$$+\left(\int_N^\infty \frac{dt'}{(t'-t)(t'-t_0)^{1/2}}\bigg/\int_N^\infty \frac{dt'}{t'(t'-t_0)^{1/2}}\right)\left(-\int_{t_0}^N \frac{dt'\ln H(t')}{t'(t'-t_0)^{1/2}}+\int_N^\infty \frac{dt'\ln w(t')}{t'(t'-t_0)^{1/2}}\right]\Bigg\} . \quad \text{(A1)}$$

We take the branch of the square-root function with positive real part where it occurs in this equation.

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Test for Fractionally Charged Partons from Deep-Inelastic Bremsstrahlung in the Scaling Region*

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We show that measurements of deep-inelastic bremsstrahlung, $e^\pm + p \rightarrow e^\pm + \gamma + \text{anything}$, in the appropriate scaling region will provide a definitive test for fractionally charged constituents in the proton, provided the parton model is valid. More precisely, measurement of the difference between the scaling inclusive bremsstrahlung cross sections of the positron and electron will allow the determination of a proton structure function $V(x)$ which, unlike the deep-inelastic e-p structure functions, obeys an exact sum rule based on conserved quantum numbers. In particular, we show that $\int_0^1 dx\, V(x) = \frac{1}{3}Q + \frac{2}{9}B$ ($= \frac{5}{9}$ for a proton target) in the quark model, whereas $\int_0^1 dx\, V(x) = Q$ in the case of integrally charged constituents. Since the result is independent of the momentum distribution of the partons, the sum rule holds for nuclear targets as well. Since $V(x)$, which involves the cube of the parton charge, is related to odd-charge-conjugation exchange in the t channel, Pomeranchukon, and other C-even contributions are not present, so that $V(x)$ should have a readily integrable quasielastic peak. This, combined with the fact that there exists a simple kinematic region in which the difference is of the same order as the inclusive bremsstrahlung cross sections themselves, and the fact that there is no hadronic-decay background, should make this a feasible experiment on proton and nuclear targets.

INTRODUCTION

The observation of scaling in the highly inelastic limit of electron-proton scattering has excited considerable interest in constituent models of hadrons. The existence of charged, structureless "partons" in the nucleon, together with an assumption limiting the partons' momentum distribution, is sufficient to derive scaling.[1] It is also well known that to account for scaling it is not necessary to postulate the full apparatus of a parton model but instead only to abstract from such a theory the singular behavior of current commutators in the vicinity of the light cone.[2]

Since they are more specific, however, parton models make concrete predictions which cannot be obtained from more general light-cone considerations. An example is the prediction of scaling in the process $p+p \rightarrow \mu^+ + \mu^- + \text{anything}$[3] at high energy and large $(\mu^+\mu^-)$ invariant mass. A test of this prediction will be central in establishing the parton model independently of the light-cone approach.[4] More recently the parton model has been found to provide a particularly simple explanation of large-angle exclusive scattering.[5] Although the parton model may be only an abstraction of a more complete theory, it is important to obtain and test all of its predictions, particularly in cases where the number of new assumptions is minimal.[6]

If partons are taken seriously it is important to find ways of determining their quantum numbers. Although the electroproduction structure functions $\nu W_2^{ep}(x)$ and $\nu W_2^{en}(x)$ are sensitive to the squared charges of the partons, it is impossible to extract from them values of the charges without making additional, strong assumptions regarding the distribution of partons within the nucleon.[7] Our object is to describe an experiment which admits a parton-model description and which provides a definitive probe of the partons' charges. The experiment involves the process

$$e^{\pm}+p\to e^{\pm}+\gamma+\text{anything}$$

in an appropriate "scaling" region. More precisely, measurement of the difference between the scaling inclusive bremsstrahlung cross sections of the positron and electron will allow determination of a structure function dependent upon the charge cubed of the various partons.[8] As we shall see, this provides a definitive test for fractionally charged partons. This process avoids the complications of Pomeranchukon subtractions and hadronic decay backgrounds. The assumption of a particular longitudinal-momentum distribution for the partons is not necessary in the derivation of sum rules.

If the parton model is correct and scaling is observed, then the corresponding structure function depends only on the odd-charge-conjugation piece of the parton distribution functions

$$V(x)\equiv\sum_a \lambda_a{}^3U_a(x)$$
$$=\sum_a \lambda_a{}^3U_a^{\text{odd}}(x)\,,$$
$$U_a^{\text{odd}}(x)=\tfrac{1}{2}[U_a(x)-U_{\bar a}(x)]\,,$$

where $U_a(x)$ is the probability to find a parton of type a with charge λ_a and fraction x of the proton's momentum in an infinite-momentum reference frame. Unlike $\nu W_2^{ep}(x)$, which obtains contributions from even-charge-conjugation (e.g., Pomeranchukon) t-channel exchange terms, the new structure function should show a quasielastic peak (vanish as $x\to 0$); sum rules involving the integral of $V(x)$ can be expected to converge in a finite experimentally accessible region. Moreover, integrals over $V(x)$ are determined by the normalization of various odd-charge-conjugation form factors (e.g., charge, baryon number, hypercharge) and thus provide a definitive test for fractional charge. We also note that since the $U_a^{\text{odd}}(x)$ are related in parton models to the structure functions for highly inelastic neutrino scattering, $V(x)$ should be completely determined by the results of neutrino experiments.

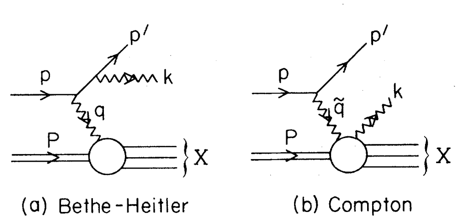

FIG. 1. Diagrams which contribute to inclusive bremsstrahlung, $e^{\pm}p\to e^{\pm}\gamma X$. The Bethe-Heitler amplitude also receives a contribution from the amplitude in which the photon is emitted from the incident lepton. The Compton amplitude changes sign with the lepton charge.

THE BREMSSTRAHLUNG CROSS SECTION

The diagrams relevant to the inclusive bremsstrahlung process $e^{\pm}+p\to e^{\pm}+\gamma+\text{anything}$ are shown in Fig. 1. In general there are contributions from both the standard Bethe-Heitler bremsstrahlung amplitude and the virtual inelastic Compton amplitude. The difference of the inclusive cross sections

$$d\sigma(e^+ +p\to e^+ +\gamma+X)-d\sigma(e^- +p\to e^- +\gamma+X)$$

is due, in order α^3, to the interference of these two amplitudes (see Fig. 2), which is a particular discontinuity of the 3-photon "double" Compton amplitude[9]

$$V_{\mu\nu\lambda}=\frac{4\pi^2E_P}{M}\int d^4x\,d^4y\,e^{iq\cdot y+ik\cdot x}\times\langle P|J_\nu(y)T^*(J_\lambda(0)J_\mu(x))|P\rangle\,. \qquad (1)$$

We shall work in the Bjorken kinematic region[10]

$$2P\cdot q=2P\cdot(\tilde q-k)\gg M^2\,,$$
$$Q^2\equiv -q^2=-(\tilde q-k)^2\gg M^2\,, \qquad (2a)$$

with $x\equiv Q^2/2P\cdot q$ fixed. In addition we require that

$$\tilde Q^2\equiv-\tilde q^2\gg M^2\,,$$
$$2P\cdot\tilde q\gg M^2\,, \qquad (2b)$$
$$Q^2-\tilde Q^2=2k\cdot q\gg M^2\,.$$

In the parton model the leading contribution to $V_{\mu\nu\lambda}$ in this kinematic region arises when all three pho-

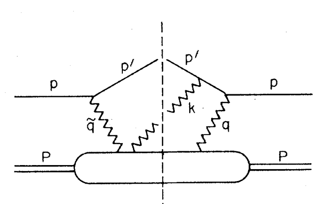

FIG. 2. The absorptive amplitude contributing to the $e^{\pm}p\to e^{\pm}\gamma X$ cross section difference, from the interference of the diagrams of Fig. 1.

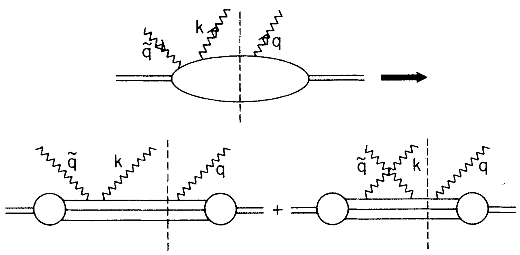

FIG. 3. The surviving single-parton contribution to the interference amplitude in the Bjorken scaling limit. The kinematical restrictions require that all three photons interact with the same parton. The result is proportional to the charge cubed of the parton.

tons scatter on an individual parton (see Fig. 3) and is given by kinematical factors multiplying the scale-invariant function $V(x)$.

This result is derived from the following considerations:

(a) As long as both spacelike photons, q and $\tilde{q}$, are massive (i.e., have large transverse momenta in an infinite-momentum frame) and are such that $Q^2 - \tilde{Q}^2 = 2k\cdot q \gg M^2$ (which constrains k also to have large transverse momentum in an infinite-momentum frame), then all diagrams in which photons interact with more than one parton line are strongly suppressed. [This assumption is, in general, not satisfied for inelastic Compton processes. In the case of small transverse momentum transfer, $P_T{}^2 \equiv ut/s$, where

$$t \equiv (\tilde{q} - k)^2\,,\quad s \equiv (P + \tilde{q})^2\,,\quad u \equiv (P - k)^2\,,$$

multiple parton processes can be important even in the scaling region.[11] This has been shown explicitly for the case of[12]

$$\gamma + p \to \text{“}\gamma\text{”}(Q^2) + \text{anything}\,,\quad Q^2 \gg M^2\,.$$

On the other hand, for large $P_T{}^2$ the elementary parton process calculated by Bjorken and Paschos[7] can be shown to dominate.[13]]

Since the interference contribution requires that both the Bethe-Heitler and Compton amplitudes have the same final state, we see that our kinematic restriction requires large transverse momentum in the hadronic wave function unless q, k, and $\tilde{q}$ all interact with the same parton, as in Fig. 3.[14] Of course, if the photon were taken to be in the soft, infrared region ($k\cdot P \ll M^2$) then bremsstrahlung can take place off of any of the constituent partons. This generates the usual target bremsstrahlung term in the soft-photon radiative correction formulas.

(b) As in the usual application of the parton model, the requirements of large q^2, $\tilde{q}^2$, $P\cdot q$, and $P\cdot\tilde{q}$ are assumed to justify the neglect of interparton interactions during the time period of the photon processes (the impulse approximation) and final-state interactions (incoherence approximation).

Thus the standard assumptions of parton models imply that the difference of positron and electron inclusive bremsstrahlung cross sections scales and is weighted by the cube of the partons' charges. Denoting, as usual, the fraction of the proton's momentum in an infinite-momentum frame carried by parton i as η_i we find that ($|p\rangle = \sum_n \mathfrak{a}_n |n\rangle$)

$$V_{\mu\nu\lambda} = \frac{1}{2P\cdot q}\,\frac{1}{x}\sum_{n,i} |\mathfrak{a}_n|^2 \langle n|\delta(\eta_i - x)\lambda_i{}^3|n\rangle M^i_{\mu\nu\lambda}\,, \tag{3}$$

with

$$M^i_{\mu\nu\lambda} = \tfrac{1}{2}\mathrm{Tr}\,\not{p}_i\gamma_\nu(\not{p}_i + \not{q}) \times[\gamma_\mu(\not{p}_i + \not{q} + \not{k})^{-1}\gamma_\lambda + \gamma_\lambda(\not{p}_i - \not{k})^{-1}\gamma_\mu]\,, \tag{4}$$

where $x \equiv Q^2/2P\cdot q$. We have written $M^i_{\mu\nu\lambda}$ for the case of spin-$\frac{1}{2}$ partons; the spin-zero case is analogous. From $V_{\mu\nu\lambda}$ we may extract the structure function $V(x)$:

$$V(x) \equiv \sum_{n,i} |\mathfrak{a}_n|^2 \langle n|\delta(\eta_i - x)\lambda_i{}^3|n\rangle \equiv \sum_a U_a(x)\lambda_a{}^3\,, \tag{5}$$

the sum being over parton and antiparton of different types, a.

The cross section is a function of six independent variables,

$$\begin{aligned} P\cdot p &\equiv ME \equiv \tfrac{1}{2}Q^2\alpha\,,\\ P\cdot p' &\equiv ME' \equiv \tfrac{1}{2}Q^2\alpha'\,,\\ P\cdot k &\equiv Mk_0 \equiv \tfrac{1}{2}Q^2\gamma\,,\\ p\cdot k &\equiv \tfrac{1}{2}Q^2\beta\,,\\ p'\cdot k &\equiv \tfrac{1}{2}Q^2\beta'\,, \end{aligned} \tag{6}$$

and

$$Q^2 \equiv -(p - p' - k)^2\,.$$

The difference of cross sections may be written as ($s = 2p\cdot P$)

$$\frac{d^6\sigma_+}{(d^3p'/p_0')(d^3k/k_0)} - \frac{d^6\sigma_-}{(d^3p'/p_0')(d^3k/k_0)} = \frac{(e^2/4\pi)^3}{\pi^2 sQ^2}\sum_a |T^a_{\rm int}|^2 U_a(x)\lambda_a{}^3\,, \tag{7}$$

with

$$|T^a_{\rm int}|^2 = \frac{-M^a_{\mu\nu\lambda}L^{\mu\nu\lambda}}{q^2\tilde{q}^2}\bigg|_{\eta_a = x} \tag{8}$$

and

$$L_{\mu\nu\lambda} = \tfrac{1}{2}\,{\rm Tr}\,\not{p}'\gamma_\lambda\not{p}[\gamma_\nu(\not{p}-\not{q})^{-1}\gamma_\mu + \gamma_\mu(\not{p}-\not{k})^{-1}\gamma_\nu]\,. \tag{9}$$

The expression for $M^a_{\mu\nu\lambda}$ changes depending on whether partons of type a have spin 0 or $\frac{1}{2}$. The product $M^a_{\mu\nu\lambda}L^{\mu\nu\lambda}$ was computed using Hearn's program REDUCE.[15] The complete expressions are given in the Appendix for both spin-0 and spin-$\frac{1}{2}$ partons.

Here we will concentrate on a particularly simple region, namely,

$$\frac{1}{\beta-\beta'} \gg \alpha, \alpha', \gamma, \beta, \beta' \gg 1\,, \quad \text{with} \quad \alpha - \alpha' - \gamma \equiv 1/x\,,$$

of order unity, in which the formulas of the Appendix simplify considerably. To preserve the condition $Q^2 - \tilde{Q}^2 = Q^2(\beta-\beta') \gg M^2$, we are required to take Q^2 quite large. We choose this region for illustrative purposes only – in general Q^2 need not be larger than the onset of scaling in inelastic electroproduction (e.g., 1 GeV^2), and the full formulas of the Appendix must be used. In this region we find

$$\frac{d^6\sigma_+}{(d^3p'/p_0')(d^3k/k_0)} - \frac{d^6\sigma_-}{(d^3p'/p_0')(d^3k/k_0)} \cong \frac{-8(e^2/4\pi)^3}{\pi^2\alpha\beta\gamma x Q^6}\Big\{[\alpha^2x^2 - \alpha\beta x - \alpha x + \beta^2 - \beta\alpha' x + x^2\alpha'^2 + x\alpha' + 1]\sum_{\text{spin }1/2}\lambda_a^3 U_a(x)$$
$$+ [\alpha\beta x - 2\alpha\alpha' x^2 - \alpha x + \alpha'\beta x + \alpha' x + 1 - x^2\gamma^2]\sum_{\text{spin }0}\lambda_a^3 U_a(x)\Big\}\,. \tag{10}$$

Clearly the different dependences of spin-0 and spin-$\frac{1}{2}$ terms on the invariants allows one in principle to distinguish the parton's spin.

Besides simplifying our formulas this kinematic region satisfies the important experimental requirement that the interference be a substantial fraction of the signal. To estimate the individual electron and positron cross section we have calculated the squared amplitudes for the Bethe-Heitler and inelastic Compton processes off of a single parton. We find (e.g., for spin-$\frac{1}{2}$ partons) for the squared Compton amplitude

$$|T_C|^2 \cong \frac{8}{x^2\gamma^2Q^2}(\alpha^2x^2 - \alpha\beta x - \alpha x + \beta^2 - \alpha'\beta x + \alpha'^2x^2 + \alpha' x + 1) \tag{11}$$

and for the squared Bethe-Heitler amplitude

$$|T_{\rm BH}|^2 \cong \frac{8}{\beta^2Q^2}(\alpha^2x^2 - \alpha\beta x - \alpha x + \beta^2 - \alpha'\beta x + \alpha'^2x^2 + \alpha' x + 1)\,. \tag{12}$$

From these we can construct the interference-to-signal ratio

$$\frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \cong -2\frac{\sum\lambda_a^3U_a(x)}{\left[\frac{x\gamma}{\beta}\sum_a\lambda_a^2U_a(x) + \frac{\beta}{x\gamma}\sum_a\lambda_a^4U_a(x)\right]} \tag{13}$$

which is clearly of order unity. The choice of $\beta \cong \beta'$ yields this result because with that restriction the denominators which enter the inelastic Compton as well as the Bethe-Heitler amplitudes are approximately equal. It is therefore experimentally quite feasible to measure the quantity $\sum_a U_a(x)\lambda_a^3$ from the e^+-e^- cross section difference.

SUM RULES

To realize the particular utility of the interference measurement, one must recall that the usual sum rules for sums over the squares of the partons' charges involving $F_2(x) = \nu W_2(x)$ depend on a variety of questionable assumptions.[7] Rigorous sum rules must derive from quantum-number conservation. Specifically we have

$$Q = \int_0^1 dx \sum_a \lambda_a U_a(x)\,,$$
$$Y = \int_0^1 dx \sum_a y_a U_a(x)\,, \tag{14}$$
$$B = \int_0^1 dx \sum_a b_a U_a(x)\,,$$

where Q, Y, and B (λ_a, y_a, and b_a) are the charge, hypercharge, and baryon numbers of the target hadron (parton) of interest. All of these sum rules depend only on the odd-charge-conjugation part of $U_a(x)$:

$$U_a^{\rm odd}(x) \equiv \tfrac{1}{2}[U_a(x) - U_{\bar a}(x)]\,.$$

In general it is possible to reduce λ_a^3 (which is odd under charge conjugation) to a linear combination of λ_a, y_a, and b_a so that the integral

$$\int_0^1 V(x)dx = \int_0^1 dx \sum_a \lambda_a^3 U_a(x) \qquad (15)$$

is determined by quantum-number conservation.

This is in striking contrast to the sum rules involving the electroproduction structure functions $\nu W_2^{ep}(x)$ and $\nu W_2^{en}(x)$, defined by

$$\begin{aligned} \nu W_2^{ep}(x) &= x\sum_a \lambda_a^2 U_a(x)\,, \\ \nu W_2^{en}(x) &= x\sum_a \lambda_a^2 U_{\hat a}(x)\,, \end{aligned} \qquad (16)$$

where $\hat a$ is the isospin reflection of the parton a. νW_2^{ep} and νW_2^{en} depend only on the combination $U_a^{\rm even} = \tfrac{1}{2}[U_a(x) + U_{\bar a}(x)]$ and are therefore unrelated to the conserved quantum numbers. The following sum rules are easily constructed:

$$\int_0^1 \frac{dx}{x}\,\nu W_2(x) = \sum_a \lambda_a^2 N_a\,, \qquad (17a)$$

$$\int_0^1 dx\,\nu W_2(x) = \sum_a \lambda_a^2 \bar x_a N_a\,, \qquad (17b)$$

where $N_a \equiv \int_0^1 dx\, U_a(x)$ is the mean multiplicity for a parton of type a, and $\bar x_a N_a \equiv \int_0^1 x dx\, U_a(x)$ is the momentum fraction for partons of type a. The right-hand side of (17a) is completely unknown without strong assumptions. If it is possible to define a distribution function for the momenta of partons in each constituent state $|n\rangle$ ($|p\rangle \equiv \sum_n \mathfrak{a}_n |n\rangle$), and, if *one assumes* the distribution function to be symmetric in all its variables, then the right-hand side of (17b) reduces to the mean square charge of the partons.[16] The usefulness of (17b) is further diminished in the presence of neutral gluons for which case the mean square charge defined by the sum rule will be anomalously low. Similar remarks apply to sum rules for $\nu W_2^{ep} - \nu W_2^{en}$: They are valid only with specific assumptions about the distribution of partons in the nucleons.

Sum rules for $V(x) \equiv \sum_a \lambda_a^3 U_a(x)$ suffer from none of these difficulties:

(a) In all models with partons of charge 0 or ±1 (e.g., Drell, Levy, Yan; Han, Nambu; σ model; etc.) $\lambda_a^3 \equiv \lambda_a$ so that

$$\int_0^1 dx\, V(x) = Q = \begin{cases} 1 \text{ for protons} \\ 0 \text{ for neutrons}\,. \end{cases} \qquad (18)$$

(b) In the standard quark parton model $\lambda_a^3 = \tfrac{1}{3}\lambda_a + \tfrac{2}{9}b_a$ so that

$$\int_0^1 dx\, V(x) = \tfrac{1}{3}Q + \tfrac{2}{9}B = \begin{cases} \tfrac{5}{9} \text{ for protons} \\ \tfrac{2}{9} \text{ for neutrons} \end{cases}. \qquad (19)$$

The sum rule provides a striking test for fractionally charged partons. Since the sum rule is independent of the parton distribution, similar results hold for nuclear targets as well:

$$\int_0^1 dx\, V(x) = \frac{3Z + 2A}{9} \text{ (Quark model)}\,.$$

For nuclei with $A = 2Z$, the quark-model sum rule gives $\tfrac{7}{9}$ of the corresponding result for integrally charged constituents. Thus tests of the sum rule and the parton model can be performed on nuclear targets with the additional benefit of large cross sections.

Lastly, in various models it is possible to extract most of the functions $U_a(x)$ from deep-inelastic neutrino and electron scattering off of protons and neutrons,[17] and thereby relate $V(x)$ back to these processes. In the quark model, for example, one obtains

$$V(x) = \frac{1}{108}\left\{\frac{9}{x}\,[F_2^{\nu n}(x) - F_2^{\nu p}(x)] - 7[F_3^{\nu n}(x) + F_3^{\nu p}(x)]\right\} - \tfrac{1}{27}[U_\lambda(x) - U_{\bar\lambda}(x)]\,.$$

If one assumes $U_\lambda(x) = U_{\bar\lambda}(x)$ in nonstrange baryons the last term drops out; if not it may be expressed in terms of $\Delta S = 1$ deep-inelastic neutrino scattering.[17] Of course the integral $\int_0^1 dx[U_\lambda(x) - U_{\bar\lambda}(x)] \equiv S$ vanishes for nonstrange baryons. Similar analyses may be performed in other models.

CONCLUSION

In conclusion, we have shown that the parton model predicts a very specific scaling form for deep-inelastic bremsstrahlung. The prediction that the right-hand side of Eq. (A7) depends in the scaling region only on the variable x and not on any of the four other dimensionless ratios of invariants provides a strong test of the validity of the parton model. Second, since the structure function $V(x)$ depends on the cube of the parton charge, it is possible to obtain exact sum rules, Eqs. (18) and (19), which provide a definitive test of whether the constituents of the proton have fractional versus integral charge.

Since $V(x)$ does not receive contributions from diffractive, Pomeranchukon, or other C-even exchange components, it should have a readily integrable quasielastic peak. This, combined with the fact that there exists a simple kinematic region in which the Bethe-Heitler–Compton interference signal is maximal, and with the absence of a hadronic decay background, should make deep-inelastic bremsstrahlung a feasible experiment for proton and nuclear targets.

ACKNOWLEDGMENT

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APPENDIX

In this appendix we give the complete parton-model prediction for deep-inelastic bremsstrahlung in the scaling region. The inclusive cross section assuming spin-$\frac{1}{2}$ partons is ($e^2/4\pi=\alpha=1/137.036$, $Q^2=-q^2$, $M\nu=P\cdot q$, $s=2p\cdot P$)

$$\frac{d\sigma(e^{\pm}p\to e^{\pm}\gamma X)}{(d^3p'/p_0')(d^3k/k_0)}=\frac{s}{2M\pi}\frac{d\sigma}{dQ^2d\nu(d^3k/k_0)}$$
$$=\left(\frac{e^2}{4\pi}\right)^3\frac{1}{4\pi^2sQ^2}\left[\sum_a\lambda_a{}^2U_a(x)\,|T_{\rm BH}|^2\pm2\sum_a\lambda_a{}^3U_a(x)\,|T_{\rm int}|^2+\sum_a\lambda_a{}^4U_a(x)\,|T_C|^2\right], \tag{A1}$$

where

$$\begin{aligned}|T_{\rm BH}|^2=\frac{-4}{Q^2}[&-2x^2L_2{}^2\alpha\beta'(\alpha-\alpha')-2x^2L_1L_2(\alpha^2\beta-\alpha^2-\alpha\beta\alpha'+\alpha\alpha'\beta'-\alpha'^2\beta'-\alpha'^2)\\&-2x^2L_1{}^2\beta\alpha'(\alpha-\alpha')-xL_2{}^2\beta'(\alpha\beta-2\alpha\beta'-3\alpha+\alpha'\beta'+\alpha')\\&-xL_1L_2(\alpha\beta^2-3\alpha\beta\beta'-3\alpha\beta+3\alpha\beta'+2\alpha+3\beta\alpha'\beta'+3\beta\alpha'-\alpha'\beta'^2-3\alpha'\beta'-2\alpha')\\&+xL_1{}^2\beta(\alpha\beta-\alpha-2\beta\alpha'+\alpha'\beta'+3\alpha')-L_2{}^2\beta'(\beta'+1)+L_1L_2(\beta^2-2\beta\beta'-3\beta+\beta'^2+3\beta'+2)-L_1{}^2\beta(\beta-1)],\end{aligned} \tag{A2}$$

$$\begin{aligned}|T_C|^2=\frac{-4Q^2}{\bar{Q}^4}[&-x^2D_1{}^2(\alpha^2\beta'+\alpha\beta\alpha'-\alpha\alpha'\beta'-\beta\alpha'^2)-2x^2D_1D_2(\alpha^2\beta-\alpha^2+\alpha\beta\alpha'-\alpha\alpha'\beta'-\alpha'^2\beta'-\alpha'^2)\\&-x^2D_2{}^2(\alpha^2\beta'+\alpha\beta\alpha'-\alpha\alpha'\beta'-\beta\alpha'^2)-xD_1{}^2(\alpha\beta\beta'-\alpha\beta'^2-\alpha\beta'+\beta^2\alpha'-\beta\alpha'\beta'-\beta\alpha')\\&+xD_1D_2(\alpha\beta^2+\alpha\beta\beta'+\alpha\beta-\alpha\beta'-2\alpha-\beta\alpha'\beta'-\beta\alpha'-\alpha'\beta'^2+\alpha'\beta'+2\alpha')\\&+xD_2{}^2(\alpha\beta^2-\alpha\beta+\alpha\beta'^2+2\alpha\beta'-\beta^2\alpha'+2\beta\alpha'-\alpha'\beta'^2-\alpha'\beta')\\&+D_1D_2(\beta^2-2\beta\beta'-3\beta+\beta'^2+3\beta'+2)-D_2{}^2(\beta^2-\beta+\beta'^2+\beta')],\end{aligned} \tag{A3}$$

$$\begin{aligned}|T_{\rm int}|^2=\frac{-2}{\bar{Q}^2}[&4x^3P_{22}\alpha^2\alpha'+4x^3P_{21}\alpha^2\alpha'+4x^3P_{12}\alpha\alpha'^2+4x^3P_{11}\alpha\alpha'^2\\&-x^2P_{22}(\alpha^2\beta+2\alpha^2\beta'-\alpha^2+\alpha\beta\alpha'+3\alpha\alpha'\beta'+4\alpha\alpha'+\alpha'^2\beta'+\alpha'^2)\\&-x^2P_{21}(3\alpha^2\beta-2\alpha^2\beta'-3\alpha^2-\alpha\beta\alpha'+5\alpha\alpha'\beta'+4\alpha\alpha'-\alpha'^2\beta'-\alpha'^2)\\&-x^2P_{12}(\alpha^2\beta-\alpha^2+3\alpha\beta\alpha'+\alpha\alpha'\beta'-4\alpha\alpha'+2\beta\alpha'^2+\alpha'^2\beta'+\alpha'^2)\\&+x^2P_{11}(\alpha^2\beta-\alpha^2-5\alpha\beta\alpha'+\alpha\alpha'\beta'+4\alpha\alpha'+2\beta\alpha'^2-3\alpha'^2\beta'-3\alpha'^2)\\&+xP_{22}(\alpha\beta^2-\alpha\beta+2\alpha\beta'^2+3\alpha\beta'+\beta\alpha'\beta'+2\beta\alpha')\\&+xP_{21}(3\alpha\beta\beta'+2\alpha\beta-2\alpha\beta'^2-4\alpha\beta'-2\alpha-2\beta\alpha'\beta'-\beta\alpha'+\alpha'\beta'^2+3\alpha'\beta'+2\alpha')\\&+xP_{12}(\alpha\beta\beta'-2\alpha\beta'+2\beta^2\alpha'-3\beta\alpha'+\alpha'\beta'^2+\alpha'\beta')\\&+xP_{11}(\alpha\beta^2-2\alpha\beta\beta'-3\alpha\beta+\alpha\beta'+2\alpha-2\beta^2\alpha'+3\beta\alpha'\beta'+4\beta\alpha'-2\alpha'\beta'-2\alpha')\\&-P_{22}(\beta^2-\beta\beta'-2\beta+2\beta'^2+3\beta'+1)-P_{21}(\beta\beta'+\beta-\beta'^2-2\beta'-1)\\&+P_{12}(2\beta^2-\beta\beta'-3\beta+\beta'^2+2\beta'+1)-P_{11}(\beta^2-\beta\beta'-2\beta+\beta'+1)].\end{aligned} \tag{A4}$$

For the case of spin-0 partons

$$\begin{aligned}|T_{\rm int}|^2=\frac{-1}{\bar{Q}^2}[&8x^3P_{22}\alpha^2\alpha'+8x^3P_{21}\alpha^2\alpha'+8x^3P_{12}\alpha\alpha'^2+8x^3P_{11}\alpha\alpha'^2\\&-4x^2P_{22}\alpha(\alpha\beta-\alpha+3\alpha'\beta'+3\alpha')-4x^2P_{21}\alpha(\alpha\beta-\alpha+\alpha'\beta'+\alpha')\\&-4x^2P_{12}\alpha'(3\alpha\beta-3\alpha+\alpha'\beta'+\alpha')-4x^2P_{11}\alpha'(\alpha\beta-\alpha+\alpha'\beta'+\alpha')\\&+4xL_1(\alpha\beta-\alpha-2\beta\alpha'+\alpha'\beta'+\alpha')-4xL_2(\alpha\beta-2\alpha\beta'-\alpha+\alpha'\beta'+\alpha')\\&+xP_{22}(5\alpha\beta\beta'+8\alpha\beta-9\alpha\beta'-8\alpha+5\alpha'\beta'^2+9\alpha'\beta'+4\alpha')\end{aligned}$$

$$
\begin{aligned}
&+xP_{21}(\alpha\beta\beta' + 4\alpha\beta - 5\alpha\beta' - 4\alpha + \alpha'\beta'^2 + \alpha'\beta') \\
&+xP_{12}(5\alpha\beta^2 - 9\alpha\beta + 4\alpha + 5\beta\alpha'\beta' + 9\beta\alpha' - 8\alpha'\beta' - 8\alpha') \\
&+xP_{11}(\alpha\beta^2 - \alpha\beta + \beta\alpha'\beta' + 5\beta\alpha' - 4\alpha'\beta' - 4\alpha') \\
&+4L_1(\beta^2 - \beta\beta' - 2\beta + \beta' + 1) + 4L_2(\beta\beta' + \beta - \beta'^2 - 2\beta' - 1) \\
&-P_{22}(2\beta\beta'^2 + 5\beta\beta' + 4\beta - 5\beta'^2 - 9\beta' - 4) - P_{21}\beta'(\beta - \beta' - 1) \\
&-P_{12}(2\beta^2\beta' + 5\beta^2 - 5\beta\beta' - 9\beta + 4\beta' + 4) - P_{11}\beta(\beta - \beta' - 1)] \,. \qquad \text{(A5)}
\end{aligned}
$$

The quantities α, β, α', β' and kinematics are defined in Eq. (6). The propagators are (mass terms are neglected)

$$
\begin{aligned}
&L_1{}^{-1} = \frac{(p-k)^2}{Q^2} = -\beta\,, \qquad L_2{}^{-1} = \frac{(p'+k)^2}{Q^2} = \beta'\,, \\
&D_1{}^{-1} = \frac{(xP+q+k)^2}{Q^2} = x(\alpha - \alpha') - 1 + \beta - \beta'\,, \\
&D_2{}^{-1} = \frac{(xP-k)^2}{Q^2} = -x\gamma\,,
\end{aligned} \qquad \text{(A6)}
$$

and $P_{ij} = L_i \cdot D_j$, $i,j = 1,2$. Also note the relation $x = (\alpha - \alpha' - \gamma)^{-1}$.

The odd-conjugation structure function $V(x) = \sum_a \lambda_a{}^3 U_a(x)$ is thus obtained from experiment by the relation

$$
V(x) = \left[\frac{d\sigma(e^+p \to e^+\gamma X)}{(d^3p'/p_0)(d^3k/k_0)} - \frac{d\sigma(e^-p \to e^-\gamma X)}{(d^3p'/p_0)(d^3k/k_0)}\right] \Big/ \left[\left(\frac{e^2}{4\pi}\right)^3 \frac{|T_{\text{int}}|^2}{sQ^2\pi^2}\right] .
$$

A severe test of the parton model is obtained from the requirement that the right-hand side of (A7) is in fact a function of x alone. Note that hadronic decay processes, e.g., $ep \to e\pi^0 X \to e\gamma\gamma X$, contribute to the sum but not to the difference of $e^\pm$ cross sections. The nominal order of the total scaling inelastic bremsstrahlung e-p cross section is (α/π) times the total scaling inelastic e-p cross section.

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[8]We also note that measurements of the Bethe-Heitler–Compton interference contribution from the difference of positron and electron *elastic* bremsstrahlung determines the real part of the Compton amplitude and a check of the Kramers-Kronig dispersion relation (for photon mass $\tilde{Q}^2 \to 0$, $t = -Q^2 \to 0$). Furthermore, the parton model predicts the existence of an energy-independent ("seagull") contribution to the real part of the amplitude, which is independent of the photon mass $\tilde{Q}^2$ at fixed t. See Brodsky, Close, and Gunion (Ref. 6).

[9]The proton state is normalized to $\langle P|P'\rangle = (2\pi)^3\delta^3(P-P')$. We use the metric $(a\cdot b = a^0b^0 - \vec{a}\cdot\vec{b})$ and other conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[10]The early scaling observed in inelastic e-p scattering indicates that $Q^2 \geq 1$ GeV^2 should be sufficient here.

[11]S. J. Brodsky and P. Roy, Phys. Rev. D 3, 2914 (1971); M. Bander, University of California, Irvine report, 1972 (unpublished).

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[14]These results may be equally well obtained in the covariant approach of Landshoff, Polkinghorne, and Short (Ref. 1).

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[16]This is proved as follows: For any constituent state of the proton, $|n\rangle$, containing N_n particles we define a probability function $f^n(x_1, \ldots, x_{N_n})$ (integrated over trans-

verse momenta), normalized by

$$\int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_{N_n} f^n(x_1, \ldots, x_{N_n}) = 1.$$

Then νW_2 is given by

$$\nu W_2(x) = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \cdots dx_{N_n} x_i \delta(x_i - x) f^n(x_1, \ldots, x_{N_n}),$$

where P_n is the probability for finding the state $|n\rangle$ in the proton. Integrating from 0 to 1,

$$\int_0^1 \nu W_2(x) dx = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \cdots dx_{N_n} x_i f^n(x_1, \ldots, x_{N_n})$$

$$= \sum_n P_n \sum_i \lambda_i^2 (\bar{x}_i)_n,$$

where $(\bar{x}_i)_n$ is the average momentum of particle i in the state $|n\rangle$. Since for any values of $x_1, \ldots, x_{N_n}$ momentum conservation requires $\sum_{i=1}^{N_n} x_i = 1$ we find

$$\sum_{i=1}^{N_n} (\bar{x}_i)_n = 1.$$

If $f(x_1, \ldots, x_{N_n})$ is symmetric under interchange of any indices i and j, $(\bar{x}_i)_n$ is independent of i. This implies $(\bar{x}_i)_n = (\bar{x})_n = 1/N_n$ which proves the contention that $\int_0^1 \nu W_2(x) dx$ is the mean square charge.

[17]Llewellyn Smith (Ref. 6).

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A Model of Leptons*†

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A new model of leptons based on discrete scale transformations is proposed. It is shown that this model predicts a lepton mass spectrum consisting of an infinite series of electron-like and muonlike particles whose masses and charges are given by $m_n = m_e \rho^n$ $(\rho = m_\mu / m_e)$ $Q = \frac{1}{2}e(1 + n/|n|)$. Particles with n positive are charged, those with n negative neutral. Possible weak coupling schemes of charged and neutral leptons are considered. The lepton with $n = 2$ is a heavy electron at 22 GeV. Decay modes and production mechanisms of this particle are discussed. It is shown that, apart from high production cross sections needed to fit experimental data, some of the recently observed anomalies in cosmic-ray muons can be effects due to heavy leptons predicted by this model.

I. INTRODUCTION

Ever since its unexpected discovery, the muon has remained as a tantalizing puzzle in elementary particle physics.[1] Except for the discovery of the muon neutrino the situation today regarding leptons is the same as in 1947. (Even this was anticipated in the paper of Sakata and Inouye[2] written in 1946 to clarify the π-μ puzzle.) No new charged leptons were found. So far no experimental or theoretical clue has been found to suggest a difference between the electron and the muon, apart from the mass. All experiments carried out up to this date, viz., measurement of the branching ratios for decays of hadrons into electrons and muons,[3] precision measurements of the muon magnetic moment,[4] electron-proton and muon-proton scattering[5] and the recent experiment that demonstrated that the muon obeys the same statistics as the electron,[6] show that the muon is a heavy electron or in other words, that the so-called electron-muon universality is strictly obeyed.

Many attempts have been made to understand the muon puzzle, but none of them are entirely satisfactory. One class of such theories attempts to derive the muon from quantum electrodynamics. Some of these derivations are based on the observation that muon-electron mass ratio is almost exactly $\frac{3}{2}(1/\alpha)$, this being taken as an indication that the key to the muon-electron puzzle may lie entirely within the realm of ordinary quantum electrodynamics. It is possible to start with bare (zero-mass) electron and muon fields interacting with the electromagnetic field and get two distinct masses by renormalization.[7] But these arguments were cutoff-dependent. In theories with spontaneous breakdown,[8] the necessity of a cutoff is removed, and it is possible to obtain two renormalized masses. Furthermore, the heavier lepton remains stable, i.e., $\mu \to e + \gamma$ remains forbidden, which is nice[9]; the mass ratio, however, remains arbitrary.[10]

Another approach to the muon problem depends on higher-order wave equations. Markov[11] showed that the two linear equations $(i\not\partial + m)\psi_1 = 0$ and $(i\not\partial - m)\psi_2 = 0$ (where ψ is a four-component spinor), together are equivalent to a second-order equation that can describe two Fermi particles con-