verse momenta), normalized by

$$\int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_{N_n} f^n(x_1, \ldots, x_{N_n}) = 1.$$

Then νW_2 is given by

$$\nu W_2(x) = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \cdots dx_{N_n} x_i \, \delta(x_i - x) f^n(x_1, \ldots, x_{N_n}),$$

where P_n is the probability for finding the state $|n\rangle$ in the proton. Integrating from 0 to 1,

$$\int_0^1 \nu W_2(x) dx = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \cdots dx_{N_n} x_i f^n(x_1, \ldots, x_{N_n})$$

$$= \sum_n P_n \sum_i \lambda_i^2 (\bar{x}_i)_n,$$

where $(\bar{x}_i)_n$ is the average momentum of particle i in the state $|n\rangle$. Since for any values of $x_1, \ldots, x_{N_n}$ momentum conservation requires $\sum_{i=1}^{N_n} x_i = 1$ we find

$$\sum_{i=1}^{N_n} (\bar{x}_i)_n = 1.$$

If $f(x_1, \ldots, x_{N_n})$ is symmetric under interchange of any indices i and j, $(\bar{x}_i)_n$ is independent of i. This implies $(\bar{x}_i)_n = (\bar{x})_n = 1/N_n$ which proves the contention that $\int_0^1 \nu W_2(x) dx$ is the mean square charge.

[17]Llewellyn Smith (Ref. 6).

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A Model of Leptons*†

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A new model of leptons based on discrete scale transformations is proposed. It is shown that this model predicts a lepton mass spectrum consisting of an infinite series of electron-like and muonlike particles whose masses and charges are given by $m_n = m_e \rho^n$ $(\rho = m_\mu / m_e)$ $Q = \frac{1}{2}e(1 + n/|n|)$. Particles with n positive are charged, those with n negative neutral. Possible weak coupling schemes of charged and neutral leptons are considered. The lepton with $n = 2$ is a heavy electron at 22 GeV. Decay modes and production mechanisms of this particle are discussed. It is shown that, apart from high production cross sections needed to fit experimental data, some of the recently observed anomalies in cosmic-ray muons can be effects due to heavy leptons predicted by this model.

I. INTRODUCTION

Ever since its unexpected discovery, the muon has remained as a tantalizing puzzle in elementary particle physics.[1] Except for the discovery of the muon neutrino the situation today regarding leptons is the same as in 1947. (Even this was anticipated in the paper of Sakata and Inouye[2] written in 1946 to clarify the π-μ puzzle.) No new charged leptons were found. So far no experimental or theoretical clue has been found to suggest a difference between the electron and the muon, apart from the mass. All experiments carried out up to this date, viz., measurement of the branching ratios for decays of hadrons into electrons and muons,[3] precision measurements of the muon magnetic moment,[4] electron-proton and muon-proton scattering[5] and the recent experiment that demonstrated that the muon obeys the same statistics as the electron,[6] show that the muon is a heavy electron or in other words, that the so-called electron-muon universality is strictly obeyed.

Many attempts have been made to understand the muon puzzle, but none of them are entirely satisfactory. One class of such theories attempts to derive the muon from quantum electrodynamics. Some of these derivations are based on the observation that muon-electron mass ratio is almost exactly $\frac{3}{2}(1/\alpha)$, this being taken as an indication that the key to the muon-electron puzzle may lie entirely within the realm of ordinary quantum electrodynamics. It is possible to start with bare (zero-mass) electron and muon fields interacting with the electromagnetic field and get two distinct masses by renormalization.[7] But these arguments were cutoff-dependent. In theories with spontaneous breakdown,[8] the necessity of a cutoff is removed, and it is possible to obtain two renormalized masses. Furthermore, the heavier lepton remains stable, i.e., $\mu \to e + \gamma$ remains forbidden, which is nice[9]; the mass ratio, however, remains arbitrary.[10]

Another approach to the muon problem depends on higher-order wave equations. Markov[11] showed that the two linear equations $(i\not{\partial} + m)\psi_1 = 0$ and $(i\not{\partial} - m)\psi_2 = 0$ (where ψ is a four-component spinor), together are equivalent to a second-order equation that can describe two Fermi particles conjugate in the sense of Konopinski and Mahmoud[12] (e^-, μ^+ particles and e^+, μ^- antiparticles). Markov interpreted the above equations as representing the electron and the muon. Although the fact that the Konopinski-Mahmoud hypothesis is a consequence of the theory is attractive, the bare masses of the particles are the same and no straightforward way is found to excite one of them to a higher level. Second-order equations are also considered by Rosen,[13] who proposes a quantum-mechanical equation for the electron with a radiative reaction similar to the one in the classical electron theory. Larger representations of the Lorentz group have been used by some authors to describe rest-frame states of leptons leading to new wave equations. A theory of this type was proposed recently by Kurşunoğlu,[14] using the six-dimensional representations of the group SU(3, 1); he has predicted two new spin-$\frac{3}{2}$ leptons. Barut[15] proposes to solve the muon problem by giving up one of the standard assumptions of the Dirac theory: the proportionality between the conserved quantum mechanical probability current and the conserved electromagnetic current. It is shown that a very small deviation of the probability current from the charge current suffices to give the observed masses of μ and e.

Another class of theories attempts to solve the muon problem by assigning an anomalous interaction to the muon. Schwinger[16] has proposed a scalar boson which interacts exclusively with the muon while a vector boson with nonderivative coupling was suggested by Kobzarev and Okun.[17] Ne'eman[18] suggested a connection between the SU(3)-breaking medium-strong interaction (fifth force) and a vector meson coupled anomalously to the muon. Recently discovered anomalous properties of cosmic-ray muons have aroused a fresh interest in theories of this type: Ng and Sugano[19] have used derivative couplings of a vector boson with muon field to explain the origin of muon mass. The main difficulties that theories of this type face have been summarized by Feinberg and Lederman.[20]

Another frequent speculation has been that the origin of muon mass may have to do with gravitation. A quantitative theory of this type was put forward by Motz.[21] According to his theory, an elementary particle is regarded as an object of finite radius in which there is a sharp discontinuity in the gravitational field across the boundary. Electromagnetic repulsive forces are balanced by the gravitational force. The metric ds^2 for the space near the particle is written as a function of the radius r of the particle. The condition that the path of a photon is given by $ds^2=0$ leads to a quadratic equation, whose two roots are taken to correspond to the electron and the muon. The mass of the muon obtained this way is too large by a factor 2.5. Finally we mention an interesting early attempt by Dirac[22] to solve the muon problem. Dirac proposed that the electron should be considered classically as a charged conducting sphere similar to a soap bubble, with surface tension to prevent it from flying apart under the repulsive forces of the charge. States of stable equilibrium of the system with spherical symmetry were investigated after quantizing the action integral for the system in the manner of Bohr and Sommerfeld. An excited state of the system of mass 53 times the electron mass was obtained. The large deviation from the experimental value may be attributed to the fact that spin is neglected in the theory.

In this work[23] an entirely different approach is used to seek a solution to the problem of leptons. No attempt is made to understand the dynamical origin of the muon mass. Instead the strict electron-muon universality observed by experiment is used as the basis for the theory. The fact, that apart from the mass, the muon is exactly similar to the electron, strongly suggests that some form of scale invariance is connected with leptons.

Investigations on the possible relevance of scale transformations to physical theories are not at all new. Soon after the formulation of the special theory of relativity, it was shown by Cunningham[24] and by Bateman[25] that Maxwell's equations are invariant not only under the 10-parameter Lorentz group, but also under the larger 15-parameter conformal group. The conformal group contains, besides the Lorentz group, scale transformations and an inversion as subgroups. Wess[26] considered the possibility of constructing a conformally invariant field theory and showed explicitly that a field theory can be invariant under dilations only if its quanta have zero rest mass. Heisenberg[27] attempted to show that lepton conservation follows from the scale invariance of his nonlinear spinor theory. Recently possible applications of scale invariance to theories describing elementary particles have attracted much attention.

Broken scale invariance, realized by Nambu-Goldstone bosons, and its possible relation to the breaking of chiral SU(3)×SU(3) has been a popular theme in hadron physics[28] for the last few years. The relation of scale invariance to the scaling laws in deep-inelastic electron-proton scattering[29] has also been of great interest.

The scale transformations considered above are continuous transformations which form the dilation group. In this investigation scale transformations of a different kind are used as the basis for constructing a theory of leptons. These transformations are discontinuous and form an Abelian group.

The relevance of discrete scale transformations to leptons is suggested by the extreme precision with which the electron-muon universality is realized. The origin of discrete scale invariance may be dynamical or geometrical. When all interactions are taken into account, it may turn out that the theory is invariant for one particular scale transformation. Clearly, this will lead to discrete scale invariance. Or it may be a fundamental property of the space-time manifold itself. It is assumed that the scale transformations relevant to the theory of leptons are $x_\mu \to \rho x_\mu$ with $\rho = m_\mu / m_e = 206.765$. In this work no attempt will be made to explain the number ρ or to understand the origin of scale invariance. Instead, the consequences that follow from such invariance will be discussed. (There have been very few uses of discrete scale transformations in the literature. Mitter[30] uses it to determine the asymptotic behavior of the propagator in nonlinear field theory, and Daboul[31] uses it to relate processes involving the electron and muon through e-μ universality.)

The theory predicts the existence of an infinite series of charged and neutral leptons, which includes all the known leptons. The series is in one-to-one correspondence with the set of positive and negative integers, i.e., leptons are a representation of the group of all integers with addition as the group property. Leptons that correspond to zero or positive integers are charged, those corresponding to negative integers are neutral. Particles with n even are electronlike, those with n odd are muonlike. Masses and charges of leptons are given by $m_n = m_e \rho^n$ and $Q = (e/2)(1 + n/|n|)$, respectively. The lightest new charged lepton predicted by the theory is an electronlike particle of mass 22 GeV. The heaviest muonlike and electronlike neutrinos are predicted to have masses 2.5 KeV and 12.1 eV, respectively. From discrete scale invariance alone, it is not possible to deduce an unambiguous coupling scheme for neutrinos and charged leptons. Two of the simplest possibilities are (1) each charged lepton is coupled to one specific neutrino, (2) each charged lepton of a given type (muonlike or electronlike) is coupled to all neutrinos of the same type (muonlike or electronlike). The second coupling scheme is possible only if the coupling constants for processes involving different neutrinos are different and satisfy a relation of the form $\sum_i g_i^2 = \text{constant}$. Otherwise the decay rates of leptonic and semileptonic processes would be infinite. The alternative possibility, that the lepton mass spectrum consists of two zero-mass neutrinos and an infinite family of charged leptons, is also considered. Implications of nonzero-rest-mass neutrinos are briefly considered. The measurement of neutrino rest masses and the detection of the predicted heavy electron are decisive tests for the theory. A number of searches for heavy leptons have been carried out,[32-37] although none of these experiments is conclusive. One might say that all laboratory experiments carried out to date tend to support the statement that heavy charged leptons less massive than 1 GeV with standard couplings probably do not exist. At energies accessible at NAL or ISR, detection of the e^* seems to be possible. Decay modes and production mechanisms of the e^* are discussed in detail. Finally, an attempt is made to interpret the recently observed anomalies in cosmic rays as manifestations of heavy leptons (e^*, μ^*) predicted by this theory. If one is willing to accept the high cross sections needed to fit the experimental data, then the anomalous scattering of cosmic-ray muons observed by the Utah cosmic-ray group,[38] the large rate of energy loss of cosmic ray muons,[39] and the anomalies observed in the stopping rate of underground cosmic-ray muons[40] as well as horizontal and muon-poor air showers can all be explained as effects due to the presence of heavy leptons e^* and μ^*.

II. DISCRETE SCALE TRANSFORMATIONS

The scale transformations conventionally considered in the literature are of the form

$$x_\mu \to x'_\mu = \lambda x_\mu , \tag{1}$$

where λ is a continuous parameter. Under this transformation, an operator S transforms as

$$S(x) \to S'(x') = \lambda^l S(\lambda x) , \tag{2}$$

where l is defined as the dimension of the operator S.

The dimension of a free-field operator is determined as follows. Consider, as an example, the scalar field ϕ whose free-Lagrangian density is

$$\mathcal{L} = -\frac{1}{2}\left(\frac{\partial \phi}{\partial x_\mu}\,\frac{\partial \phi}{\partial x_\mu} + m^2 \phi^2\right). \tag{3}$$

$\mathcal{L}$, being an energy density, has dimension $l = -4$. In the limit of scale invariance, i.e., $m = 0$, only the kinetic-energy term survives and this gives the dimensions of the scalar field as $l = -1$. In other words, under the transformation (1) ϕ transforms as

$$\phi(x) \to \phi'(x') = \lambda^{-1} \phi(\lambda x) . \tag{4}$$

In the same way, it can be shown that Fermi fields have dimension $l = -\frac{3}{2}$. The mass terms are not invariant under scale transformations but transform in a well-defined manner.

Instead of considering continuous scale transformations corresponding to continuous λ, as is customary, we consider discrete transformations

$$x_\mu \to x'_\mu = \rho x_\mu\ , \tag{5}$$

where ρ is now not a variable parameter but takes some fixed value. The dimensions of field quantities corresponding to discrete transformations are chosen to be the same as in the continuous case, i.e., Bose fields have dimension $l=-1$ and Fermi fields $l=-\frac{3}{2}$.

We assume that the discrete scale transformations relevant to the theory of leptons are of the form (5) with $\rho = m_\mu/m_e \simeq 207$. The choice of this value for ρ is suggested by experiment (electron-muon universality). No attempt will be made to explain the number ρ, which is regarded as a fundamental constant. (We do not rule out the possibility that ρ can be explained later in terms of fundamental coupling constants such as the fine-structure constant α and the gravitational constant κ.)

The most general discrete scale transformation of the form (5) is

$$x_\mu \to x'_\mu = \rho^n x_\mu\ , \tag{6}$$

where $n = 0, \pm 1, \pm 2, \ldots$. Under (6), the Dirac field $\psi(x)$, whose Lagrangian density is

$$\mathfrak{L} = -\overline{\psi}\gamma_\mu \partial_\mu \psi - m\overline{\psi}\psi\ , \tag{7}$$

transforms as

$$\psi(x) \to \psi'(x') = \rho^{-3n/2}\,\psi(\rho^n x)\ . \tag{8}$$

We can define an operator $U(n)$ such that

$$U(n)\psi(x)U^{-1}(n) = \rho^{-3n/2}\psi(\rho^n x)\ .$$

It is easily seen that for any two integral values n and m

$$\begin{aligned} U(n)U(m) &= U(m)U(n) \\ &= U(m+n)\ . \end{aligned} \tag{9}$$

Thus our discrete scale transformations leads to an Abelian group.

The transformation defined by (6) and (8) takes $\mathfrak{L}$ given by (7) into $\mathfrak{L}'$:

$$\begin{aligned} \mathfrak{L} \to \mathfrak{L}' &= U(n)\mathfrak{L}U^{-1}(n) \\ &= \rho^{-4n}\left[-\overline{\psi}'\gamma_\mu \partial_\mu \psi' - (m\rho^n)\overline{\psi}'\psi'\right]. \end{aligned} \tag{10}$$

$\mathfrak{L}'$ is now of same form as $\mathfrak{L}$ with m replaced by $m\rho^n$. This result is interpreted as follows. "The existence of a Dirac field whose quanta have masses m implies the existence of other Dirac fields with quanta of masses $m_n = m\rho^n$." Thus fields of different masses are obtained from the form invariance of $\mathfrak{L}$ given by (7) under (8).

The parameter ρ need not be restricted to $+(m_\mu/m_e)$. If we assume instead that $\rho = -(m_\mu/m_e)$, then for odd values of n, above requirement leads to negative masses $m_n = m\rho^n$. Negative masses can be avoided if we define the transformation for $\rho < 0$ to be

$$\begin{aligned} &x_\mu \to x'_\mu = \rho^n x_\mu\ , \\ &\psi(x) \to \psi'(x') = \rho^{-3n/2}\gamma_5^{|n|}\,\psi(\rho^n x)\ ; \end{aligned} \tag{11}$$

then it takes $\mathfrak{L}$ given by (7) into Lagrangian density for another Dirac field of mass $m|\rho|^n$ for both even and odd values of n. To see this, note that the transformation (11) reduces to (8) for n even, and it reduces to

$$\begin{aligned} &x_\mu \to -\sigma^n x_\mu\ , \\ &\psi(x) \to \psi'(x') = i\sigma^{-3n/2}\gamma_5\psi(-\sigma^n x) \end{aligned} \tag{12}$$

(where $\sigma = -\rho$) when n is odd. This changes $\mathfrak{L}$ to $\mathfrak{L}'$ given by

$$\mathfrak{L}' = -\sigma^{4n}\left[-\overline{\psi}'\gamma_\mu\partial_\mu\psi' - (m\sigma^n)\overline{\psi}'\psi'\right]. \tag{13}$$

It is clear that transformation (11) also satisfies the group property (9). The factor $i\gamma_5$ in front of $\psi(-\sigma^n x)$ in (12) mixes the field components in the spinor. Hence even and odd fields differ by a factor $i\gamma_5$. If we assume that when $n=0$ $\mathfrak{L}$ is the Lagrangian density for the electron field, then all fields corresponding to even n are electronlike and carry the electron lepton number, while all the odd fields are muonlike and carry the muon lepton number. [Apart from the dilation factor σ, the transformation (12) which takes the electron field into the muon field is same as the strong reflection operation defined by Lüders in connection with the CPT theorem.[41]] The charge on a lepton is defined to be $Q = \frac{1}{2}e(1 + n/|n|)$ with $Q = e$ for $n=0$, which is the analog of Gell-Mann–Nishijima relation for leptons. The number n specifies a lepton completely and is a universal quantum number for leptons. The masses of leptons are given by m_n

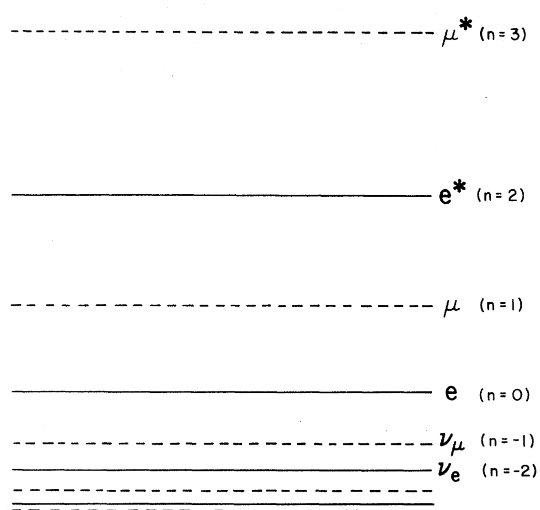

FIG. 1. Lepton mass spectrum.

$=m_e\rho^n$ (see Fig. 1). The leptons with zero or even values of n are electronlike, while those with odd n are muonlike; leptons with positive n are charged and those with negative n are neutral (neutrinos). The lepton just above the muon, denoted by e^*, is an electronlike particle of mass 22 GeV; the next lepton, μ^*, is muonlike and of mass 4554 GeV, and so on. Also, according to this theory, the heaviest neutrino is muonlike and has a mass 2.5 keV. The neutrino just below it is electronlike and has a mass of 12.1 eV. The current experimental upper limit for the mass of muon neutrino is 600 keV,[42] and for the mass of electron neutrino is 55 eV.[43]

The existence of leptons with the masses as predicted by this theory does not lead to any inconsistency with experimental data. If charged leptons with masses close to that of the muon exist, they would have observable effects on the magnetic moments of the electron and muon, on the Lamb shift, and on Compton scattering. Consideration of the anomalous magnetic moment of the muon leads to the result that new charged leptons less massive than $30m_e$ cannot exist.[44] The change in the electron magnetic moment due to presence of the above infinite family of charged leptons can be deduced from the result of Lautrup and de Rafael[45] for the contribution to the electron magnetic moment due to the muon (Fig. 2). Their result is

$$\Delta\mu=(\alpha^2/45\pi^2)(m_e/m_\mu)^2 .$$

When all diagrams of the above form with μ replaced by every possible charged lepton are summed, a convergent result,

$$\Delta\mu'=\frac{\alpha^2}{45\pi^2\rho}=0.3\times10^{-11},$$

is obtained – a result clearly consistent with the experimental value 0.001 159 646(7) for the anomalous electron magnetic moment.[46] However, if the nonminimal interaction

$$\bar\psi_{e^*}\sigma_{\mu\nu}\psi_e F_{\mu\nu} \tag{14}$$

exists between the heavy electron e^*, the electron, and the electromagnetic field, then there can be considerable deviation from quantum electrodynamics at energies comparable to the mass of the e^*.

It is amusing to note that the fourth-order contribution to the magnetic moments of leptons increases as we go up in the series. The fourth-order contribution to the magnetic moment of e^* from the diagram in Fig. 3 is $2.84\,\alpha^2/\pi^2$ in units $e\hbar/2m_{e^*}c$, while for μ^*, it is $4.6\,\alpha^2/\pi^2$ in units $e\hbar/2m_{\mu^*}c$. The contribution to the muon magnetic moment from a similar diagram is $0.75(\alpha^2/\pi^2)e\hbar/2m_\mu c$. In leptons whose universal quantum number is greater than ~132, the fourth-order contribution to the anomalous magnetic moment will exceed the second-order contribution.

III. WEAK AND ELECTROMAGNETIC INTERACTIONS

A. Electromagnetic Interactions

Since we have defined the charge to be

$$Q=\tfrac{1}{2}e(1+n/|n|),$$

the minimal electromagnetic interaction of leptons in the series is fixed to be

$$\mathcal{L}^I_{\text{e.m.}}=\tfrac{1}{2}e\sum_{n=0}^{\infty}\bar\psi_n\gamma_\mu(1+n/|n|)\psi_n A_\mu . \tag{15}$$

If we define a column vector ψ as

$$\psi=\begin{bmatrix}\vdots\\ \psi_2\\ \psi_1\\ \psi_0\\ \psi_{-1}\\ \psi_{-2}\\ \vdots\end{bmatrix}, \tag{16}$$

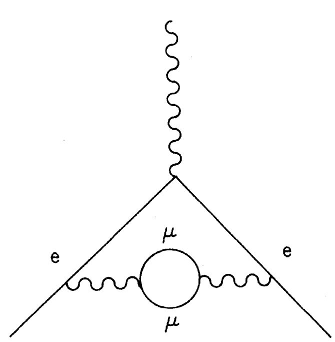

FIG. 2. Feynman diagram for muon contribution to the electron anomalous magnetic moment.

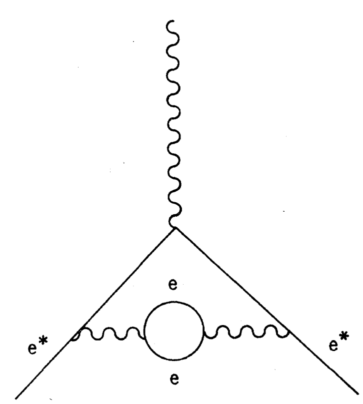

FIG. 3. Feynman diagram for electron contribution to the e^* anomalous magnetic moment.

where ψ_n $(n = \ldots, -2, -1, 0, 1, 2, \ldots)$ denotes the field operator for the lepton whose universal quantum number is n, Eq. (15) can then be written in the form

$$\mathcal{L}^I_{\mathrm{e.m.}} = \overline{\psi}\gamma_\mu C \psi A_\mu \,, \tag{17}$$

where C is the matrix defined by

$$C = \left[\begin{array}{ccc|c} \ddots & & & \\ & 1 & & 0 \\ & & 1 & \\ & & & 1 \\ \hline & 0 & & 0 \end{array}\right]. \tag{18}$$

The Lagrangian density for the free electromagnetic field is invariant under discrete scale transformations, but the interaction term (17) breaks discrete scale invariance. There is also the possibility of a nonminimal, nondiagonal interaction of the form

$$\frac{\lambda e}{m_{e^*}}\, \overline{\psi}_{e^*}\sigma_{\mu\nu}\psi_e F_{\mu\nu} \,, \text{ etc.,} \tag{19}$$

with λ arbitrary. This could be due to a new interaction or due to virtual weak interactions. (In the case that e^* and e couple to the same neutrinos.)

B. Weak Interactions

All the neutrinos of a given type cannot be permitted to couple weakly to a single charged lepton of the same type, with identical coupling constants, since this would lead to infinite decay rates for leptonic and semileptonic processes. For example, the muon cannot be allowed to couple to all muonlike neutrinos via a V-A interaction with the same coupling constant. This difficulty can be avoided by either

(a) assigning one specific neutrino to each charged lepton, or

(b) assuming that different neutrinos are coupled to a given charged lepton with coupling constants of different strengths in such a way that the total decay rate is still finite.

The simplest coupling scheme of the former type is as follows: To each charged muonlike lepton with universal quantum number n, we assign the lepton whose universal quantum number is $-n$ as the neutrino; to each charged electronlike lepton with the universal quantum number n, we assign the lepton whose universal quantum number is $-(n+2)$ as the neutrino. Thus the selection rule for the leptonic current in weak interactions is

$$\Delta n = 2n_c \quad (\mu\text{-like particles}),$$
$$= 2(n_c + 1) \quad (e\text{-like particles}),$$

which is the same as

$$\Delta n = 2n_c \quad (n_c \text{ odd}),$$
$$= 2(n_c + 1) \quad (n_c \text{ even}),$$

where we have denoted the universal quantum number of the charged lepton as n_c. With the above selection rule we can write the lepton current J^l_λ as

$$J^l_\lambda = J^\mu_\lambda + J^e_\lambda$$
$$= \sum_{n\,\mathrm{odd}} \overline{\psi}_n\gamma_\lambda(1+\gamma_5)\psi_{-n} + \sum_{n\,\mathrm{even}} \overline{\psi}_n\gamma_\lambda(1+\gamma_5)\psi_{-(n+2)} \,. \tag{20}$$

Using the notation (16), J^l_λ can be written more concisely as

$$J^l_\lambda = \overline{\psi}\gamma_\lambda(1+\gamma_5)W\psi \,, \tag{21}$$

where W is the matrix defined by

$$W = \left[\begin{array}{cccccccc} & & & & & & & \cdot^{\cdot^{\cdot}} \\ & & & & & & 1 & \\ & & & & & 0 & 0 & \cdot^{\cdot^{\cdot}} \\ & & & & 1 & 0 & 0 & \\ & & & 0 & 0 & 1 & & \\ & & 1 & 0 & 0 & & & \\ 0 & 0 & 0 & 1 & & & & \\ & & 0 & & 0 & & & \\ & & & \ddots & & & & \\ & & & & & & & \ddots \end{array}\right] \tag{22}$$

and the interaction Hamiltonian density takes the form

$$H = \frac{G}{\sqrt{2}}\, J^\dagger_\lambda J_\lambda + \mathrm{H.c} \,, \tag{23}$$

where $J_\lambda = J^l_\lambda + J^h_\lambda$, with J^h_λ = weak hadronic current. If we define lepton charges K_+, K_-, and K_3 as

$$K_+ = \tfrac{1}{2}\int d^3x\, J^{l\,\dagger}_4(x) \,,$$
$$K_- = \tfrac{1}{2}\int d^3x\, J^l_4(x) \,, \tag{24}$$
$$K_3 = \tfrac{1}{4}\int d^3x \left[\sum_{n=-1}^{-\infty} \overline{\psi}_n\gamma_4(1+\gamma_5)\psi_n - \sum_{n=0}^{\infty} \overline{\psi}_n\gamma_4(1+\gamma_5)\psi_n\right],$$

then SU(2) algebra

$$[K_+, K_-] = 2K_3\,,\quad [K_3, K_+] = K_+\,,\quad [K_3, K_-] = -K_- \tag{25}$$

is satisfied, showing that even in presence of an infinite family of leptons universality in Gell-Mann sense can be realized.

It is also interesting to note that according to the above coupling scheme, the mass of the charged lepton and the mass of its neutrino satisfy

$$m_c m_N = m_e^2 \quad (\mu\text{-like particles})\,,\qquad m_c m_N = m_{\nu_\mu}^2 \quad (e\text{-like particles})\,. \tag{26}$$

The physical significance of these relations is not yet clear.

Since the nature of neutrinos emitted in leptonic and semileptonic processes is largely unknown, we cannot rule out more complicated coupling schemes of charged and neutral leptons. Each charged lepton might couple to all neutrinos of the same type (muonic or electronic) via a V-A interaction, but with coupling constants dependent on the masses or on the universal quantum numbers of the neutrinos. For example, the muon might couple to all muonlike neutral leptons (i.e., to those neutrinos whose universal quantum number is odd), and the electron might couple to all electronlike neutral leptons (i.e., to those neutrinos whose universal quantum number is even). We define lepton currents as

$$J^\mu_\lambda = \sum_{n\,\mathrm{odd}} f_n[\bar{\mu}(x)\gamma_\lambda(1+\gamma_5)\nu^n_\mu]\,,\qquad J^e_\lambda = \sum_{n\,\mathrm{even}} f_n[\bar{e}(x)\gamma_\lambda(1+\gamma_5)\nu^n_e]\,, \tag{27}$$

where $f_n = f(n)$ = a function of the universal quantum number of the neutrino coupled to the charged lepton, and where ν^n denotes neutrino whose universal quantum number is $-n$. To make leptonic decay rates finite, $f(n)$ should be decreasing function of n. The interaction Hamiltonian density for muon decay takes the form

$$H = \frac{G_0}{\sqrt{2}} J^{\mu\dagger}_\lambda J^e_\lambda + \mathrm{H.c.}\,, \tag{28}$$

where G_0 is some constant, and if G_μ is the experimentally observed muon decay coupling constant, then we have

$$G_\mu{}^2 = G_0{}^2(f_1{}^2 + f_2{}^2 + f_3{}^2 + \cdots)(f_2{}^2 + f_4{}^2 + f_6{}^2 + \cdots)\,. \tag{29}$$

If both infinite series in (29) are convergent, the decay rate of the muon will be finite. Similarly, the interaction Hamiltonian densities for two semileptonic ($\Delta S = 0$) processes involving the muon and the electron can be written as

$$H_\mu = \frac{G_0}{\sqrt{2}} J^{h\dagger} J^\mu + \mathrm{H.c.}\,,\qquad H_e = \frac{G_0}{\sqrt{2}} J^{h\dagger} J^e + \mathrm{H.c.} \tag{30}$$

Then if G_1 and G_2 are the experimentally observed coupling constants for the above processes, we have

$$G_1{}^2 = G_0{}^2(f_1{}^2 + f_3{}^2 + f_5{}^2 + \cdots)\,,\qquad G_2{}^2 = G_0{}^2(f_2{}^2 + f_4{}^2 + f_6{}^2 + \cdots)\,. \tag{31}$$

Experimentally observed electron-muon universality demands $G_1 = G_2$, giving

$$f_1{}^2 + f_3{}^2 + f_5{}^2 + \cdots = f_2{}^2 + f_4{}^2 + f_6{}^2 + \cdots\,, \tag{32}$$

so that

$$G_\mu{}^2 = G_0{}^2(f_1{}^2 + f_3{}^2 + f_5{}^2 + \cdots)^2 = G_0{}^2(f_2{}^2 + f_4{}^2 + f_6{}^2 + \cdots)^2\,. \tag{33}$$

At present, we have not been able to think of any method (experimental or theoretical) for determining the constants, f_n, or equivalently the functional dependence of f_n on n apart from its monotonic decreasing nature.

IV. ALTERNATIVE POSSIBILITY

Neutrinos of zero rest mass were not included in the earlier scheme. According to that scheme it is possible to find a neutrino whose rest mass is smaller than any given positive number. Yet zero-mass neutrinos are not members of the infinite family.

In this section we consider another model in which two zero-mass neutrinos are generated together with an infinite family of charged leptons. We regard the electron and a zero-mass electron neutrino as the basic fields. All of the other fields are derived from these basic fields using discrete scale transformations.

We first consider the Lagrangian density $\mathcal{L}$ given by

$$\mathcal{L} = -\bar{\psi}_e\gamma_\lambda\partial_\lambda\psi_e - m\bar{\psi}_e\psi_e - \bar{\psi}_\nu\gamma_\lambda\partial_\lambda\psi_\nu - \tfrac{1}{4}F_{\lambda\delta}F_{\lambda\delta} + e\bar{\psi}_e\gamma_\lambda\psi_e A_\lambda\,, \tag{34}$$

where

ψ_e = electron field,

ψ_ν = electron neutrino field of zero-rest mass,

A_λ = photon field,

$$F_{\lambda\delta} = \frac{\partial A_\delta}{\partial x_\lambda} - \frac{\partial A_\lambda}{\partial x_\delta}\,.$$

Under the discrete scale transformation,

$$x_\mu \to \rho^n x_\mu, \quad n>0, \tag{35}$$

A_μ, being a Bose field, transforms as

$$A_\mu \to \rho^{-n} A_\mu . \tag{36}$$

As in the previous case, we have

$$\begin{aligned} \psi_e &\to \rho^{-3n/2}\psi_e(\rho^n x), \\ \psi_\nu &\to \rho^{-3n/2}\psi_\nu(\rho^n x). \end{aligned} \tag{37}$$

Also, when ρ is negative we assume that the correct transformation for a Dirac field is

$$\begin{aligned} \psi_e &\to \rho^{-3n/2}\gamma_5^{|n|}\psi_e(\rho^n x), \\ \psi_\nu &\to \rho^{-3n/2}\gamma_5^{|n|}\psi_\nu(\rho^n x). \end{aligned} \tag{38}$$

As in Sec. II, we can define an operator $U(n)$ to perform the above transformation. Under $U(n)$, Eq. (34) transforms as

$$\begin{aligned} \mathcal{L} \to U(n)\mathcal{L}U(n)^{-1} &= \rho^{-4n}(-\bar\psi_e'\gamma_\mu\partial_\mu\psi_e' - m\rho^n\bar\psi_e'\psi_e' - \bar\psi_\nu'\gamma_\mu\partial_\mu\psi_\nu \\ &\quad - \tfrac{1}{4}F_{\mu\nu}'F_{\mu\nu}' + e\bar\psi_e'\gamma_\mu\psi_e'A_\mu'), \end{aligned} \tag{39}$$

where the primes denote the transformed quantities. It is clear that the above transformation has generated only a series of charged leptons; the massless neutrino field repeats itself as the electron neutrino or as the muon neutrino. In this scheme, the electromagnetic current remains form-invariant under $U(n)$ in the sense that

$$U(n)\bar\psi_e\gamma_\mu\psi_e U(n)^{-1} = (\text{constant})\bar\psi_n\gamma_\mu\psi_n, \tag{40}$$

where ψ_n is the nth charged lepton field in the series. Similarly the weak lepton current also remains form-invariant,

$$\begin{aligned} U(n)[\bar\psi_e\gamma_\lambda(1+\gamma_5)\psi_{\nu_e}]U(n)^{-1} &= (\text{constant})\bar\psi_n\gamma_\lambda(1+\gamma_5)\psi_{\nu_e} \quad (n \text{ even}) \\ &= (\text{constant})\bar\psi_n\gamma_\lambda(1+\gamma_5)\psi_{\nu_\mu} \quad (n \text{ odd}). \end{aligned} \tag{41}$$

Again, if we define the total lepton current J_λ and the lepton charges K_+, K_-, K_3 as

$$J_\mu = \sum_{n\text{ even}} \bar\psi_n\gamma_\mu(1+\gamma_5)\psi_{\nu_e} + \sum_{n\text{ odd}} \bar\psi_n\gamma_\mu(1+\gamma_5)\psi_{\nu_\mu}, \tag{42}$$

$$\begin{aligned} K_+ &= \tfrac{1}{2}\int d^3x J_4^\dagger(x), \\ K_- &= \tfrac{1}{2}\int d^3x J_4(x), \\ K_3 &= \tfrac{1}{4}\int\Big[\bar\psi_{\nu_e}\gamma_4(1+\gamma_5)\psi_{\nu_e} + \bar\psi_{\nu_\mu}\gamma_4(1+\gamma_5)\psi_{\nu_\mu} \\ &\quad - \sum_{n=0}^{\infty}\bar\psi_n\gamma_4(1+\gamma_5)\psi_n\Big]d^3x, \end{aligned} \tag{43}$$

then it is clear that the SU(2) algebra,

$$\begin{aligned} [K_+, K_-] &= 2K_3, \\ [K_3, K_+] &= K_+, \\ [K_3, K_-] &= -K_-, \end{aligned} \tag{44}$$

is satisfied.

In the earlier scheme only K_3 was invariant under discrete scale transformations. Here all the four quantities J_λ, K_-, K_+, K_3 are invariant under discrete scale transformations.

If we assume that the weak interaction is mediated by an intermediate boson in the above scheme, then it is possible to demand form invariance of weak interactions under discrete scale transformations. This require a series of intermediate bosons. (This is not possible in the earlier scheme with an infinite series of neutrinos because there the transformation $x_\mu \to \rho^n x_\mu$ with $n<0$ is allowed. That would lead to bosons of arbitrarily small mass, which are ruled out by experiment.) Suppose we add to Eq. (34) $\mathcal{L}_1$ given by

$$\begin{aligned} \mathcal{L}_1 = &-\tfrac{1}{2}(\partial_\lambda B_\delta^\dagger - \partial_\delta B_\lambda^\dagger)(\partial_\lambda B_\delta - \partial_\delta B_\lambda) - M_0^2 B_\lambda^\dagger B_\lambda \\ &+ g\bar\psi_e\gamma_\lambda(1+\gamma_5)\psi_{\nu_e}B_\lambda, \end{aligned} \tag{45}$$

where B_λ = intermediate boson field; then, as in (39), $L = \mathcal{L}+\mathcal{L}_1$ transforms under $x_\mu \to \rho^n x_\mu$ $(n>0)$ into a Lagrangian density L' for another field involving a boson of mass $M_n = M_0\rho^n$ $(n=0, 1, 2, \ldots)$. If we suppose that all these bosons take part universally in any weak process, then the effective Fermi coupling constant is given by

$$G = \sum_{n=0}^{\infty} \frac{g^2\sqrt2}{M_0^2(\rho^2)^n}. \tag{46}$$

Summation gives

$$G = \frac{g^2\sqrt2}{M_0^2}\left(\frac{1}{1-1/\rho^2}\right). \tag{47}$$

Thus $G \simeq g^2\sqrt2/M_0^2$, and the change in the Fermi coupling constant due to the existence of the above infinite family of bosons is negligible.

V. NEUTRINOS OF NONZERO REST MASS

For the first time, our model provides theoretical values for the masses of neutrinos. The mass of the heaviest muonic neutrino is 2.5 keV and the heaviest electron neutrino is 12.1 eV. Thus, according to the first coupling scheme discussed in Sec. III, the muon and electron neutrinos emitted in muon decay should have the above masses. Even if the second coupling scheme is realized, the mean value for the masses of neutrinos from muon decay should have values close to those given above, because the probability of observing successive neutrinos in such a case will fall as fast as $f_n^{\,2}$.

Thus a crucial test for the model is measurement of neutrino masses. Except for the upper limits for masses [600 keV for ν_μ (Ref. 42) and 55 eV for ν_e (Ref 43)] the question whether neutrinos have a rest mass as large as this theory predicts is not yet settled by experiment. The above limit for ν_μ, obtained by measuring the μ momentum in π decay, is very poor because the μ momentum is insensitive to the mass of the highly relativistic neutrino. The limit for the muon neutrino mass may be further improved by studying the low-energy neutrino ends of the $K_{\mu 3}$ or radiative pion-decay[47-49] spectra. Unfortunately, the most optimistic estimates of the precision of future experiments of the above type to measure the muon neutrino mass cannot be expected to lower the limit beyond about 100 keV.[50] The present upper limit for the electron neutrino mass (55 eV) is obtained from an estimate of the end-point energy of the β spectrum of tritium.[43] There are two circumstances preventing a significant further improvement of the limit for electron neutrino mass by this method.[43] The decrease in the intensity of the β spectrum towards the end point makes the improvement of the above limit by a factor 5 or 10 extremely difficult. A more fundamental obstacle is the perturbation in the β decay amplitude caused by atomic electrons, which tends to smear out the effect of a finite neutrino mass at the end point of the β spectrum. For a free tritium atom the correction needed to allow for this effect could possibly be estimated. However, chemical bindings of the tritium atom in an actual β source will make such computations extremely difficult. It seems, therefore, that measurements of the electron neutrino mass or further improvement of the upper limit of its mass must involve entirely different techniques. An interesting possibility is the measurement of the velocity of neutrinos of known energy. Because of the smallness of the neutrino mass, the velocity of detectable neutrinos will be so high that it is probably impossible to design a laboratory experiment to measure the neutrino velocity with sufficient accuracy. However, the use of astronomical methods for measuring neutrino velocity seems to be promising. A lower limit for the neutrino mass can be obtained by observing neutrino pulses from a collapsing star.[51] According to some theoretical calculations, during the collapse of a star of mass M in the range $3M_\odot \geq M \geq 1.2M_\odot$, there will be a pulse of neutrinos lasting for about 10^{-2} sec. Energy carried away by neutrinos is about one percent of the star mass, and the average energy of neutrinos is about 30 MeV. Thus if the neutrino is massive, there will be a time delay between the arrival of photons and neutrinos from the explosion. Measurement of this time delay will give a lower limit for the neutrino mass. The main difficulty of performing such an experiment is the small probability (about one event per century) of observing a collapsing star in our galaxy. Recently Bogatyrev has suggested that construction of detectors to sense neutrinos from remote galaxies at distance of 7–10 million light years is not beyond present day technology.[52] Observation of the time of arrival of neutrinos at detectors placed at different points on the earth will determine the direction of the exploding star as well as the velocity of neutrinos. Also, if there is any correlation between Weber pulses and neutrino fluxes on the earth,[53] the time delay in the arrival of these two signals could also be used to estimate the neutrino mass. Since, in collapse of more massive stars, copious muon-neutrino emission is expected,[54] the above methods may also be used to measure the mass of the muon neutrino.

It is also interesting to note that for masses of neutrinos given by our theory, the possibility of neutrino oscillations $\nu_e \rightleftarrows \nu_\mu$ due to lepton nonconservation are ruled out. According to Pontecorvo[55] the lengths of these oscillations are given by

$$l \leq \frac{2E}{{m_{\nu_\mu}}^2 - {m_{\nu_e}}^2}\,, \tag{48}$$

where E is the energy of the neutrino. For masses of neutrinos given by our theory, the oscillation length turns out to be 10^{-8} cm for neutrinos of energy 10 MeV. Due to the smallness of the oscillation length, electron events would be expected in muon neutrino interactions. However, such events are not experimentally observed. Hence, if our predictions are correct, the separate conservation of the muon and electron lepton numbers should probably be strictly valid.

Another interesting question is whether the neutrinos of different rest masses can interact with each other. If we assume a four-fermion interaction of the form

$$H = \frac{F}{\sqrt{2}}[\overline{\nu}_\mu^1 \gamma_\mu (1+\gamma_5)\nu_\mu^2][\overline{\nu}_e^1 \gamma_\mu (1+\gamma_5)\nu_e^2]\,, \tag{49}$$

then the decay $\nu_\mu^1 \to \nu_\mu^2 + \nu_e^1 + \overline{\nu}_e^2$ (we have denoted the two heaviest muon neutrinos by ν_μ^1 and ν_μ^2, and the electron neutrinos by ν_e^1 and ν_e^2) can occur with an lifetime of about 10^{10} years, when $F = G \simeq 10^{-5}{M_p}^{-2}$. Bardin *et al.*[56] have pointed out that a relatively strong interaction between neutrinos cannot be ruled out and that a possible upper limit for F is $10^6 G$. Hence, if the heaviest muon neutrino is unstable, a lower limit for its lifetime is about four days. Similarly, a lower limit for the lifetime of the electron neutrino is 10^6 years.

The existence of an infinite series of neutrinos, coupled to heavy leptons, would have important astrophysical and cosmological implications. Neutrinos from the decay of heavy leptons produced in an astrophysical process will serve as efficient carriers of energy, because the thresholds for inverse processes involving these neutrinos are extremely high.

VI. EXPERIMENTAL DETECTION OF HEAVY LEPTONS

A. Decay Modes of Heavy Charged Leptons

The allowed decay modes of a heavy lepton L depend on its mass.[57]

(1) If the mass m_L of the lepton lies in the region $m_\mu \leqslant m_L \leqslant m_\pi$, then the only allowed decay modes are

$$L \to \mu\nu_\mu \nu_L$$

and

$$L \to e\nu_e\nu_L \,.$$

(2) In the mass region $m_\pi \leqslant m_L \leqslant m_K$ the decay $L \to \pi\nu_L$ is also possible, in addition to the above modes. However, heavy leptons less massive than the kaon, if they exist, cannot have standard weak-interaction properties, for if the heavy lepton couples to the hadron current with the usual weak-interaction coupling strength, the decay width of the kaon is affected beyond the experimental limit.

(3) If the mass of the lepton lies in the region $m_K \leqslant m_L \leqslant m_N$ the kaonic mode $L \to K\nu_L$ is also possible. The predominant decay mode in this case is $L \to \pi\nu_L$.

Very massive leptons such as the e^* predicted by our theory can decay into

(1) leptons of lower mass in the series,
(2) mesons + leptons,
(3) vector mesons + leptons,
(4) leptons and γ rays (electromagnetic decays),
(5) baryons + antibaryons + leptons,
(6) leptons + intermediate boson (if the intermediate boson exists and is less massive than the lepton).

1. *Leptonic Decay Modes of the e^**

The heavy lepton e^* can have following leptonic decay modes:

(a) $e^* \to \mu + \bar{\nu}_\mu + \nu_{e^*}$,
(b) $e^* \to e + \bar{\nu}_e + \nu_{e^*}$.

Assuming that the weak-interaction Hamiltonian density is given by (23), we get the decay rate for both of these modes as

$$\Gamma_1 = G^2 m_{e^*}^5/192\pi^3 \,. \tag{50}$$

Since the e^* is very much heavier than either the electron or the muon, the above results obtained by neglecting their masses are very precise and yield $\Gamma_1 = 1.5\times 10^{17}\ \mathrm{sec}^{-1}$.

2. *Mesonic Decay Modes of the e^**

The decay $e^* \to \pi + \nu_{e^*}$ can take place at a rate Γ_2 given by[57]

$$\Gamma_2 = \frac{G^2 f_\pi^{\,2} m_{e^*}^3 \cos^2\theta}{8\pi}\,(1 - m_\pi^{\,2}/m_{e^*}^{\,2})^2 \,, \tag{51}$$

where f_π = charged pion decay constant = 94 MeV, yielding $\Gamma_2 = 5.6\times 10^{14}\ \mathrm{sec}^{-1}$. Similarly, the decay mode $e^* \to K + \nu_{e^*}$ has a rate[57]

$$\Gamma_3 = \frac{G^2 f_K^{\,2} m_{e^*}^3}{8\pi}\sin^2\theta\,(1 - m_K^{\,2}/m_{e^*}^{\,2})^2 \,, \tag{52}$$

yielding $\Gamma_3 = 4.8\times 10^{13}\ \mathrm{sec}^{-1}$. Multimeson modes such as $e^{+*} \to \pi^+\pi^0\nu_{e^*}$, $e^{+*} \to K^+K^0\nu_{e^*}$ are also possible. The decay rate for the two-pionic mode of a heavy lepton has been calculated by Thacker and Sakurai.[58] For the e^* the decay rates of above modes are slightly greater than the corresponding single-meson mode. If there are neutral currents, the decays $e^* \to e + \pi^0$ and $e^* \to e + K^0$ are also possible.

3. *Decays into Vector Mesons*

The decay rates of heavy leptons for a number of interesting modes were evaluated by Tsai.[59] From his results the decay rate of the mode $e^* \to \rho + \nu_{e^*}$ is given by

$$\Gamma_4 = 18\times 10^{10} m_{e^*}^3 (1 - m_\rho^{\,2}/m_{e^*}^{\,2})^2 \times (1 + 2m_\rho^{\,2}/m_{e^*}^{\,2})\quad \mathrm{sec}^{-1} \,, \tag{53}$$

where all masses are in GeV, yielding $\Gamma_4 = 1.9\times 10^{15}\ \mathrm{sec}^{-1}$. Using specific models the rates for the decay of a heavy lepton into $A_1(1070)$, $K^*(892)$, and $Q(1300)$ were also evaluated by the same author[59]; for the e^* these rates are given by

$$\Gamma_5(e^* \to K^*\nu_{e^*}) = 1.29\times 10^{10} m_{e^*}^3\left(1 - \frac{m_{K^*}^{\,2}}{m_{e^*}^{\,2}}\right)^2\left(1 + 2\frac{m_{K^*}^{\,2}}{m_{e^*}^{\,2}}\right)\ \mathrm{sec}^{-1} = 1.4\times 10^{14}\ \mathrm{sec}^{-1} \,, \tag{54}$$

$$\Gamma_6(e^* \to Q + \nu_{e^*}) = 0.614\times 10^{10} m_{e^*}^3\left(1 - \frac{m_Q^{\,2}}{m_{e^*}^{\,2}}\right)\left(1 + 2\frac{m_{K^*}^{\,2}}{m_{e^*}^{\,2}}\right)\ \mathrm{sec}^{-1} = 6.5\times 10^{13}\ \mathrm{sec}^{-1} \,, \tag{55}$$

$$\Gamma_7(e^* \to A_1 + \nu_{e^*}) = 7.2\times 10^{10}\, m_{e^*}{}^3 \left(1 - \frac{m_A{}^2}{m_{e^*}{}^2}\right)^2 \left(1 + 2\frac{m_A{}^2}{m_{e^*}{}^2}\right) \text{ sec}^{-1} = 7.8\times 10^{14} \text{ sec}^{-1}. \tag{56}$$

(In all of the above expressions, the masses are expressed in GeV.)

4. *Electromagnetic Decays*

A possible form of the interaction of e^* and e with the electromagnetic field is given by (19). This is the only possible form of interaction of two charged particles of unequal mass with the electromagnetic field since the interaction $\bar{\psi}_{e^*}\gamma_\mu \psi_e A_\mu$ violates gauge invariance. The decay rate for the mode $e^* \to \gamma + e$ due to the interaction (19) can be given approximately by the formula

$$\Gamma_8 = \frac{16 e^2 \omega^3 \lambda^2}{m_{e^*}{}^2}, \tag{57}$$

where ω is the angular frequency of the emitted photon and when $\lambda = 1$ we get $\Gamma_8 = 10^{18}$ sec^{-1}. A crude limit for the coupling constant λ in the interaction (19) can be obtained from a consideration of the contribution $\Delta\mu$ to the anomalous magnetic moment of the electron due the diagram of Fig. 4. The interaction (19) is nonrenormalizable. Using a cutoff, Terazawa[60] has obtained the following result:

$$\Delta\mu = (\lambda e \Lambda)^2 / 2\pi m_{e^*}{}^2 + O(\ln(\Lambda^2/m_{e^*}{}^2)). \tag{58}$$

The cutoff parameter Λ must satisfy the condition $\Lambda > m_{e^*}$, implying $\Delta\mu > (\lambda e)^2/2\pi^2$. Since $|\Delta\mu| < |\mu_{\text{exp}} - \mu_{\text{QED}}| \simeq 6\times 10^{-8}$, we get $\lambda < 10^{-4}$, giving $\Gamma_8 < 10^{14}$ sec^{-1}. More complete calculations made by de Rujula and Lautrup[61] to evaluate the contribution to the anomalous magnetic moment of the electron due to the existence of a heavy lepton yield a limit of the same order. The nonminimal electromagnetic interaction (19) may not exist. In such a case, the decay $e^* \to e + \gamma$ will be strictly forbidden if e^* and e are coupled only to their own neutrinos. However, if there are only two zero-mass neutrinos, as discussed in the Sec. IV, then the decay $e^* \to e + \gamma$ can occur via the diagrams (a)–(f) of Fig. 5. A rough estimate[62] gives a decay rate less than 10 sec^{-1}. If the intermediate boson exists the decay $e^* \to e + \gamma$ can occur with a much larger rate, $\simeq 10^{15}$ sec^{-1}, via the diagrams (g), (h), and (i) of Fig. 5. [Diagrams (d) and (e) cancel exactly.] Hence, in absence of the interaction (19) and the intermediate boson, the e^* is almost stable against electromagnetic decay.

5. *Decay of the e^* into the Hadron Continuum*

The rate of decay of a heavy lepton into a neutrino plus the hadron continuum was estimated by Tsai.[59] His result gives as the rate for the decay mode $e^* \to \nu_{e^*} +$ hadron continuum

$$\Gamma_9 = 3.47\times 10^{10}\, m_{e^*}{}^5 (1 - 2/m_{e^*}{}^2) \quad (m_{e^*} \text{ is in GeV})$$
$$= 1.7\times 10^{17} \text{ sec}^{-1}. \tag{59}$$

Thus the leptonic and the total hadronic decay modes of the e^* have approximately the same branching ratios. The total width of the e^* is roughly 5×10^{17} sec^{-1}. Branching ratios for various modes are given in Table I.

6. *Decay of the e^* into the Intermediate Boson*

If both the e^* and the intermediate boson exist and if $m_W < 22$ GeV, then the decay $e^* \to W + \nu_{e^*}$ will

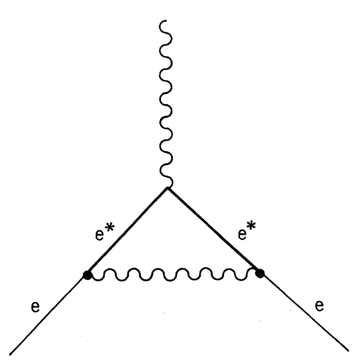

FIG. 4. Feynman diagram for contribution to the electron anomalous magnetic moment from the interaction (19).

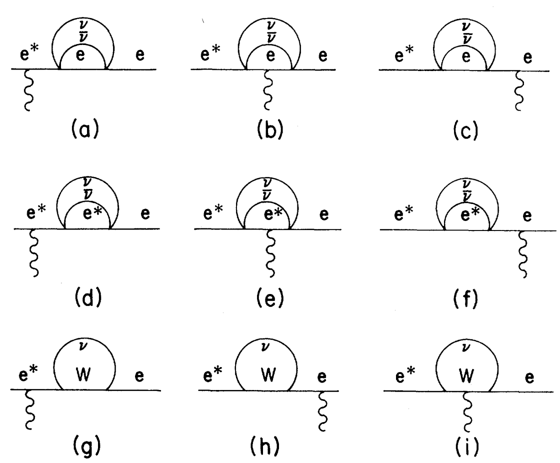

FIG. 5. (a)–(f) Second-order Feynman diagrams for $e^* \to e + \gamma$. (g)–(i) Possible Feynman diagrams for the same process in the presence of an intermediate boson.

completely dominate the widths given in the Table I, except for the values of m_W very close to m_{e*}. The width of the above mode is given by[59]

$$\Gamma = (G/8\pi\sqrt{2}\ m_{e*}^3)(1 - m_W^2/m_{e*}^2)^2(1 + 2m_W^2/m_{e*}^2)$$
$$= 46 \times 10^{25} f(x)\,, \qquad (60)$$

where $x = m_W/m_{e*}$ and $f(x) = (1-x^2)^2(1+2x^2)$. The function $f(x)$ is plotted in Fig. 6, and the widths of $e*$ for various values of m_W are given in Table II.

B. Production Mechanisms for $e*$

So far there is no strong experimental evidence that suggests the existence of any heavy leptons. If one assumes that weak and electromagnetic interactions of heavy leptons are analogous to those of the known charged leptons, then experimental data on the decay of the kaon imply that the masses of heavy leptons cannot be lower than that of the kaon. Otherwise, the decay of the kaon into a heavy lepton and a neutrino would be more probable than its decay into the muon. In recent years a number of searches were made for heavy leptons. A heavy electronlike lepton of the type proposed by Low[63] would modify the electron-positron pair-production cross section. An experiment to detect such an effect was analyzed by Gutbrod and Schildknecht.[34] They found no evidence for such a particle in the mass range $120 \leqslant m \leqslant 1000$ MeV. A similar experiment to detect a heavy muonlike lepton was conducted by Wilson *et al.*[35] They measured the muon bremsstrahlung cross section with muons of energies up to 13 GeV. If the heavy muon is coupled to the ordinary muon via Low's nonminimal interaction,[63] the bremsstrahlung cross section will be modified. No such modification was found, indicating that there is no significant evidence for a heavy muon with a mass less than 600 MeV. Alles-Borelli *et al.*[36] have looked for heavy leptons using electron-positron colliding beams. According to these authors heavy leptons with masses less than 780 MeV decaying into electrons or muons are probably ruled out. The only experiment thus far carried out which claims to have detected a heavy lepton is the one conducted by Ramm.[64] He has arrived at the result that the muon-pion invariant-mass distributions observed in neutrino interactions and in decays of K_L^0 are compatible with the existence of a neutral lepton with a mass in the range $0.422 < m < 0.437$ GeV. In another recent paper the same author claims that there is also evidence for a charged lepton in the same mass range.[65] However, these results are very doubtful because of poor statistics, and have not been confirmed.

Small production cross sections and rapid decay rates make detection of heavy leptons extremely difficult. Among the many possible production mechanisms, the most promising are

(1) electron-proton collisions,
(2) electron-positron colliding beams,
(3) high-energy photon-nucleon collisions,
(4) proton-nucleon collisions,
(5) p-p colliding beams,
(6) neutrino interactions.

TABLE I. Decay widths and branching ratios of $e*$.

Mode	Γ (in $\sec^{-1}$)	Γ/Γ_{tot}
$e* \to e\nu_e\nu_{e*}$	1.5×10^{17}	33%
$e* \to \mu\bar{\nu}_\mu\nu_{e*}$	1.5×10^{17}	33%
$e* \to \pi\nu_{e*}$	5.6×10^{14}	(8×10^{-2})%
$e* \to K\nu_{e*}$	4.8×10^{13}	(1×10^{-2})%
$e* \to \rho\nu_{e*}$	1.9×10^{15}	(40×10^{-2})%
$e* \to K^*\nu_{e*}$	1.4×10^{14}	(3×10^{-2})%
$e* \to Q\nu_{e*}$	6.5×10^{13}	(1.3×10^{-2})%
$e* \to A_1\nu_{e*}$	7.8×10^{14}	(0.5×10^{-2})%
$e* \to \nu_{e*}$ + hadron continuum	1.7×10^{17}	33%

TABLE II. Decay rates of the $e*$ into an intermediate vector boson of mass m_W.

m_W (in GeV)	Γ (in $\sec^{-1}$)
5.0	45×10^{25}
10.0	24×10^{25}
15.0	18×10^{25}
20.0	7×10^{25}
21.0	2×10^{23}

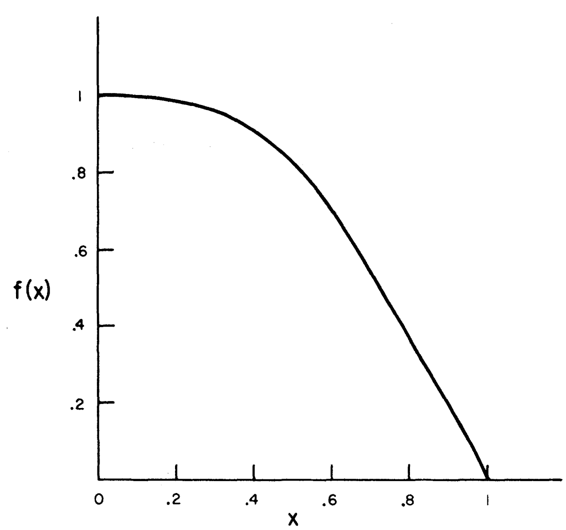

FIG. 6. Plot of $f(x)$ [see Eq. (60)] vs x for $|x| < 1$.

1. *Electron-Proton Collisions*

The process $e+p \to e^* + p$ can occur if the interaction (19) exists (see Fig. 7), and the differential cross section has been calculated by Gutbrod *et al.*[34] Neglecting form factors, it can be written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{\lambda\alpha}{m_{e*}}\right)^2 \frac{kx}{q^4E^2m[1-(E^*/K)\cos\theta]}, \qquad (61)$$

where

$$x = 2q^2(m_{e*}{}^2q^2 + 4m_{e*}{}^2mE - 8m^2E^2 - 4q^2mE - 2m_{e*}{}^4) - 4m_{e*}{}^4m^2,$$

with m = electron mass, m_{e*} = mass of e^*, E = energy of the electron in the lab, E^* = energy of e^* in the lab, $k = (E^{2*} - m_{e*}{}^2)^{1/2}$, and q = momentum transfer to the proton. Using the limit on λ obtained in Sec. VI, we find near the threshold that $(d\sigma/d\Omega)_{\theta=0} < 10^{-30}$ cm^2.

Similarly, the e^* might also be produced in electron-electron collisions $e+e \to e+e^*$ with a cross section comparable to the above.

If the proton target is at rest in the lab frame, then production of the e^* can be easily detected through high-transverse-momenta muons and electrons produced in the leptonic decay mode. Another way to detect the e^* would be to scan the recoil-proton-momentum distribution.[66] Since the final state is a quasi-two-body state, e^* excitation would lead to a peak in the recoil-proton momentum distribution. Other competing reactions, such as electroproduction or photoproduction of one or several pions, cannot give rise to such peaks.

2. *Colliding Electron-Positron Beams*

The differential and total cross sections for production of a heavy lepton of mass m_l in the process $e^+e^- \to l^+l^-$ are given by[67] (Fig. 8)

$$\frac{d\sigma}{d(\cos\theta)} = \pi\alpha^2\lambda^2\beta[\tfrac{1}{2}(1+\cos^2\theta) + (m_l/2E^2)(1-\cos^2\theta)], \qquad (62)$$

$$\sigma_{\text{tot}} = (1/m_l{}^2)(2.1\times10^{-32}\ \text{cm}^2)f(x), \qquad (63)$$

where λ and E are the wavelength and energy of each incident particle in the center-of-mass system and where β is the velocity of the heavy lepton in the center-of-mass system. The function $f(x)$ is defined as

$$f(x) = x^{-2}(1-1/x^2)^{1/2}(1+1/2x^2),$$

with

$$x = E/m_l.$$

From the behavior of the function $f(x)$, it follows that a maximum occurs when $x \simeq 1.2$, so that the most suitable center-of-mass energy for production of e^* is 26 GeV. At this energy, $\sigma_{\text{tot}} = 1.3 \times 10^{-35}$ cm^2. The total cross section for production of a muon pair of the same energy is 1.5×10^{-35} cm^2. Production of the e^* can be detected through its leptonic decay modes $e^* \to \mu\bar{\nu}_\mu\nu_{e*}$, $e^* \to e\bar{\nu}_e\nu_{e*}$, each of which has a decay probability of 33%. The events of the type $e^+ + e^- \to \mu^-(e^-) + e^+(\mu^+)$ + neutrals, which can easily be detected, corresponds to one lepton in the pair decaying via the muonic mode and the other via the electronic mode.

Tsai[59] has shown that the spins of heavy leptons l^+l^- produced in $e^+ + e^- \to l^+ + l^-$ are strongly correlated. In addition, there is also a strong correlation between the energy and angular distributions of the decay products of l^+ and l^-, which might also be used as a means to detect the l.

If the intermediate vector boson exists with a mass close to that of e^*, then detection of e^* will become extremely difficult, because the intermediate boson will also decay into $\mu\nu_\mu$ and $e\nu_e$ with a comparable probability. The total cross section for production of a boson of mass m_B in e^+e^- collisions (Fig. 9) is given by[67]

$$\sigma = (1/m_B{}^2)(2.1\times10^{-32}\ \text{cm}^2)B(x), \qquad (64)$$

where

$$B(x) = \tfrac{3}{4}(1-1/x^2)^{3/2}(\tfrac{4}{3} + 1/x^2),$$

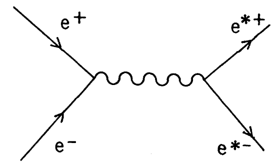

FIG. 8. Feynman diagram for the process $e^+e^- \to e^{*+}e^{*-}$.

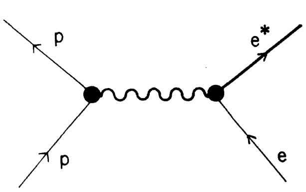

FIG. 7. Feynman diagram for the process $e+p \to e^* + p$.

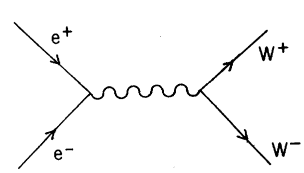

FIG. 9. W-boson production in e^+e^- collisions.

with $x=E/m_B$. When $m_B = 22$ GeV we have $\sigma = 1.2 \times 10^{-35}$ cm^2. Thus the production cross sections for the e^* and W are also not very different.

3. *Photon-Nucleon Collisions*

Photons impinging on a high-Z target will produce lepton pairs in the Coulomb field of the nucleus. The cross section for the process, $\gamma + Z \to e^{+*} + e^{-*} + Z'$ (Fig. 10), can be calculated exactly by making use of the known nuclear form factors. An approximate calculation ignoring nucleon form factors gives the total cross section as[68]

$$\sigma = \left(\frac{4}{137}\right)\left(\frac{7}{9}\right) r_0^{\,2} \frac{m_e^{\,2}}{m_{e*}^{\,2}} \ln \frac{k}{m_{e*}}, \qquad (65)$$

where m_e = mass of the electron, m_{e*} = mass of e^*, E = energy of the heavy lepton e^*, k = energy of the incident photon, and r_0 = classical radius of the electron. For 300-GeV photons (maximum useful energy of the NAL tagged-photon beam), σ turns out to be 0.3×10^{-35} cm^2. Production of the e^* can be detected by observing high-transverse-momenta muons and electrons produced in the leptonic decay of e^*.

As in the case of e^+e^- collisions, if the intermediate vector boson has a mass close to that of the e^*, it will be very difficult to distinguish e^* production from W production via the reaction $\gamma + Z \to W^+ + W^- + Z'$. (A Columbia-Harvard-Hawaii group has proposed an experiment to search for photoproduction of pairs of heavy leptons or intermediate bosons using NAL photon beams.[69] However, this experiment is sensitive only to the production of heavy lepton pairs in the mass range $2 < m_l < 10$ GeV.)

4. *Proton-Nucleon Collisions*

Proton-nucleon collisions can produce lepton pairs by virtual photons (Fig. 11), i.e.,

$$p + Z \to \langle\gamma\rangle + Z'$$
$$\searrow e^{*+} + e^{*-},$$

where $\langle\gamma\rangle$ denote the virtual photon. This experiment can be carried out with protons incident on a high-Z target.

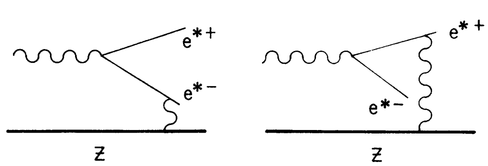

FIG. 10. e^* production in photon-nucleon collisions.

5. *p-p Colliding Beams*

If it were not for the low luminosity, another possible method for production of the e^* would be p-p colliding beams. (A center-of-mass energy of 56 GeV will be available at the CERN Intersecting Storage Rings.[70]) The cross section for production of the e^* in p-p collisions at energies much greater than the rest mass of the e^* can be deduced from calculations of muon pair production cross sections in high-energy proton-proton collisions. Such calculations are model-dependent. For the purpose of obtaining an estimate of the differential cross section, we use the results of Sanda and Suzuki.[71] Using a current-algebra sum rule derived from an equal-time commutator between charged densities, they have set the lower bound for muon pair production cross section in p-p collisions as

$$\frac{d\sigma}{d|q|^2} \geq \frac{\alpha^2}{36\pi |q^2|^2} \sigma_T(s) I(s) s, \qquad (66)$$

where s = energy of p-p in the c.m. system, $(|q^2|)^{1/2}$ = invariant mass of the muon pair, $\sigma_T(s) \simeq 30$ mb (approximate total p-p cross section at high energies), and

$$I(s) = \left[\left(1 - \frac{2M}{s} + \frac{|q|^2 - 4m_N^{\,2}}{s}\right)^2 - \frac{4|q^2|}{s}\right]^{3/2},$$

with M = cutoff parameter. But when the collision energy is sufficiently high, the results are insensitive to the cutoff. Assuming that the same result is valid for e^* pair production in p-p collisions, we obtain

$$\frac{d\sigma}{d|q|^2} \geq 10^{-30} \text{ cm}^2 \text{ GeV}^{-1} \qquad (67)$$

at c.m. energies much greater than the rest mass of the e^*.

6. *Neutrino Interactions*

The types of neutrino interactions that might produce the e^* depend on the nature of the weak couplings of the e^* with neutrinos. If there exist only two neutrinos, as discussed in the Sec. IV,

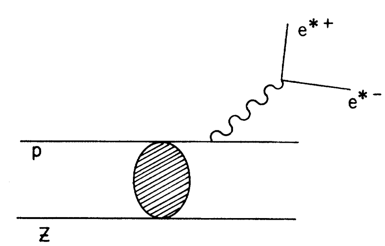

FIG. 11. e^* production in proton-nucleon collisions.

or if the second coupling scheme discussed in the Sec. III is realized, then the e^* can be produced in the reactions

$$\nu_e + p \to n + e^{+*},$$
$$\bar{\nu}_e + n \to p + e^{-*}.$$

However, since high-energy electron-neutrino beams are not available, the above reactions have no practical value. Even if the e^* is coupled to its own neutrino, it can be produced in the reaction

$$\bar{\nu}_e + e^- \to \bar{\nu}_{e*} + e^{-*}.$$

Again, this process has no practical value, at least for terrestrial experiments.

Promising neutrino interactions that might create the e^* at useful rates are nucleon-muon neutrino collisions

$$\nu_\mu + Z \to Z + \mu^- + e^{+*} + \bar{\nu}_{e*},$$
$$\bar{\nu}_\mu + Z \to Z + \mu^+ + e^{-*} + \nu_{e*}.$$

The total cross sections for the parallel processes

$$\bar{\nu}_\mu + Z \to Z + \mu^+ + \mu^- + \nu_\mu,$$
$$\nu_\mu + Z \to Z + \mu^- + e^+ + \nu_e,$$
$$\bar{\nu}_\mu + Z \to Z + \mu^+ + e^- + \nu_e$$

have been calculated by several authors,[72] assuming a four-fermion point interaction and single-photon exchange with the nucleus (Fig. 12). Differential cross sections have also been recently computed by Løvseth and Radomski.[73] For sufficiently high energies, the total cross section can be given approximately by[74]

$$\sigma = \frac{5(G\,\alpha Z)^2}{54\pi^2} E q_0 \ln \frac{2Eq_0}{m_{e*}^2}, \tag{68}$$

where q_0 = recoil momentum of the nucleus and E = neutrino energy. For $E = 300$ GeV, the total cross section is approximately 10^{-42} cm^2.

7. *Heavy Leptons as Decay Products of Intermediate Bosons*

Numerous experiments have been proposed to detect the intermediate vector boson (IVB). If it exists and is more massive than the heavy electron e^* (most theoretical models predict intermediate bosons more massive than 22 GeV), then one might hope to see the boson decaying into e^* and ν_{e*} some fraction of time. The branching ratio for the two modes $W \to e^* + \nu_e$ and $W \to \mu + \nu_\mu$ is given by[57]

$$\frac{\Gamma_{e*}}{\Gamma_\mu} = \left(1 - \frac{m_{e*}^2}{2m_W^2} - \frac{m_{e*}^4}{2m_W^4}\right)\left(1 - \frac{m_{e*}^2}{m_W^2}\right). \tag{69}$$

For an IVB of mass 37.3 GeV (mass of the IVB as predicted by Schechter and Ueda[75] and also by Lee[76]) the above yields 44%.

8. *Indirect Tests for Detection of Heavy Leptons*

Heavy leptons, if coupled to the electron and the electromagnetic field via the interaction (19), will modify quantum electrodynamics at high energies, and this will serve as an indirect test for presence of heavy leptons coupled to the electromagnetic field as above. The influence of the e^* on pair production [if the coupling (19) exists] may be seen by considering the diagrams (Fig. 13) for pair production by an external field. In the presence of the e^*, we have to consider not only diagrams (a) and (b), but also (c) and (d) (Fig. 13), where the electron propagator is replaced by the e^* propagator. (Gutbrod and Schildknecht[34] have calculated the electron-pair-production cross section supposing the existence of a heavy electron.) In the same way, bremsstrahlung cross sections are also modified due to the presence of a heavy electron coupled to the electromagnetic field via (19).

9. *Can the μ^* be Detected?*

Production of the μ^* in laboratory experiments is out of the question. However, there is a remote

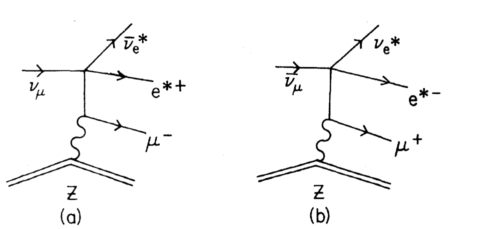

FIG. 12. Feynman diagrams for e^* production by ν_μ interactions.

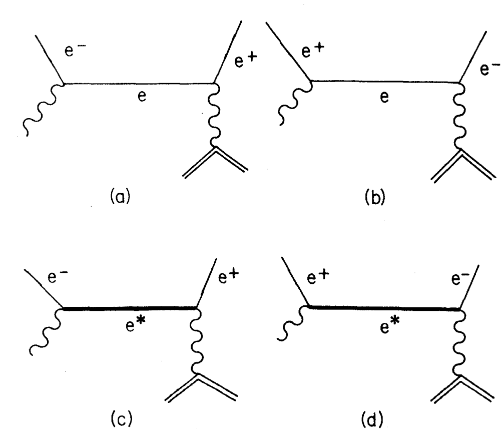

FIG. 13. Feynman diagrams for pair production in the presence of the coupling (19).

possibility that effects due to its existence may be detected in deep-underground cosmic-ray-muon experiments. Analogous to production of e^* in electron-nucleon collisions, the reaction

$$\mu + Z \to \mu^* + Z'$$

might be detected in underground cosmic-ray-muon interactions. The heavy muon might also be produced by cosmic-ray neutrinos via the reactions

$$\bar{\nu}_\mu + n \to p + \mu^*,$$

$$\nu_\mu + p \to n + \mu^*.$$

Even if μ^* is coupled only to its own neutrino some flux of $\nu_{\mu*}$ can be expected from decays of μ^* or other heavy objects.

VII. HEAVY LEPTONS AND COSMIC-RAY ANOMALIES

In this section we consider the possibility that at least some of the recently observed anomalies in cosmic-ray muons are manifestations of our heavy leptons (e^* and μ^*). The most important anomalous cosmic-ray effects widely discussed in the literature are

(1) the Utah effect,[38]

(2) the anomalous stopping rate of underground muons,[40]

(3) horizontal and muon-poor air showers,[77]

(4) a large rate of energy loss of muons in the TeV region.[39]

We do not claim that all the above effects are due to the existence of heavy leptons predicted by us. Most of the popular explanations given for these effects are in terms of postulated particles. In many cases, a particle is postulated and properties are assigned to explain the observed anomaly. On the contrary, we consider to what extent our particles, whose masses and certain properties are already fixed, can account for the above observations.

A. Utah Effect

The experiments conducted by the Utah group[38] examine the zenith-angle distribution of underground cosmic-ray muons at fixed slant depths of 2000–8000 hg cm^{-2}. Due to the competition between decay and absorption processes, the pions and kaons produced at large zenith angles by primary cosmic-ray interactions in the upper atmosphere have a greater probability of decaying into muons than do the mesons which plunge into the denser parts of the atmosphere immediately. This leads to the $\sec\theta$ law for the intensity of underground muons.[78] What is observed by the Utah group is a distribution less strong than a $\sec\theta$ dependence, showing the existence of an isotropic muon flux. An isotropic muon flux can arise from the rapid leptonic decay of a heavy particle or from direct production of muons in primary cosmic-ray interactions. Direct production of muons is unlikely for the following reason. To fit the Utah data, a total cross section of 0.3 mb is necessary, and the cross section of muons of the same energy (a few TeV) on nucleons should also have the same magnitude. Observed attenuation of underground muons rules out such a high cross section for muons on nucleons.

A number of explanations have been given for the Utah effect. The simplest one that does not contradict other cosmic-ray observations is the so-called X process suggested by Bjorken *et al.*[79] According to this model, the Utah observation is explained as follows:

(a) New particles X are formed in primary cosmic-ray interactions,

$$Np \to X_1 \bar{X}_2 + \text{hadrons};$$

(b) the X particle decays into states containing muons, and the decays of the X^- must give rise to left-handed negative muons, which guarantees that the muons from π and K decay are not absorbed by the X process;

(c) muon-nucleon interactions lead to shower formation via

$$\mu p \to X_1 \bar{X}_2 + \mu + \text{hadrons}.$$

If we assume that $X_1 = X_2 = e^*$, then all of above are fulfilled since the decay $e^* \to \mu^- \nu_{e*} \nu_\mu$ has a rate 1.5×10^{17} sec, and the μ^-'s produced are predominantly left-handed. (Compare the electrons produced in μ^- decay; they are predominantly left-handed.) The threshold effect shows that the mass of the X particle is around 20 GeV,[80] which is very close to the mass of the heavy electron predicted by us. There is also some evidence for the muon bundles expected from the process (c).

The main difficulty in identifying e^* as X is the high cross section for the process (a) (about 0.3 mb) needed to fit the Utah data. If the e^* has standard electromagnetic and weak-interaction properties, then electromagnetic or weak production of e^* in pN collisions is ruled out. It seems that if $X = e^*$, then some as yet unknown interaction is responsible for its production, or the e^* may be strongly interacting. But at present, there is no reason to assign a strong interaction to the e^*.

The alternative possibility, $X_1 = e^*$ and $X_2 = e$, is ruled out by the mass of the X pair obtained from Utah data.

B. Anomalous Stopping Rate of Underground Cosmic-Ray Muons

Closely related to the Utah effect are the findings of Baschiera *et al.*[40] They have found some indication for an anomaly in the stopping rate of underground cosmic-ray muons. Namely, the ratio R of stopping muons S_μ to the traversing muons N_μ at a given depth was higher than the value predicted by conventional processes. Normal photoproduction, muon production by decay of pions and kaons, muon production of muon pairs, or the neutrino production of muons seem unable to account for this discrepancy. One explanation given for the above observation is that muons are also produced as decay products of a short-lived parent.[40] According to the estimates of the above authors, the cross section for production of such a particle in muon interactions should be 10^{-28}–10^{-29} cm^2. If it were not for the uncertainty in the cross section, e^* production by $\mu+N \to e^{+*}+e^{*-}+\mu+N$ could be responsible for the above effect. Another possible mechanism is $\mu+N\to\mu^*+N$, because the μ^* can decay into states containing muons. If the nonminimal interaction (19) between the μ^* and the μ exists, it can lead to the above process. Consideration of the anomalous magnetic moment of the muon, as in Sec. VI, gives the limit for λ in (19) as $\lambda \leq 3\times10^{-2}$, while an estimate of the cross section for μ^* production using this limit yields $\sigma \leq 10^{-29}$ cm^2.

C. Horizontal and Muon-Poor Air Showers

Ordinary cosmic-ray air showers are well known. In addition to these, it is also known that there are two other distinct types: horizontal air showers (HAS) and muon-poor air showers (MPS).[77] HAS contain abundant muons whereas MPS contain less muons. The rate of MPS is one thousandth of the ordinary air showers but is three orders of magnitude higher than HAS. It is speculated that both HAS and MPS originate from the same object. Mikamo *et al.*[81] have shown that existence of a heavy object with electromagnetic and muonic decay modes will explain HAS, MPS, as well as the Utah spectrum – provided the production cross section of the heavy particle is around 10^{-29} cm^2. The muonic decay mode will give rise to HAS and the γ rays from the electromagnetic decay to MPS. To explain the observed shower rate, the branching ratio for the two modes must be nearly unity. All of the above requirements, except the high production cross section, can be satisfied if we assume that the responsible particle is the e^*. The limit we have obtained for the electromagnetic decay rate of the e^* will not exclude the above branching ratio for electromagnetic and muonic decay modes.

Connected with air showers is the existence of high-transverse-momenta muons or large-angle scattering of cosmic-ray muons. High-transverse-momenta muons can originate from the decay of a heavy object like the e^*. Since the e^* has equal probabilities of decaying into muons and electrons, observation of the lateral distribution of electrons and muons in extensive air showers might be a test for our hypothesis. Although their results are statistically not very significant, the Tokyo[77] cosmic-ray group has observed one shower event of approximate energy 1.5 TeV which is incident on emulsion chambers from downward. The possibility that this shower was induced by the bremsstrahlung of a muon or from a neutrino interaction was estimated to be less than 10^{-5}. Events of this type are usually regarded as large-angle scattering of horizontal muons. Nevertheless, other possibilities cannot be ruled out; the shower might have been initiated by the decay product of a heavy object such as the e^* or μ^* formed in muon interactions. In this way we can connect the above observation to the effect observed by Baschiera *et al.*, discussed earlier in this section.

D. Energy Loss of Cosmic-Ray Muons

Cosmic-ray-muon experiments indicate that the rate of energy loss of cosmic-ray muons in the TeV region is somewhat larger than the rate expected from conventional processes.[39] At muon energies less than 1 TeV, the dominant mechanisms of energy loss are ionization and excitation. At higher energies (above 1 TeV) bremsstrahlung, pair production, and photonuclear interactions become more important. Horizontal-air-shower studies indicate that shower rate is too high by a factor of 6 if only the above processes are taken into account.[77] If the μ^* exists and is coupled to the muon and the electromagnetic field via the interaction (19), then an increase of the bremsstrahlung cross section at energies comparable to the mass of μ^* can be expected. Because the presence of μ^* will modify the muon propagator the bremsstrahlung process (Fig. 14) will be enhanced as a resonance effect. The resonance will occur when $|p'+k|$ = mass of $\mu^* \simeq 4.5$ TeV. The modified

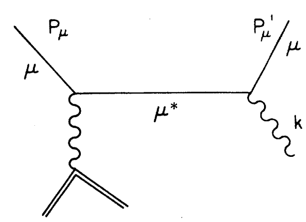

FIG. 14. Feynman diagrams for muon bremsstrahlung.

formula for the cross section for muon bremsstrahlung can be given very approximately by the expression

$$\sigma = \lambda^2 \sigma_{BH}(1+p^2/\Lambda^2), \qquad (70)$$

where Λ is of the order of the μ^* mass and σ_{BH} is the bremsstrahlung cross section as given by Bethe and Heitler.[82]

Before concluding this section, we would like to point out another instance of suspected-heavy-particle production in cosmic rays. Cosmic-ray flux measurements in the region 10^{10}–10^{14} eV obtained by calorimeters on Proton I and Proton II satellites have indicated that the cross section for protons on carbon increase by about 20% [83] in the interval between 2×10^{10} and 10^{12} eV. Kaufman and Mongan have interpreted this result as production of a heavy particle in carbon-proton collisions.[83] The mass of the particle is estimated to be between 15 and 29 GeV. Also, the particle should have a muonic decay mode in which most of the energy is transferred to the muon. The production cross section was estimated to be 55 mb. The main difficulty we encountered in trying to interpret anomalies in cosmic-ray muons as manifestations of our heavy leptons was the problem of obtaining a high production cross section. It is not easy to think of any mechanism involving standard electromagnetic and weak interactions which will account for the needed high production cross section required. Two possibilities are that the heavy leptons may have strong interactions or that the weak interaction might become strong at the high energies needed to produce these objects.

VIII. CONCLUSION

It is not claimed that the model of leptons based on discrete scale symmetry has offered a complete solution to the muon puzzle. The model is also inadequate in another respect. The relation $Q=\frac{1}{2}e\times(1+n/|n|)$ is empirical; it remains to be explained why a lepton, less massive than the electron, is uncharged.

Nevertheless, if the lepton mass spectrum predicted by the model is experimentally demonstrated to exist by detection of the e^* or by measurement of neutrino masses, then the problem of the muon is also partly resolved because in such a case, the muon appears as a consequence of discrete scale invariance, and all that is needed in addition is an explanation for the origin of the number ρ. When all interactions are taken into account, it might turn out that the theory is scale-invariant for one particular scale transformation. Clearly, this would lead to discrete scale invariance. If the neutrino mass is not zero, the above transformation should also provide an explanation for the relation $Q=\frac{1}{2}e(1+n/|n|)$ and the fact that a lepton less massive than the electron is uncharged.

We have suggested also the possible discrete scale invariance of the intermediate boson fields, with a universal quantum number n taking only positive values. There may be instances in which this type of discrete scale invariance is realized even in hadron physics. In fact, the measurements of fireball masses by the Japanese-Brazilian cosmic-ray group[84] strongly suggest that the type of discrete scale invariance we have discussed in Sec. IV is realized by fireballs detected in cosmic-ray experiments. They have found that fireball masses satisfy a relation of the form $M_n = m_0 a^n$, with $m_0 \simeq 230$ MeV and $a \simeq 10$.

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‡On leave of absence from Vidyodaya University, Ceylon.

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