

## Estimates of Invariant Amplitudes for Reactions with $\rho$ Exchange

J. Randa

*Physics Department, University of Illinois, Urbana, Illinois 61801*

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The hypothesis that  $\rho$ -exchange invariant amplitudes for different reactions have the same ratio in the scattering region as they do at the  $\rho$  pole is considered. The hypothesis is found to be in reasonable agreement with existing data. It can then be used to predict, to within a factor of about 2, the size of  $\rho$ -exchange cross sections in future experiments.

Theoretical prediction of magnitudes of high-energy differential cross sections has seldom kept pace with experimental developments. The simplest such predictions, from single-particle exchange, yield cross sections which are too large and which have a disastrous energy dependence. Regge-pole (and cut) models are normally *post factum* affairs; rather than having known residue functions with which experimental results can be predicted, the experimental results are used to determine what the residue functions must be. In order to predict cross sections, we must have some way of determining the residues.

Experimental data are quite obstinate in their rejection of a simple factorizable Regge-pole explanation of individual helicity amplitudes.<sup>1</sup> We shall not attempt such an explanation. What we shall attempt is to develop a simple method of predicting the size of differential cross sections, using the Regge-pole model as a sort of motivating framework, but not accepting the onus of predicting (or even separately considering) individual helicity amplitudes.

The amplitude due to the exchange of a Regge trajectory reduces to the amplitude from single-particle exchange at the particle's pole. Consequently, since we know how to calculate Feynman graphs, we can calculate numbers for the residue functions at the pole if we know the relevant coupling constants. This tells us nothing about the shape of the residue functions as functions of  $t$ , but it does give us a handle on their size. The simplest thing which could happen is that between  $t=m^2$  and  $t=0$  the invariant amplitudes for different reactions proceeding via the same exchange have the same behavior. That is, if  $|\mathfrak{M}_1|^2 = |\mathfrak{M}_2|^2$  at  $t=m^2$ , then  $|\mathfrak{M}_1|^2 = |\mathfrak{M}_2|^2$  in the scattering region also.

Obviously we do not expect this relation to be exact. Amplitudes for different reactions will have different  $t$  dependence due to different kinematic factors; the presence or absence of dips at  $t \approx -0.6 \text{ GeV}^2/c^2$  also varies from reaction to

reaction. We can, however, hope that the overall size of the amplitudes will be about the same and that the presence of major kinematic effects can be simply predicted.

To test this method we choose  $\rho$  exchange since it is relatively easy to isolate (which is essential) and is one of the best understood trajectories. We choose  $\pi^-p \rightarrow \pi^0n$  as our reference and consider the reaction  $\pi^-p \rightarrow \omega^0n$ . For the  $\rho\pi\pi$  coupling we use  $g_{\rho\pi\pi}^2/4\pi = 2.16$ . This value is obtained from the  $\rho \rightarrow 2\pi$  decay. The  $\omega\rho\pi$  coupling constant is obtained using the method of Gell-Mann, Sharp, and Wagner.<sup>2</sup> We then obtain  $g_{\omega\rho\pi}^2/4\pi = 28.4 \text{ GeV}^{-2}$ . The coupling constants at the  $\rho pn$  vertex are irrelevant here since at  $t=m_\rho^2$  the Feynman graph factors into the product of a term from the  $\rho pn$  vertex and a term from the other vertex.<sup>3</sup> Since we are interested in the ratio

$$\left| \frac{\mathfrak{M}_\rho(\pi^-p \rightarrow \omega^0n)}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2,$$

the factor from the  $\rho pn$  vertex drops out.

Using these coupling constants one finds

$$\left| \frac{\mathfrak{M}_\rho(\pi^-p \rightarrow \omega^0n)}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2 = \begin{cases} 1.83, & p_{\text{lab}} = 4.5 \text{ GeV}/c \\ 1.94, & p_{\text{lab}} = 20 \text{ GeV}/c \\ 1.97, & p_{\text{lab}} \rightarrow \infty. \end{cases} \quad (1)$$

This is at  $t=m_\rho^2$ . The  $\mathfrak{M}_\rho$ 's are the invariant amplitudes, as defined in Ref. 4. Figure 1 compares experimental results for  $1.85 \times |\mathfrak{M}(\pi^-p \rightarrow \pi^0n)|^2$  (Ref. 5) and the natural-parity-exchange contribution to  $|\mathfrak{M}(\pi^-p \rightarrow \omega^0n)|^2$  (Ref. 6) at  $p_{\text{lab}} = 5.85 \text{ GeV}/c$  and  $5.5 \text{ GeV}/c$ , respectively.

The  $\pi^-p \rightarrow \omega^0n$  amplitude has a pronounced dip in the forward direction and lacks the shoulder exhibited by the  $\pi^-p \rightarrow \pi^0n$  amplitude beginning at  $t = -0.7 \text{ GeV}^2/c^2$ , but in the region between, the agreement is reasonably good. The discrepancy in the forward direction can be understood as the result of unequal-mass kinematics.<sup>7</sup> In the  $t$  channel, natural-parity exchanges require helicity flip

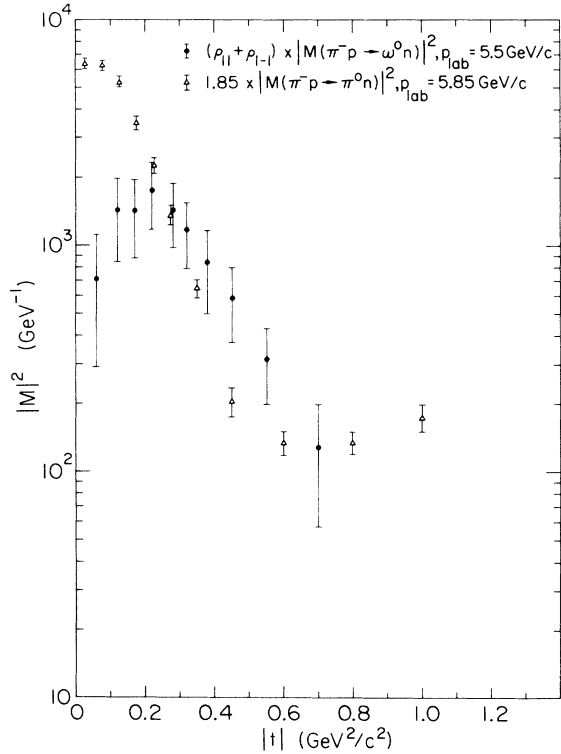


FIG. 1. Comparison of  $(\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^- p \rightarrow \omega^0 n)|^2$  at  $p_{\text{lab}} = 5.5 \text{ GeV}/c$  (Ref. 6) and  $1.85 \times |\mathfrak{M}(\pi^- p \rightarrow \pi^0 n)|^2$  at  $p_{\text{lab}} = 5.85 \text{ GeV}/c$  (Ref. 5).

at the  $\omega$ - $\pi$ -Reggeon vertex. Consequently, all helicity amplitudes to which natural parity exchange contributes contain half-angle factors. In the case of  $\pi^- p \rightarrow \omega^0 n$ ,

$$\begin{aligned} f_{10, \frac{1}{2}} &\propto \cos \frac{1}{2} \theta_t \sin \frac{1}{2} \theta_t = \frac{1}{2} \sin \theta_t, \\ f_{10, \frac{1}{2} - \frac{1}{2}} &\propto (\cos \frac{1}{2} \theta_t)^2. \end{aligned} \quad (2)$$

The magnitude of each of these factors decreases sharply in the forward ( $t' > -0.2 \text{ GeV}^2/c^2$ ) direction, leading one to expect a pronounced dip in the differential cross section at  $t' = 0$ . The dip seen is consistent, in shape and position, with the dip predicted by the kinematics.

Since the Feynman graph factorizes at the pole, we also have

$$\left| \frac{\mathfrak{M}_\rho(\pi^+ p \rightarrow \omega^0 \Delta^{++})}{\mathfrak{M}_\rho(\pi^+ p \rightarrow \pi^0 \Delta^{++})} \right|^2 = \left| \frac{\mathfrak{M}_\rho(\pi^- p \rightarrow \omega^0 n)}{\mathfrak{M}_\rho(\pi^- p \rightarrow \pi^0 n)} \right|^2 \quad (3)$$

at  $t = m_\rho^2$ . As in the  $\pi^- p \rightarrow \omega^0 n$  case, we expect a noticeable forward dip in the  $\pi^+ p \rightarrow \omega^0 \Delta^{++}$  differential cross section, arising from the half-angle factors contained in all the natural-parity-exchange amplitudes.

Figure 2 compares  $|\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$  and  $1.87 \times |\mathfrak{M}(\pi^+ p \rightarrow \pi^0 \Delta^{++})|^2$ , both at  $p_{\text{lab}} = 8 \text{ GeV}/c$ .<sup>8</sup> It should be noted that in Fig. 2 the  $|\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$

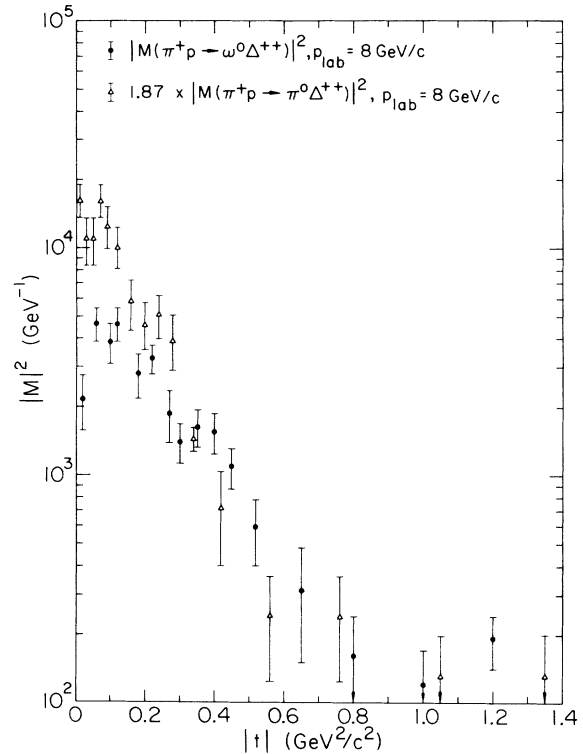


FIG. 2. Comparison of  $|\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$  and  $1.87 \times |\mathfrak{M}(\pi^+ p \rightarrow \pi^0 \Delta^{++})|^2$  at  $p_{\text{lab}} = 8 \text{ GeV}/c$  (Ref. 8).

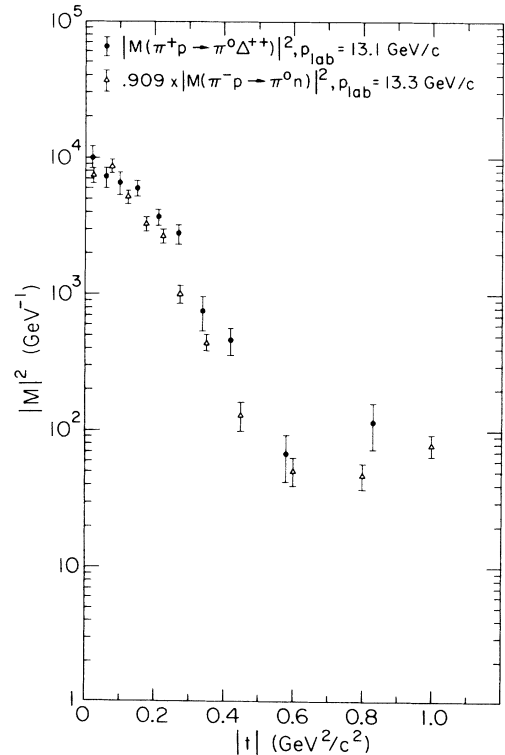


FIG. 3. Comparison of  $|\mathfrak{M}(\pi^+ p \rightarrow \pi^0 \Delta^{++})|^2$  at  $p_{\text{lab}} = 13.1 \text{ GeV}/c$  (Ref. 11) and  $0.909 \times |\mathfrak{M}(\pi^- p \rightarrow \pi^0 n)|^2$  at  $p_{\text{lab}} = 13.3 \text{ GeV}/c$ . (Ref. 5).

plotted contains both natural- and unnatural-parity-exchange parts. Unlike the  $\pi^-p \rightarrow \omega^0n$  case, we do not have the density-matrix elements needed to extract the natural-parity part of  $|\mathfrak{M}(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$ . Consequently the  $\rho$  exchange amplitude  $|\mathfrak{M}_\rho(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$  will be smaller than what is plotted for  $\pi^+p \rightarrow \omega^0\Delta^{++}$ . We do not encounter this problem in  $\pi^+p \rightarrow \pi^0\Delta^{++}$  since the  $\pi\pi$  system in the  $t$  channel is pure natural parity.

We can next compare  $\pi^+p \rightarrow \pi^0\Delta^{++}$  to  $\pi^-p \rightarrow \pi^0n$ . At the  $\rho NN$  vertex we use<sup>9</sup>

$$G_{\rho pp}^v = 4.0, \quad G_{\rho pp}^T = 12.0, \quad G_{\rho pn} = \sqrt{2}G_{\rho pp}. \quad (4)$$

At the  $\rho N\Delta$  vertex we assume magnetic dipole coupling and use the coupling constant  $G_{\rho p\Delta^{++}}/4\pi = 30$ .<sup>10</sup> We find (at  $t = m_\rho^2$ , of course)

$$\left| \frac{\mathfrak{M}_\rho(\pi^+p \rightarrow \pi^0\Delta^{++})}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2 = \begin{cases} 0.892, & p_{\text{lab}} = 8 \text{ GeV}/c \\ 0.909, & p_{\text{lab}} = 13.1 \text{ GeV}/c \\ 0.922, & p_{\text{lab}} = 25 \text{ GeV}/c \\ 0.934, & p_{\text{lab}} \rightarrow \infty. \end{cases} \quad (5)$$

Again using the factorization property,

$$\left| \frac{\mathfrak{M}_\rho(\pi^+p \rightarrow \omega^0\Delta^{++})}{\mathfrak{M}_\rho(\pi^-p \rightarrow \omega^0n)} \right|^2 = \left| \frac{\mathfrak{M}_\rho(\pi^+p \rightarrow \pi^0\Delta^{++})}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2. \quad (6)$$

Figure 3 compares  $|\mathfrak{M}(\pi^+p \rightarrow \pi^0\Delta^{++})|^2$  and  $0.909 \times |\mathfrak{M}(\pi^-p \rightarrow \pi^0n)|^2$  at 13.1 GeV/c (Ref. 11) and 13.3 GeV/c (Ref. 5), respectively. Figure 4(a) compares  $|\mathfrak{M}(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$  at 8 GeV/c (Ref. 8) with  $0.892 \times (\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^-p \rightarrow \omega^0n)|^2$  at 6.95 GeV/c.<sup>12</sup>

Again we encounter the problem that  $|\mathfrak{M}(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$  contains unnatural-parity exchange, whereas the  $(\rho_{11} + \rho_{1-1})$  factor extracts the natural-parity part of  $|\mathfrak{M}(\pi^-p \rightarrow \omega^0n)|^2$ . To point out the difference this makes, in Fig. 4(b) we compare  $|\mathfrak{M}(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$  at 8 GeV/c and  $0.892 \times |\mathfrak{M}(\pi^-p \rightarrow \omega^0n)|^2$  at 6.95 GeV/c. At a lower energy, we do have the density matrices needed. Figure 4(c) compares

$$0.88 \times (\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^-p \rightarrow \omega^0n)|^2$$

at 5.5 GeV/c (Ref. 6) and

$$(\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^+p \rightarrow \omega^0\Delta^{++})|^2$$

at 5.45 GeV/c.<sup>13</sup>

We next turn to  $A_2$  production. Using  $g_{A_2\rho^+ \pi^+}^2/4\pi = 41.0/\text{GeV}^4$  (obtained with  $\Gamma = 112 \text{ MeV}$ ,  $M_{A_2} = 1.31 \text{ GeV}$ , branching ratio = 76%) we find

$$\left| \frac{\mathfrak{M}_\rho(\pi^-p \rightarrow A_2^0n)}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2 = \left| \frac{\mathfrak{M}_\rho(\pi^+p \rightarrow A_2^0\Delta^{++})}{\mathfrak{M}_\rho(\pi^+p \rightarrow \pi^0\Delta^{++})} \right|^2 = \begin{cases} 0.14, & p_{\text{lab}} = 4.5 \text{ GeV}/c \\ 0.23, & p_{\text{lab}} = 20 \text{ GeV}/c \\ 0.24, & p_{\text{lab}} \rightarrow \infty. \end{cases} \quad (7)$$

From the kinematic factors associated with the exchange of a natural-parity object in production of a  $2^+$  meson, we again expect a forward dip. Figure 5 compares  $|\mathfrak{M}(\pi^+p \rightarrow A_2^0\Delta^{++})|^2$  with  $0.2 \times |\mathfrak{M}(\pi^+p \rightarrow \pi^0\Delta^{++})|^2$  at  $p_{\text{lab}} = 8 \text{ GeV}/c$ .<sup>14</sup>

We notice here a definite discrepancy. The discrepancy itself is expected since, as in Fig. 4(a), we are comparing the differential cross section to the contribution from the exchange of the  $\rho$  trajectory alone. The question is whether the size of the discrepancy is correct. Since elements of the spin density matrix have not been measured for this reaction, we cannot determine the amounts of natural- and unnatural-parity exchange. If we assume that about half of  $A_2$  production goes by unnatural-parity exchange, as is the case with  $\omega$  production, then our prediction for the  $\rho$  contribution is about the right size. (We do not worry about the shoulder structure around  $t = -0.6$ .) The assumption that only about half of the cross section results from natural-parity exchange receives some support from comparison of energy dependence of the total cross sections for  $\pi^+ + n \rightarrow A_2^0 + p$  (Ref. 15) and  $\pi^- + p \rightarrow \omega^0 + n$ .<sup>6</sup> A problem remains, however. The expected forward dip is not present in the  $A_2$  production data. It is possible that the unnatural-parity contribution is washing out this dip. Until the spin density matrix elements are available we cannot be sure.

In the case considered thus far, we have obtained reasonably good estimations of the size of  $\rho$ -exchange contributions to differential cross sections by using

$$|\mathfrak{M}_\rho(t)|^2 = \left| \frac{\mathfrak{M}_\rho(m_\rho^2)}{\mathfrak{M}'_\rho(m_\rho^2)} \right|^2 |\mathfrak{M}'_\rho(t)|^2,$$

where  $\mathfrak{M}'_\rho(t)$  is the invariant matrix element of some reference reaction. One can now estimate the cross sections of unmeasured processes in which  $\rho$  exchange should dominate, such as  $\varphi$  and  $A_1$  production in charge exchange processes. In determining the coupling constants we use  $m_{A_1} = 1.07 \text{ GeV}$ ,  $\Gamma_{A_1} = 0.095 \text{ GeV}$ ,  $\Gamma(A_1 \rightarrow 3\pi) = \Gamma(A_1 \rightarrow \rho\pi) = \Gamma(\text{total})$ ,  $m_\varphi = 1.019 \text{ GeV}$ ,  $\Gamma_\varphi = 4.0 \text{ MeV}$ ,  $\Gamma(\varphi \rightarrow \rho\pi) = \Gamma(\varphi \rightarrow 3\pi) = 0.183 \Gamma(\text{total})$ . The results obtained are

$$\left| \frac{\mathfrak{M}_\rho(\pi^-p \rightarrow \varphi^0n)}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2 = \begin{cases} 0.006, & p_{\text{lab}} = 4.5 \text{ GeV}/c \\ 0.008, & p_{\text{lab}} \rightarrow \infty \end{cases} \quad (8)$$

and

$$\left| \frac{\mathfrak{M}_\rho(\pi^-p \rightarrow A_1^0n)}{\mathfrak{M}_\rho(\pi^-p \rightarrow \pi^0n)} \right|^2 = \begin{cases} 0.08, & p_{\text{lab}} = 4.5 \text{ GeV}/c \\ 0.05, & p_{\text{lab}} = 20 \text{ GeV}/c \\ 0.05, & p_{\text{lab}} \rightarrow \infty \end{cases} \quad (9)$$

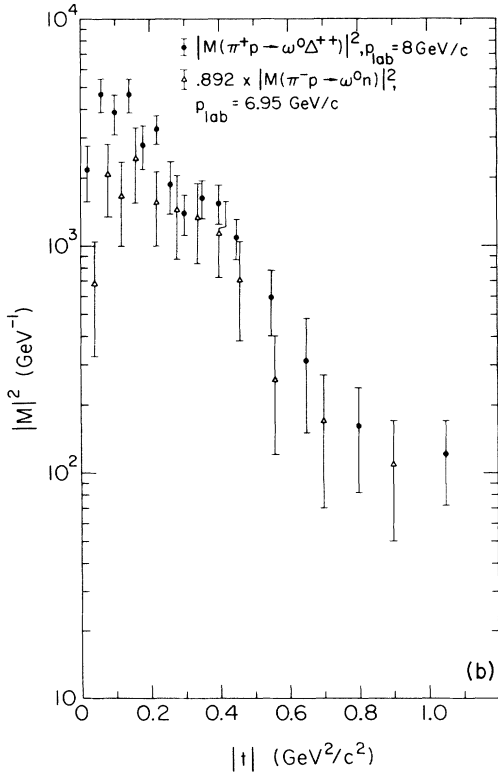
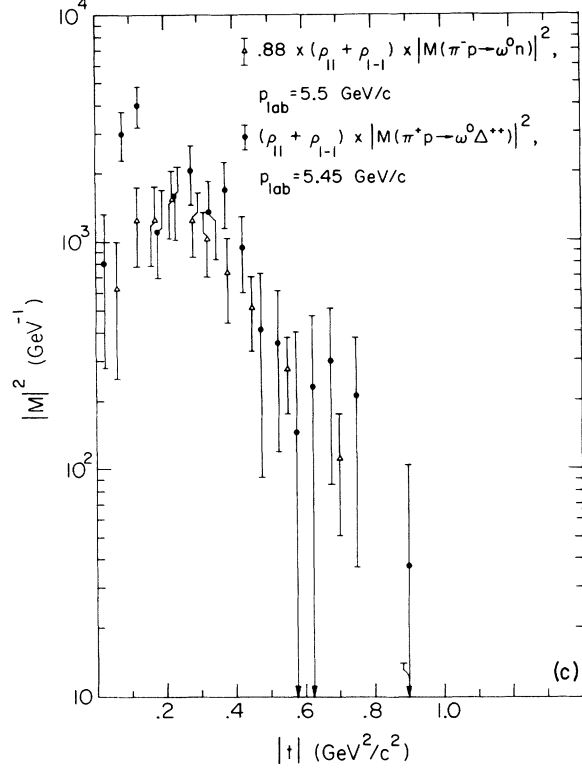
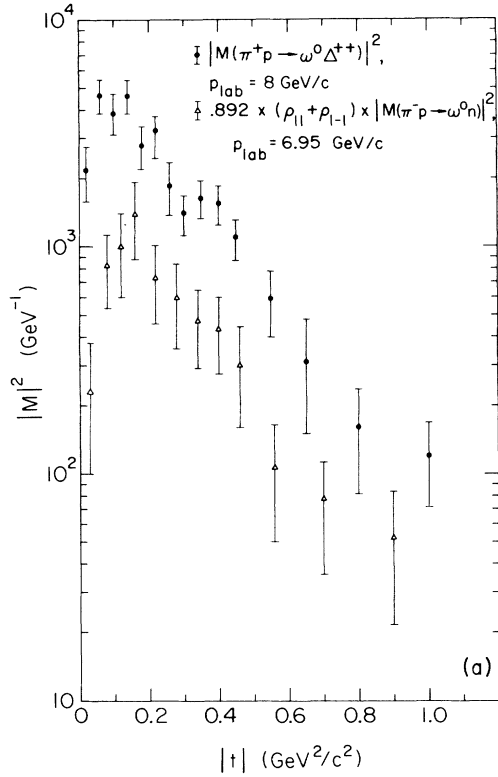


FIG. 4. (a) Comparison of  $0.892 \times$  natural parity part of  $|\mathfrak{M}(\pi^- p \rightarrow \omega^0 n)|^2$  at  $p_{\text{lab}} = 6.95$  GeV/c (Ref. 12) and  $|\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$  at  $p_{\text{lab}} = 8$  GeV/c (Ref. 8). (b) Comparison of  $0.892 \times |\mathfrak{M}(\pi^- p \rightarrow \omega^0 n)|^2$  at  $p_{\text{lab}} = 6.95$  GeV/c (Ref. 12) and  $|\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$  at  $p_{\text{lab}} = 8$  GeV/c (Ref. 8). (c) Comparison of  $0.88 \times (\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^- p \rightarrow \omega^0 n)|^2$  at  $p_{\text{lab}} = 5.5$  GeV/c (Ref. 6) and  $(\rho_{11} + \rho_{1-1}) \times |\mathfrak{M}(\pi^+ p \rightarrow \omega^0 \Delta^{++})|^2$  at  $p_{\text{lab}} = 5.45$  GeV/c (Ref. 13).

and of course

$$\begin{aligned} \left| \frac{\mathfrak{M}_\rho(\pi^+ p \rightarrow A_1^0 \Delta^{++})}{\mathfrak{M}_\rho(\pi^- p \rightarrow A_1^0 n)} \right|^2 &= \left| \frac{\mathfrak{M}_\rho(\pi^+ p \rightarrow \varphi^0 \Delta^{++})}{\mathfrak{M}_\rho(\pi^- p \rightarrow \varphi^0 n)} \right|^2 \\ &= \left| \frac{\mathfrak{M}_\rho(\pi^+ p \rightarrow \pi^0 \Delta^{++})}{\mathfrak{M}_\rho(\pi^- p \rightarrow \pi^0 n)} \right|^2. \end{aligned} \quad (10)$$

This method also enables us to predict the  $\rho$  contribution to  $pp \rightarrow n\Delta^{++}$ . Ma and co-workers<sup>16</sup> have pointed out that this reaction is still dominated by  $\pi$  exchange at  $p_{\text{lab}} = 25$  GeV/c. Using the fact that

$$\left| \frac{\mathfrak{M}_\rho(pp \rightarrow n\Delta^{++})}{\mathfrak{M}_\rho(pn \rightarrow np)} \right|^2 = \left| \frac{\mathfrak{M}_\rho(\pi^+ p \rightarrow \pi^0 \Delta^{++})}{\mathfrak{M}_\rho(\pi^- p \rightarrow \pi^0 n)} \right|^2 = 0.9$$

at  $p_{\text{lab}} = 25$  GeV/c,  $t = m_\rho^2$ , we can determine the  $\rho$  contribution to  $pp \rightarrow n\Delta^{++}$  from its contribution to  $pn$  charge-exchange scattering. The cross section for  $pn$  charge-exchange (CEX) scattering has been fitted by various authors.<sup>17</sup> Although the  $\rho$  amplitudes are small compared to the  $\pi$  amplitudes, they can be isolated from the  $\pi$  amplitudes

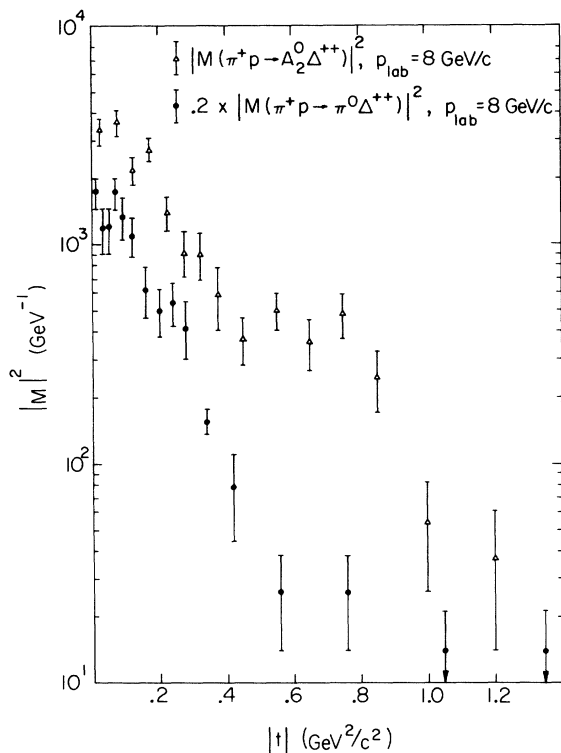


FIG. 5. Comparison of  $|\mathfrak{M}(\pi^+p \rightarrow A_2^0\Delta^{++})|^2$  and  $0.2 \times |\mathfrak{M}(\pi^+p \rightarrow \pi^0\Delta^{++})|^2$  at  $p_{\text{lab}} = 8 \text{ GeV}/c$  (Ref. 14).

by considering  $d\sigma/dt$  ( $pn$  CEX)  $- d\sigma/dt$  ( $\bar{p}p$  CEX), since the  $\rho$  trajectory has negative signature whereas the  $\pi$  (and  $A_2$ ) have positive signature. Using the fit of Phillips<sup>17</sup> one finds the  $\rho$  part of the  $pn$  charge-exchange differential cross section to be  $0.017 \text{ mb}/(\text{GeV}^2/c^2)$  at  $p_{\text{lab}} = 25 \text{ GeV}/c$  and  $t = -0.2 \text{ GeV}^2/c^2$ . At  $p_{\text{lab}} = 25 \text{ GeV}/c$ ,  $t = -0.2 \text{ GeV}^2/c^2$ ,  $d\sigma/dt$  ( $pp \rightarrow n\Delta^{++}$ )  $\cong 0.2 \text{ mb}/(\text{GeV}^2/c^2)$ . Hence it is no surprise that one finds the dependence of the cross section on  $p_{\text{lab}}$  to be still char-

acterized by  $\pi$  exchange. Assuming an exchange-degenerate  $\rho$  and  $A_2$ , and assuming that  $\pi$  exchange is responsible for about 75% of the cross section at  $25 \text{ GeV}/c$ , we expect that the  $\pi$  and the  $\rho + A_2$  contributions will become equal at  $p_{\text{lab}} \cong 120 \text{ GeV}/c$ .

Photoproduction can also be treated in this context. We consider the  $\rho$ -exchange part of  $\gamma p \rightarrow \pi^+ n$ . For the  $\gamma\rho\pi$  vertex we invoke vector dominance in order to use  $g_{\gamma\omega} \times (\omega\rho\pi \text{ vertex})$ , where the  $\omega$  is transversely polarized. The calculation then yields

$$\left| \frac{\mathfrak{M}_\rho(\gamma p \rightarrow \pi^+ n)}{\mathfrak{M}_\rho(\pi^- p \rightarrow \pi^0 n)} \right|^2 = 0.7 \times 10^{-3}, \quad (11)$$

at  $s = 11.2$  ( $p_{\text{lab}} = 5.5 \text{ GeV}/c$ ). This would constitute less than one tenth of the experimentally observed cross section at this energy.<sup>18</sup> It is appreciably bigger than the  $\rho$  contribution which Frøyland and Gordon find in their fit to the photoproduction data.<sup>19</sup> As they suggest, however, their  $\rho$  residue parameter could contain a sizeable error due to a large error in the phase and magnitude of the  $\rho$ -Pomeranchukon cut.

The approximation that the ratio of invariant amplitudes is the same in the physical region as at the  $\rho$  pole, when combined with determination of the presence of a forward dip on the basis of simple kinematic considerations, appears to give a reasonably good estimation of the  $\rho$  contribution to differential cross sections. This estimation can be useful in predicting cross sections in which  $\rho$  exchange is expected to dominate and in estimating the relative importance of  $\rho$  exchange for a reaction in which other trajectories can be exchanged.

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## *CPT*-Violating Model of Weak Interactions\*

J. P. Hsu

*Department of Physics, Rutgers - The State University, New Brunswick, New Jersey 08903†  
and Institute of Theoretical Physics, McGill University, Montreal, Province de Québec, Canada*

and

M. Hongoh

*Institute of Theoretical Physics, McGill University, Montreal, Province de Québec, Canada*

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We study a model of weak interactions in which the divergences are no worse than those of the renormalizable theory and the *CPT* invariance is maximally violated. The model is consistent with all existing data, and some data favor our predictions over those of conventional theories. In particular, the model predicts that the lifetime of  $\Lambda^0$  decaying in flight at 100 GeV will be shorter than that measured at rest by  $\sim 16\%$ , a prediction which can be tested at the National Accelerator Laboratory.

### I. INTRODUCTION

In a previous paper, we have constructed a model to explore the possible violation of *CPT* invariance in the domain of weak interactions.<sup>1</sup> It was found that one can have a maximal *CPT* violation in the weak interactions without contradicting the existing data. In the model, the divergences in the high-order amplitudes are no worse than those of the renormalizable theories; and the coupling contains the usual weak currents, a heavy intermediate boson, and a zero four-momentum "aoraton" (which means "invisible particle"). This model is consistent with the following remarkable features: (a) the smallness of the neutral leptonic decay modes and the  $K_L^0 - K_S^0$  mass difference, (b) the universality of weak interactions manifested through the usual weak vector and axial-vector currents, (c) the experimental absence of the intermediate boson. Furthermore, the neutral leptonic current can be excluded from the weak-interaction Lagrangian density  $\mathcal{L}_{\text{int}}(x)$  if one assumes that  $\int \mathcal{L}_{\text{int}}(x) d^4x$

satisfies a symmetry principle.<sup>2</sup>

Here we shall first discuss the interaction Lagrangian and Feynman rules for the model, and then discuss some of its further implications and their experimental tests.

### II. THE INTERACTION LAGRANGIAN AND THE FEYNMAN RULES

The weak-interaction Lagrangian is assumed to be<sup>1</sup> (we use the notation in Ref. 1)

$$\mathcal{L}_{\text{int}}(x) = gJ_\lambda(x)S^\dagger(x)h_\lambda^*(x) + gJ_\lambda^*(x)S(x)h_\lambda(x), \quad (1)$$

where  $g$  is the coupling constant,  $J_\lambda$  is the usual weak current,  $S$  is a scalar boson, and  $h_\lambda$  is given by

$$h_\lambda = \sum_{m=1}^4 (a_m + \chi a_m^\dagger) e_\lambda^{(m)},$$

$$h_\lambda^\dagger = \sum_{m=1}^4 (a_m^\dagger + \chi a_m) e_\lambda^{(m)*},$$