A m_{AB}p_{CD}+p_C p_{AB}m_{CD}+D(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})]$$
$$+F_8[-p_A E_{AB}\Delta_{CD}+p_C\Delta_{AB}E_{CD}-W(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})]\},$$

$$f_3=F_1(E_{AB}E_{CD}+m_{AB}m_{CD})-F_2(E_{AB}E_{CD}-m_{AB}m_{CD})+2(F_3+F_4)E_{AB}E_{CD}+2F_5m_{AB}m_{CD}$$
$$+F_6[p_A\Delta_{AB}E_{CD}+p_C E_{AB}\Delta_{CD}+D(E_{AB}m_{CD}+m_{AB}E_{CD})]$$
$$+F_7[-p_A p_{AB}m_{CD}+p_C m_{AB}p_{CD}-W(E_{AB}m_{CD}-m_{AB}E_{CD})]$$
$$+F_8[-p_A\Delta_{AB}E_{CD}+p_C E_{AB}\Delta_{CD}+D(E_{AB}m_{CD}-m_{AB}E_{CD})], \tag{9}$$

$$f_4=-F_1(p_{AB}p_{CD}+\Delta_{AB}\Delta_{CD})+F_2(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})+2(F_3+F_4)p_{AB}p_{CD}-2F_5\Delta_{AB}\Delta_{CD}$$
$$+F_6[p_A E_{AB}\Delta_{CD}+p_C p_{AB}E_{CD}+W(p_{AB}\Delta_{CD}+\Delta_{AB}p_{CD})]$$
$$+F_7[p_A m_{AB}p_{CD}-p_C p_{AB}m_{CD}-D(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})]$$
$$+F_8[p_A E_{AB}\Delta_{CD}-p_C\Delta_{AB}E_{CD}+W(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})],$$

$$f_5=-F_1(E_{AB}m_{CD}+m_{AB}E_{CD})+F_2(E_{AB}m_{CD}-m_{AB}E_{CD})-2(F_3+F_4)E_{AB}m_{CD}-2F_5m_{AB}E_{CD}$$
$$+F_6[-p_A\Delta_{AB}m_{CD}-p_C m_{AB}\Delta_{CD}-D(E_{AB}E_{CD}+m_{AB}m_{CD})]$$
$$+F_7[p_A p_{AB}E_{CD}+p_C E_{AB}p_{CD}+W(E_{AB}E_{CD}-m_{AB}m_{CD})]$$
$$+F_8[p_A\Delta_{AB}m_{CD}+p_C m_{AB}\Delta_{CD}-D(E_{AB}E_{CD}-m_{AB}m_{CD})],$$

$$f_6=-F_1(p_{AB}\Delta_{CD}+\Delta_{AB}p_{CD})+F_2(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})+2(F_3+F_4)p_{AB}\Delta_{CD}-2F_5\Delta_{AB}p_{CD}$$
$$+F_6[p_A E_{AB}p_{CD}+p_C p_{AB}E_{CD}+W(p_{AB}p_{CD}+\Delta_{AB}\Delta_{CD})]$$
$$+F_7[p_A m_{AB}\Delta_{CD}+p_C\Delta_{AB}m_{CD}-D(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})]$$
$$+F_8[p_A E_{AB}p_{CD}+p_C p_{AB}E_{CD}+W(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})],$$

$$f_7=F_1(E_{AB}m_{CD}+m_{AB}E_{CD})+F_2(E_{AB}m_{CD}-m_{AB}E_{CD})+2(F_3+F_4)m_{AB}E_{CD}+2F_5E_{AB}m_{CD}$$
$$+F_6[p_A\Delta_{AB}m_{CD}+p_C m_{AB}\Delta_{CD}+D(E_{AB}E_{CD}+m_{AB}m_{CD})]$$
$$+F_7[p_A p_{AB}E_{CD}+p_C E_{AB}p_{CD}+W(E_{AB}E_{CD}-m_{AB}m_{CD})]$$
$$+F_8[p_A\Delta_{AB}m_{CD}+p_C m_{AB}\Delta_{CD}-D(E_{AB}E_{CD}-m_{AB}m_{CD})],$$

$$f_8=F_1(p_{AB}\Delta_{CD}+\Delta_{AB}p_{CD})+F_2(p_{AB}\Delta_{CD}-\Delta_{AB}p_{CD})-2(F_3+F_4)\Delta_{AB}p_{CD}+2F_5p_{AB}\Delta_{CD}$$
$$+F_6[-p_A E_{AB}p_{CD}-p_C p_{AB}E_{CD}-W(p_{AB}p_{CD}+\Delta_{AB}\Delta_{CD})]$$
$$+F_7[p_A m_{AB}\Delta_{CD}+p_C\Delta_{AB}m_{CD}-D(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})]$$
$$+F_8[p_A E_{AB}p_{CD}+p_C p_{AB}E_{CD}+W(p_{AB}p_{CD}-\Delta_{AB}\Delta_{CD})],$$

where

$$W = \tfrac{1}{2}(E_A + E_B + E_C + E_D) = s^{1/2}, \tag{10}$$

$$D = \tfrac{1}{2}(E_A + E_C - E_B - E_D) = \tfrac{1}{2}(m_A{}^2 + m_C{}^2 - m_B{}^2 - m_D{}^2)/s^{1/2}, \tag{11}$$

and

$$\cos\theta_s = [s(t-u) - (m_A{}^2 - m_B{}^2)(m_C{}^2 - m_D{}^2)]/4s\,p_A p_C. \tag{12}$$

To describe $AB \to CB$ (e.g., $\Sigma N \to \Lambda N$), we simply put $D = B$ [not to be confused with the D in Eq. (11)]. There remain eight independent amplitudes. To arrive at elastic scattering $AB \to AB$, in addition to putting $C = A$ and $D = B$ in (9), we must also include time-reversal invariance. It requires $f_7 = -f_5$ and $f_8 = -f_6$, which in turn rules out F_7 and F_8 (not O_7 and O_8). Finally, for the case $NN \to NN$,[7] we have all $\Delta_{ij} = 0$ and $f_8 = -f_6 = 0$ (spin conservation). The last condition eliminates F_6.

Starting from expression (9) for the f_i one is able to write down kinematic-singularity-free amplitudes $\bar{f}_i$. These $\bar{f}_i$ are linear functions of the F_i with coefficients that are just polynomials in s, t, and the masses. For example, f_6 reads, in terms of s and the four masses,

$$\begin{aligned} f_6 = \tfrac{1}{4}[s-(m_A+m_B)^2]^{1/2}[s-(m_C+m_D)^2]^{1/2}s^{-1/2} \\ \times\{ -F_1(m_A - m_B + m_C - m_D) - F_2(m_A - m_B - m_C + m_D) + 2(F_3+F_4)(m_C - m_D) \\ - 2F_5(m_A - m_B) + 2F_6[s - \tfrac{1}{4}(m_A - m_B - m_C + m_D)^2] \\ - F_7\tfrac{1}{2}(m_A + m_B - m_C - m_D)(m_A - m_B - m_C + m_D) + 2F_8[s - \tfrac{1}{4}(m_A - m_B + m_C - m_D)^2]\}. \end{aligned} \tag{13}$$

Clearly $\bar{f}_6 = f_6 s^{1/2}/p_{AB}p_{CD}$ is free of all kinematic singularities if the F_i are. This can be done for the other f_i's in an analogous fashion. One then finds that the kinematic-singularity-free amplitudes $\bar{f}_i$ are given by

$$\begin{aligned} &\bar{f}_1 = f_1 s p_{AB} p_{CD}, \quad \bar{f}_2 = f_2 s E_{AB} E_{CD}, \\ &\bar{f}_3 = f_3 s/E_{AB}E_{CD}, \quad \bar{f}_4 = f_4 s/p_{AB}p_{CD}, \\ &\bar{f}_5 = f_5 s^{1/2}/E_{AB}E_{CD}, \quad \bar{f}_6 = f_6 s^{1/2}/p_{AB}p_{CD}, \\ &\bar{f}_7 = f_7 s^{1/2}/E_{AB}E_{CD}, \quad \bar{f}_8 = f_8 s^{1/2}/p_{AB}p_{CD}. \end{aligned} \tag{14}$$

That one is able to factor out of the f_i a term containing all the s kinematic singularities partly "proves" that the F_i are the correct invariant amplitudes. (A complete proof would require that the same procedure is possible also for the t- and u-channel h.a.; this will be discussed later.)

However, one has to be careful and realize that the expressions (14) for the $\bar{f}_i$ are true only for the general case in which all four masses are different. When a pair or more of the masses become equal, the expressions for the $\bar{f}_i$ will assume, as is well known, a different form. This is because some of the kinematic singularities in (9) are automatically removed if some of the masses are equal. Also one must set $F_7 = F_8 = 0$ for elastic scattering, no matter what coefficients they have in (9). In all these mass-degenerate cases, therefore, the best way to proceed is to investigate the properly reduced f_i, and then extract the correct $\bar{f}_i$ directly therefrom, rather than modifying (14).

A partial-wave expansion for the f_i is trivial, if use is made of the d functions.[6] The partial-wave amplitudes are used for analyzing experimental data in the low-energy region. Inversion of the F_i in terms of the f_i then incorporates the experimental data in the invariant amplitudes F_i for phenomenological analysis. The inversion is not easy, unfortunately.

At high energies, the asymptotic behavior for the F_i can be deduced from the Regge-pole model (Secs. III and IV). If the conjecture of s-channel helicity conservation[8] is assumed (for diffractive processes), all but F_3 and F_4 will decouple from the leading trajectories. All eight F_i then satisfy unsubtracted dispersion relations. Furthermore, only F_3 would survive asymptotically,[9] if there is asymptotic helicity independence, i.e., $\langle ++|T^s|++\rangle \simeq \langle +-|T^s|+-\rangle$.

In a perturbative calculation, one often faces the problem of developing covariants such as $i\gamma\cdot K \otimes i\gamma\cdot P$ and $i\gamma\cdot K\otimes 1 + 1\otimes i\gamma\cdot P$, etc., in terms of the eight standard O_i. These are called the *equivalence theorems*.[3] An algebraic derivation is possible, namely, by invoking the eight independent equations obtained by evaluation of the various helicity "matrix elements" for, say,

$$i\gamma\cdot K\otimes i\gamma\cdot P = \sum_{i=1}^{8} a_i O_i \tag{15}$$

on both sides, and solving for the eight a_i. This method is not practical here in the s channel, because none of the coefficients of the a_i are zero, as is evident from (9). It turns out that, if we continue Eq. (15) etc. to the t channel, many of these coefficients in the continued equations do vanish. We shall come to this point in the next section.

Likewise, for a u-channel exchange diagram, it requires a *Fierz transformation*[3] to bring the amplitude to the eight standard O_i. Again an algebraic derivation of the Fierz transformation is possible with less effort if we employ the same trick. We shall see that we will gain more if, instead of the t channel, we go to the u channel (Sec. IV).

Before closing this section, we would like to remark that, although the formalism to be used below for describing the t- and u-channel amplitudes does not differ much from the s-channel one, the physical content of each channel is certainly different. As a result, we deal with each channel separately.

III. t-CHANNEL AMPLITUDES

By t-channel $[\overline{D}(-k_2, \mu_2') + B(k_1, \mu_1) \to C(p_2, \lambda_2) + \overline{A}(-p_1, \lambda_1')]$ amplitudes we mean

$$\langle p_2, \lambda_2; -p_1, \lambda_1' | T^t | -k_2, \mu_2'; k_1, \mu_1\rangle = -\sum_{i=1}^{8} [\overline{u}_C(p_2, \lambda_2) O_i^{(1)} v_A(p_1, \lambda_1')] \times [\overline{v}_D(k_2, \mu_2') O_i^{(2)} u_B(k_1, \mu_1)] F_i(s, t) \tag{16}$$

with the same F_i, $O_i^{(1)}$, and $O_i^{(2)}$ as in Eq. (1).

We consider the crossed channel (by means of a boost plus analytic continuation) mainly in order to deduce the high-energy behavior for the F_i, rather than to actually study antihyperon-nucleon scattering. If the latter is the case, we should directly start with

$$\overline{A}(\overline{p}_1) + B(\overline{k}_1) \to \overline{C}(\overline{p}_2) + D(\overline{k}_2), \tag{17}$$

with $\overline{p}_1$, $\overline{k}_1$, $\overline{p}_2$, and $\overline{k}_2$ having nothing to do with p_1, k_1, p_2, and k_2 in (1).

The evaluation of the h.a. in the c.m. frame of the t channel [Fig. 1(b)] is expedited again by introducing eight quasivariables E_{ij}, m_{ij}, p_{ij}, and Δ_{ij}, which now read

$$\begin{aligned} E_{ij} &= [t - (m_i - m_j)^2]^{1/2}, \\ m_{ij} &= E_{ij}(m_i + m_j)/t^{1/2}, \\ p_{ij} &= [t - (m_i + m_j)^2]^{1/2}, \\ \Delta_{ij} &= p_{ij}(m_i - m_j)/t^{1/2}, \end{aligned} \tag{18}$$

where $ij = \overline{A}C$ or $B\overline{D}$.[10] Using for the antiparticle $\overline{D}$ with helicity μ_2' the spinor

$$v_D(k_2, \mu_2') = (-1)^{1/2+\mu_2'} [2m_D(E_D + m_D)]^{-1/2} \begin{pmatrix} -2p_D\mu_2' \\ E_D + m_D \end{pmatrix}, \tag{19}$$

the eight t-channel regularized parity-conserving h.a.[11] are given, in the notation of (16) and the Appendix, by

$$\begin{aligned}
g_1 &= F_1 p_{AC} p_{BD} - F_3 \Delta_{AC}\Delta_{BD} - F_6 \tfrac{1}{2}[(E_A - E_C) p_{AC} \Delta_{BD} - (E_B - E_D)\Delta_{AC} p_{BD}] \\
&\quad + \cos\theta_t [F_3 m_{AC} m_{BD} + F_5 E_{AC} E_{BD} - F_6(p_A p_{AC} m_{BD} - p_B m_{AC} p_{BD})], \\
g_2 &= -F_2 E_{AC} E_{BD} - F_4 m_{AC} m_{BD} - (F_7 + F_8)\tfrac{1}{2}(E_B - E_D) m_{AC} E_{BD} - (F_7 - F_8)\tfrac{1}{2}(E_A - E_C) E_{AC} m_{BD} \\
&\quad + \cos\theta_t [F_4 \Delta_{AC}\Delta_{BD} - F_5 p_{AC} p_{BD} - (F_7 + F_8) p_B \Delta_{AC} E_{BD} - (F_7 - F_8) p_A E_{AC}\Delta_{BD}], \\
g_3 &= F_3 E_{AC} E_{BD} + F_5 m_{AC} m_{BD}, \\
g_4 &= F_4 p_{AC} p_{BD} - F_5 \Delta_{AC}\Delta_{BD}, \\
g_5 &= -F_3 m_{AC} E_{BD} - F_5 E_{AC} m_{BD} + F_6 p_A p_{AC} E_{BD}, \\
g_6 &= -F_4 \Delta_{AC} p_{BD} + F_5 p_{AC}\Delta_{BD} + (F_7 - F_8) p_A E_{AC} p_{BD}, \\
g_7 &= F_3 E_{AC} m_{BD} + F_5 m_{AC} E_{BD} + F_6 p_B E_{AC} p_{BD}, \\
g_8 &= F_4 p_{AC}\Delta_{BD} - F_5 \Delta_{AC} p_{BD} - (F_7 + F_8) p_B p_{AC} E_{BD},
\end{aligned} \tag{20}$$

where

$$E_A = (t + m_A^2 - m_C^2)/2t^{1/2}, \tag{21}$$

$$p_A^2 = [t - (m_A - m_C)^2][t - (m_A + m_C)^2]/4t, \quad \text{etc.}, \tag{22}$$

and

$$\cos\theta_t = [t(s-u) + (m_A^2 - m_C^2)(m_B^2 - m_D^2)]/4t p_A p_B. \tag{23}$$

Obviously Eq. (20) is much simpler than Eq. (9). The simplicity is essentially due to the collinearity of p_1 and p_2 in $[\overline{u}_C(p_2)O_i^{(1)} v_A(p_1)]$ and of k_1 and k_2 in $[\overline{v}_D(k_2)O_i^{(2)} u_B(k_1)]$. Thus even for a general $\overline{D}B \to C\overline{A}$ with four different masses, the task of inverting the F_i in terms of the g_i now poses no problems. This result is quite useful.

The reduction of (20) to describe hyperon-nucleon scattering is also tremendously simplified. First, although we still need eight amplitudes for

$\overline{B}B \to C\overline{A}$ (e.g., $\overline{N}N \to \Lambda\overline{\Sigma}$), a simplification already occurs because $\Delta_{BD} = \Delta_{BB} = 0$. For the reaction $\overline{B}B \to A\overline{A}$ (e.g., $\overline{N}N \to \Sigma\overline{\Sigma}$) one has $\Delta_{BD} = \Delta_{AC} = 0$ as well as C invariance. The C and P invariance together require $g_6 = g_8 = 0$, so that $(F_7 - F_8) = (F_7 + F_8) = 0$, or $F_7 = F_8 = 0$. The well-known $\overline{N}N \to N\overline{N}$,[7] in addition, acquires an extra condition $g_7 = -g_5$; consequently F_6 also needs to vanish.

By definition the g_i's are free of the kinematic singularities in s. If the F_i are invariant amplitudes, we can factor out of the g_i in (20) the terms containing the remaining kinematic singularities in t, and obtain the regular amplitudes $\overline{g}_i$ as follows:

$$
\begin{aligned}
&\overline{g}_1 = g_1 t p_{AC} p_{BD}, \quad \overline{g}_2 = g_2 t E_{AC} E_{BD},\\
&\overline{g}_3 = g_3 t / E_{AC} E_{BD}, \quad \overline{g}_4 = g_4 t / p_{AC} p_{BD},\\
&\overline{g}_5 = g_5 t^{1/2} / E_{AC} E_{BD}, \quad \overline{g}_6 = g_6 t^{1/2} / p_{AC} p_{BD},\\
&\overline{g}_7 = g_7 t^{1/2} / E_{AC} E_{BD}, \quad \overline{g}_8 = g_8 t^{1/2} / p_{AC} p_{BD}.
\end{aligned}
\tag{24}
$$

That we are able to define these $\overline{g}_i$ is the second part of the proof that the F_i have been correctly chosen.

For the mass-degenerate reactions, $\overline{B}B \to C\overline{A}$, $\overline{B}B \to A\overline{A}$, and $\overline{N}N \to N\overline{N}$, the limit in (24) (obtained by putting $\overline{D} = \overline{B}$, $C = A$, etc.) may not yield the correct regular amplitudes $\overline{g}_i$ for the desired process. This point we have explained in the previous section. The correct procedure to follow is to make a pertinent reduction in the original g_i [Eq. (20)] and factor out the explicit t singularities from these mass-degenerate g_i directly.

Reggeization of the g_i at fixed t yields the asymptotic behavior for the F_i: $F_1, F_2 \sim s^{\alpha(t)}, F_3, F_4, F_5, F_6, F_7, F_8 \sim s^{\alpha(t)-1}$. The number of subtractions needed in a dispersion approach for each invariant amplitude can thus be inferred. It also allows for investigations such as the conjecture of asymptotic s-channel helicity conservation, in which case all F_i would satisfy unsubtracted dispersion relations (Sec. II).

The inversion of the F_i in terms of the g_i is necessary anyway, to examine kinematic constraints such as conspiracy, evasion, and so on. Since the inversion of Eq. (20) is really feasible, we are able to make some applications. That is, we are able to derive the equivalence theorem and the Fierz transformation for four spinors by an algebraic method.

Continue, say, both sides of Eq. (15), i.e.,

$$
i\gamma\cdot K \otimes i\gamma\cdot P = \sum_{i=1}^{8} a_i O_i\,, \tag{15}
$$

to the t channel, so that $u_A(p_1, \lambda_1)$ becomes $v_A(p_1, \lambda_1')$, $\overline{u}_D(k_2, \mu_2)$ becomes $\overline{v}_D(k_2, \mu_2')$, and p_1 (k_1) and p_2 (k_2) are collinear [Fig. 1(b)]. We obtain eight new independent equations, the right-hand sides of which are the same as those in Eqs. (20) with $(-F_i)$ replaced by the a_i. If we denote the left-hand sides by L_i, clearly we can first solve for a_3, a_5, and a_6 from L_3, L_5, and L_7 simultaneously. The rest can then be easily computed, one by one, with the other L_i. We find

$$
\begin{aligned}
&a_1 = 0,\\
&a_2 = \tfrac{1}{4}(m_A + m_C)(m_B + m_D),\\
&a_3 = \tfrac{1}{4}(s-u),\\
&a_4 = \tfrac{1}{4}[(m_A - m_C)^2 + (m_B - m_D)^2 - t],\\
&a_5 = \tfrac{1}{4}(m_A - m_C)(m_B - m_D),\\
&a_6 = 0,\\
&a_7 = \tfrac{1}{4}(-m_A + m_C - m_B + m_D),\\
&a_8 = \tfrac{1}{4}(m_A - m_C - m_B + m_D).
\end{aligned}
\tag{25}
$$

The same procedure, when carried out for

$$
i\gamma\cdot K \otimes 1 + 1 \otimes i\gamma\cdot P = \frac{4}{m_A + m_B + m_C + m_D} \sum_{i=1}^{8} b_i O_i\,, \tag{26}
$$

yields

$$
\begin{aligned}
&b_1 = -\tfrac{1}{4}(s-u),\\
&b_2 = -\tfrac{1}{4}(s-u),\\
&b_3 = \tfrac{1}{4}(m_A + m_C)(m_B + m_D),\\
&b_4 = \tfrac{1}{4}(m_A - m_C)(m_B - m_D),\\
&b_5 = \tfrac{1}{4}t,\\
&b_6 = -\tfrac{1}{4}(m_A + m_C - m_B - m_D),\\
&b_7 = -(m_A - m_C + m_B - m_D),\\
&b_8 = -(m_A - m_C - m_B + m_D).
\end{aligned}
\tag{27}
$$

While this kind of derivation for the *equivalence theorems* may not be the most general one, it is clearly a bonus in the course of evaluation of the t-channel h.a. Moreover, we realize that, due to the correlations among the a_i (or b_i) to yield an acceptable result like (25) [or (27)], any mistake made in the computation of the O_i in (16) and (20) can be detected easily.

It is true that a *Fierz transformation*

$$
[\overline{u}_D(k_2, \mu_2) O_i^{(3)} u_A(p_1, \lambda_1)][\overline{u}_C(p_2, \lambda_2) O_i^{(4)} u_B(k_1, \mu_1)] = \sum_{j=1}^{8} c_{ij} [\overline{u}_C(p_2, \lambda_2) O_j^{(1)} u_A(p_1, \lambda_1)][\overline{u}_D(k_2, \mu_2) O_j^{(2)} u_B(k_1, \mu_1)]
\tag{28}
$$

can be also derived with the same trick. The left-hand side of (28), however, has nothing to do with the t-channel h.a., and is not readily available. A better way is to go to the u channel (Sec. IV).

IV. u-CHANNEL AMPLITUDES

The u channel is taken as $\overline{C}(-p_2, \lambda_2') + B(k_1, \mu_1) \to \overline{A}(-p_1, \lambda_1') + D(k_2, \mu_2)$, with the amplitudes given by[10]

$$\langle -p_1, \lambda_1'; k_2, \mu_2 | T^u | -p_2, \lambda_2'; k_1, \mu_1\rangle = -\sum_{i=1}^{8} [\overline{v}_C(p_2, \lambda_2') O_i^{(1)} v_A(p_1, \lambda_1')][\overline{u}_D(k_2, \mu_2) O_i^{(2)} u_B(k_1, \mu_1)] F_i(s, t, u). \tag{29}$$

By now we can evaluate the h.a. in a straightforward way. We obtain[11] [cf. Fig. 1(c) and the Appendix]

$$\begin{aligned} h_1 = & -F_1(p_{AD}p_{BC} - \Delta_{AD}\Delta_{BC}) - F_2(p_{AD}p_{BC} + \Delta_{AD}\Delta_{BC}) - 2F_3(2p_{AD}p_{BC} + \Delta_{AD}\Delta_{BC}) \\ & - 2F_4(2p_{AD}p_{BC} - \Delta_{AD}\Delta_{BC}) - 6F_5 p_{AD}p_{BC} - F_6[-p_A E_{AD}\Delta_{BC} + p_B\Delta_{AD}E_{BC} - W(p_{AD}\Delta_{BC} - \Delta_{AD}p_{BC})] \\ & + F_7[-p_A E_{AD}\Delta_{BC} - p_B\Delta_{AD}E_{BC} - W(p_{AD}\Delta_{BC} + \Delta_{AD}p_{BC})] \\ & + F_8[p_A m_{AD}p_{BC} - p_B p_{AD}m_{BC} + D(p_{AD}\Delta_{BC} + \Delta_{AD}p_{BC})] \\ & + \cos\theta_u\{F_1(E_{AD}E_{BC} + m_{AD}m_{BC}) + F_2(E_{AD}E_{BC} - m_{AD}m_{BC}) - 2(F_3 - F_4)m_{AD}m_{BC} - 2F_5E_{AD}E_{BC} \\ & \qquad - F_6[p_A\Delta_{AD}E_{BC} - p_B E_{AD}\Delta_{BC} + D(E_{AD}m_{BC} + m_{AD}E_{BC})] \\ & \qquad + F_7[-p_A\Delta_{AD}E_{BC} - p_B E_{AD}\Delta_{BC} + D(E_{AD}m_{BC} - m_{AD}E_{BC})] \\ & \qquad + F_8[-p_A p_{AD}m_{BC} + p_B m_{AD}p_{BC} - W(E_{AD}m_{BC} - m_{AD}E_{BC})]\}, \end{aligned} \tag{30}$$

where

$$\begin{aligned} E_{ij} &= \tfrac{1}{2}[u - (m_i - m_j)^2]^{1/2}, \quad m_{ij} = E_{ij}(m_i + m_j)/u^{1/2}, \\ p_{ij} &= \tfrac{1}{2}[u - (m_i + m_j)^2]^{1/2}, \quad \Delta_{ij} = p_{ij}(m_i - m_j)/u^{1/2}, \end{aligned} \tag{31}$$

$$W = \tfrac{1}{2}(E_A + E_B + E_C + E_D) = u^{1/2}, \tag{32}$$

$$D = \tfrac{1}{2}(E_A + E_C - E_B - E_D) = \tfrac{1}{2}(m_A^2 + m_C^2 - m_B^2 - m_D^2)/u^{1/2}, \tag{33}$$

$$\cos\theta_u = [u(t-s) + (m_A^2 - m_D^2)(m_B^2 - m_C^2)]/4u p_A p_B, \tag{34}$$

with

$$E_A = [u + m_A^2 - m_D^2]/2u^{1/2}, \quad \text{etc.}, \tag{35}$$

$$p_A^2 = [u - (m_A - m_D)^2][u - (m_A + m_D)^2]/4u, \tag{36}$$

and

$$p_B^2 = [u - (m_B - m_C)^2][u - (m_B + m_C)^2]/4u. \tag{37}$$

A comparison of h_1 and f_1 shows that h_1 is the same as f_1 if a formal substitution

$$(F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8) \to (F_1, F_2, -F_3, F_4, -F_5, -F_6, F_8, F_7) \tag{38}$$

and an explicit replacement

$$(m_A, m_C, s) \to (m_C, m_A, u) \tag{39}$$

are made in f_1. In fact, a detailed calculation shows that this is true for all h_i. Many of the properties of the f_i discussed in Sec. II can thus be carried through for the h_i without change. It is worth noting from Eqs. (38) and (39) that one may prefer to use the second set O_7' and O_8' than O_7 and O_8 for s-u crossing.

Due to the analogy between the h_i and f_i it is clear that we can find $\overline{h}_i$ free of s and u kinematic singularities. Combining this with the other proofs made in the s and t channels we finally conclude that the F_i *are* invariant amplitudes.

Reggeization of the h_i at fixed u reveals that all F_i behave as $s^{\alpha(u)-1}$ asymptotically. Since the inversion of the F_i in terms of the h_i is as difficult as with the f_i, an investigation on the Regge constraints in backward scattering will be more involved.

Had we chosen in (2) and (4) the equally acceptable spinor covariants

$$[\overline{u}_D(k_2, \mu_2) O_i'^{(3)} u_A(p_1, \lambda_1)][\overline{u}_C(p_2, \lambda_2) O_i'^{(4)} u_B(k_1, \mu_1)] \tag{40}$$

where $O_i'^{(3)} \otimes O_i'^{(4)} \equiv \overline{O}_i'$, with

$$\begin{aligned} &\overline{O}_i' = O_i' \quad \text{for } i = 1 \text{ to } 6, \\ &\overline{O}_7' = \gamma_5 i\gamma \cdot \tfrac{1}{2}(p_2 + k_1) \otimes \gamma_5, \\ &\overline{O}_8' = \gamma_5 \otimes \gamma_5 i\gamma \cdot \tfrac{1}{2}(k_2 + p_1), \end{aligned} \tag{41}$$

the resulting h_i would be as simple as the g_i in (20), because k_2 (p_2) and p_1 (k_1) are collinear in the u channel. Now (2) is related to (40) by a *Fierz transformation*

$$[\overline{u}_C(p_2, \lambda_2) O_i'^{(1)} u_A(p_1, \lambda_1)][\overline{u}_D(k_2, \mu_2) O_i'^{(2)} u_B(k_1, \mu_1)] = \sum_{j=1}^{8} d_{ij} [\overline{u}_D(k_2, \mu_2) O_j'^{(3)} u_A(p_1, \lambda_1)][\overline{u}_C(p_2, \lambda_2) O_j'^{(4)} u_B(k_1, \mu_1)]. \tag{42}$$

The coefficients d_{ij} can be readily computed if we continue (42) to the u channel,

$$[\overline{v}_C(p_2, \lambda_2') O_i'^{(1)} v_A(p_1, \lambda_1')][\overline{u}_D(k_2, \mu_2) O_i'^{(2)} u_B(k_1, \mu_1)] = \sum_{j=1}^{8} d_{ij} [\overline{u}_D(k_2, \mu_2) O_j'^{(3)} v_A(p_1, \lambda_1')][\overline{v}_C(p_2, \lambda_2') O_j'^{(4)} u_B(k_1, \mu_1)], \tag{43}$$

just as we continued (15) or (26) to the t channel. It turns out that all d_{ij} are either pure numbers, or depend only on the four masses.

An inverted expression for (42),

$$[\overline{u}_D(k_2, \mu_2) O_i'^{(3)} u_A(p_1, \lambda_1)][\overline{u}_C(p_2, \lambda_2) O_i'^{(4)} u_B(k_1, \mu_1)] = \sum_{j=1}^{8} c_{ij}' [\overline{u}_C(p_2, \lambda_2) O_j'^{(1)} u_A(p_1, \lambda_1)][\overline{u}_D(k_2, \mu_2) O_i'^{(2)} u_B(k_1, \mu_1)], \tag{44}$$

is more useful for the u-channel exchange diagrams in a perturbative approach. The c_{ij}', which are related to the d_{ij} merely by relabeling the masses, are displayed as follows[3]:

$$c_{ij}' = \begin{bmatrix} \Gamma_a & 0 \\ \Gamma_b & \Gamma_c \end{bmatrix}, \qquad \Gamma_a = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & -2 & 2 & 0 \\ 4 & -4 & 2 & -2 & 0 \\ 6 & 6 & 0 & 0 & -2 \end{bmatrix}, \qquad \Gamma_b = \frac{1}{2}\begin{bmatrix} 0 & 2 & -2 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}, \tag{45}$$

$$\Gamma_c = \frac{1}{8}\begin{bmatrix} 3(m_D + m_A - m_C - m_B) & -m_D - m_A + m_C + m_B & -m_D - m_A + m_C + m_B & 3(m_D + m_A - m_C - m_B) & -m_D - m_A + m_C + m_B \\ -m_D + m_A - 2m_C + 2m_B & -m_D + m_A + 2m_C - 2m_B & m_D - m_A - 2m_C + 2m_B & m_D - m_A + 2m_C - 2m_B & m_D - m_A \\ -2m_D + 2m_A - m_C + m_B & 2m_D - 2m_A - m_C + m_B & -2m_D + 2m_A + m_C - m_B & 2m_D - 2m_A + m_C - m_B & m_C - m_B \end{bmatrix}.$$

The ordering (D, A, C, B) on the left-hand side of (44) is kept in Γ_c for convenience.

That we make an analytic continuation to the u channel with (42) rather than continue (44) or (28) to the t channel follows from the simple fact that the left-hand side of (43) has already been calculated in (29). In particular, a compact result for c_{ij}' like (45) checks also the correctness of (30) and (9), if a Fierz transformation itself involves only simple coefficients.

We believe that our c_{ij}' are more reliable than those obtained with other methods, because had we made a mistake in the h_i or in some of the already calculated c_{ij}', the progression from one step to the other for the next coefficient would be catastrophic.

V. SUMMARY

We have examined the invariant and helicity amplitudes for a general process $A + B \rightarrow C + D$ (all four particles having $J^P = \frac{1}{2}^+$), and the analytically continued $\overline{D} + B \rightarrow C + \overline{A}$ (t channel) and $\overline{C} + B \rightarrow \overline{A} + D$ (u channel). The full formalism can be immediately applied to describe typical hyperon-nucleon scattering such as $\Sigma N \rightarrow \Lambda N$, $\Sigma N \rightarrow \Sigma N$, etc., for which experiments at higher energies will soon be possible.

Due to the unequal-mass kinematics, the relation between the c.m. helicity amplitudes and the invariant amplitudes is necessarily complicated. Since we need both of them for a complete description of the process, we have introduced quasivariables like E_{ij}, p_{ij}, m_{ij}, and Δ_{ij} [Eqs. (8), (18), and (31)] to expedite the manipulation of the algebra. The reduction from the general $A + B \rightarrow C + D$ to $A + B \rightarrow C + B$, to $A + B \rightarrow A + B$, and to $A + A \rightarrow A + A$ is natural and can be easily carried out.

When the momenta of a pair of spinors are collinear, e.g., $\overline{u}_\lambda(\vec{p}) O v_\mu(-\vec{p})$, many of the helicity "matrix elements" are zero in different configurations (i.e., $\lambda\mu = ++, +-, -+, --$). We have exploited this property to derive the equivalence theorems and Fierz transformations by an algebraic method. Not only is the result reliable (because many correlations are involved), it also checks the correctness of the otherwise ugly t- and u-channel h.a. [Eqs. (20) and (30)], and, through s-u crossing, the s-channel ones [Eq. (9)] too.

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APPENDIX

The eight regularized, parity-conserving, x-channel c.m. helicity amplitudes for a general $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}$ scattering are defined in standard notations as

$$(H_1,H_2)=\tfrac{1}{2}[\langle ++|T^x|++\rangle(+,-)\langle --|T^x|++\rangle],$$

$$(H_3,H_4)=\tfrac{1}{2}\left[\frac{\langle +-|T^x|+-\rangle}{1+\cos\theta_x}(+,-)\frac{\langle -+|T^x|+-\rangle}{1-\cos\theta_x}\right],$$

$$(H_5,H_6)=\tfrac{1}{2}[\langle ++|T^x|+-\rangle(+,-)\langle --|T^x|+-\rangle]/\sin\theta_x,$$

$$(H_7,H_8)=\tfrac{1}{2}[\langle +-|T^x|++\rangle(-,+)\langle -+|T^x|++\rangle]/\sin\theta_x,$$

where $\pm$ stands for $\pm\frac{1}{2}$. In the text,

$$f_i=4M^2H_i \quad \text{for } x=s,$$

$$g_i=M^2H_i \quad \text{for } x=t,$$

and

$$h_i=4M^2H_i \quad \text{for } x=u,$$

where

$$M^2=(m_A m_B m_C m_D)^{1/2}.$$

---

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