cently worked with more sophisticated residues which can be required to give exactly the right πNN coupling constant and Δ width, as well as to fit the backward data. For the signs of interest to us, their residues imply the same results as (4.3) and (4.4).

[22]The motivation is from duality diagrams, whose prediction of a relative minus sign between the Breit-Wigner residues of the resonances belonging to Σ_α and those belonging to Σ_γ in the reaction $\bar{K}N \rightarrow \pi\Lambda$ is experimentally correct. Evidence for exchange-degenerate behavior of the Y^* contributions to the imaginary parts of invariant amplitudes has been given by C. Schmid and J. Storrow [Nucl. Phys. B29, 219 (1971)] and by R. Field and J. D. Jackson [Phys. Rev. D 4, 693 (1971)].

[23]C. Schmid and J. Storrow, Nucl. Phys. B44, 269 (1972), have recently constructed a model for the $\pi^- p \rightarrow \Lambda K^0$ polarization based on Σ_α-Σ_γ interfering with Σ_δ-Σ_β. To produce the correct polarization, such a model must, of course, conflict with the SU(3) estimate that Σ_δ-Σ_β is small compared to the observed cross section, and must involve a Σ_δ-Σ_α sign relationship differing from that suggested, via SU(3), by πN elastic scattering.

[24]Private communications from V. Barger and M. Olsson.

[25]This conjecture leads to a correct prediction for the sign of the polarization in $K^- n \rightarrow \Lambda\pi^-$.

[26]R. Eisner, R. Field, S. Chung, and M. Aguilar-Benitez, Phys. Rev. D (to be published).

[27]D. Davies *et al.*, Phys. Rev. D 5, 1 (1972).

[28]These two reactions are also exotic, as $K^+ p \rightarrow pK^+$ is. What the evidence from Sec. IV for Σ EXD breaking suggests is that *small* exotic amplitudes, such as those for these two reactions, are *not* approximately real.

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Effects of Proton–Atmospheric-Nuclei Interaction Processes on Predicted Muon Intensities*

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In numerous calculations of the muon intensity deep in the atmosphere, the details of the interactions which produce pions and kaons have been avoided by assuming a production spectrum proportional to $E^{-\alpha}$, where α is a constant. In contrast to this approach, we have used a realistic form for the primary cosmic-ray spectrum in which the spectral index varies with energy, and we have described the cosmic-ray proton–atmospheric-nuclei interactions using a statistical model for pion and kaon particle production. The resulting relative muon intensities at sea level have been calculated and are found to fall off more rapidly with momentum than intensities predicted by previous calculations. The muon charge ratio as a function of momentum (conveniently normalized to agree with experiment at low energies) is found to be sensitive to the spectral character of the primary cosmic-ray spectrum. Similar effects were found for muon intensities at large zenith angles resulting in no significant effects on predicted zenithal enhancements.

INTRODUCTION

During the past twenty years, numerous papers have been published[1-5] concerning muon differential intensities. In the earliest work by Barrett *et al.*[6] and in a relatively recent model developed by Maeda[2] and extended by Cantrell[3] the complexity of the interaction of the primary cosmic rays with the atmosphere is avoided by the use of production spectra for the muon parents, namely, pions and kaons. These spectra[6-12] are assumed to be proportional to $E^{-\alpha}$, where α is the logarithmic slope of the primary cosmic-ray spectrum and has been assumed to be a constant 2.7. Although the details of the propagation of muon parents and muons through the atmosphere are treated differently by various authors, the assumption of the above power-law production spectrum is common to most models. In this paper, pion and kaon production spectra are calculated in arbitrary units by using the two-temperature model developed by Wayland[13,14,15] to account for the interaction of the primary cosmic rays with nuclei in the atmosphere. These spectra are then used in place of the power-law production spectra in the Maeda-Cantrell formulation. The spectral character of the relative muon intensities resulting from these different production spectra are compared. In addition, the effect on muon intensities of a primary spectrum whose slope varies realistically with energy is examined.

PROCEDURE

In the two-temperature model, the transverse-momentum distribution of secondary particles is assumed to be independent of the energy of the incident proton. This assumption has been verified experimentally for nucleon-nucleon interactions above 15 GeV, and the validity of this assumption should improve with increasing energy. A characteristic temperature T_0 parametrizes the $p_\perp$ distribution, and a second temperature T governs the longitudinal-momentum distribution. Good fits to experimental machine data from 15 to 30 GeV are obtained using different T and T_0 and different interaction volumes for each particle type. These same parameters have been used in the present calculations. The multiplicity is determined by T in a complicated manner which allows agreement with measured multiplicity at cosmic-ray energies, and to a good approximation the longitudinal temperature T is proportional to $(E_0)^{1/4}$, where E_0 is the incident energy.

For a given particle type, the two-temperature model determines the differential cross section $d^2\sigma/dp\,d\Omega$. The accuracy of calculations at high energies near 1000 GeV depends on the validity of the assumed form for the multiplicity. However, it should be noted that this form shows good agreement with experimental results over the entire range of interest here, namely 20–1000 GeV.[14] Proton-nucleus collisions[14] as well as interactions of atmospheric nuclei with $Z>1$ incident primaries[16] can be related to nucleon-nucleon interactions by appropriate scale factors. Since the present calculations are made in arbitrary units to show effects on muon spectra, primary interactions with the atmosphere are considered to be nucleon-nucleon collisions only.

The differential rate at which particles are produced at a given momentum p is given by

$$\frac{dq}{dp} \propto \int_{E_\mu}^{\infty} \int_{\Omega} \frac{d^2\sigma}{dp\,d\Omega}\, j(E)\, d\Omega\, dE\,, \qquad (1)$$

where $d\Omega = \sin\theta\, d\theta\, d\phi$, E_μ is the lower bound on the energy for the production process considered (in our case, 20 GeV), and $j(E)$ is the differential energy spectrum of the primary cosmic rays. Gaussian quadrature integration was used. The azimuthal angle ϕ was integrated over the limits from 0 to 2π radians. The angle θ was integrated from 0 to $\tan^{-1}(2.0/p)$, where p is the momentum at which dq/dp is being evaluated. The energy E was integrated from $E_\mu = 20$ GeV to infinity to a precision of 0.1%. Using this procedure the π^+, π^-, K^+, and K^- intensities were each calculated independently in arbitrary units in the momentum interval 20–2000 GeV/c. The arbitrariness of units is introduced by the choice of interaction volume used in the two-temperature model. Of course, the same interaction volume was used for all intensity calculations for a given particle type. Analytical forms for these secondary spectra were found by using a least squared polynomial fit to the common logarithms of the calculated intensities in which the independent variable was the common logarithm of p. The analytical forms of the spectra were then used in place of the production spectra in the Maeda-Cantrell model to calculate the resulting muon intensities at sea level. The parameters used in the calculation are the same as those in Ref. 17.

The above calculation procedure was applied for two different assumptions about the primary spectrum: (1) a constant logarithmic slope of −2.6, and (2) a slope which varies with energy in the manner given below. The primary cosmic-ray integral spectrum[18,19] was differentiated to yield a differential intensity proportional to $E^{\alpha(E)}$, where

$$\alpha(E) = -2.15 - 0.065 \log_{10}(E) \quad \text{for } E \leq 10^8 \text{ GeV} \qquad (2)$$

and

$$\alpha(E) = -2.6 \quad \text{for } E > 10^8 \text{ GeV}.$$

TABLE I. Constants which provide best fit to production spectra with the function $I(p) \propto p^{a+b\log_{10}p}$.

Particle	Constant $p>20$ GeV/c a	b	Variable $p>20$ GeV/c a	b
π^+	−2.88	0.00	−1.84	−0.244
π^-	−2.97	0.00	−2.27	−0.168
K^+	−2.96	0.00	−2.23	−0.176
K^-	−2.98	0.00	−2.29	−0.179

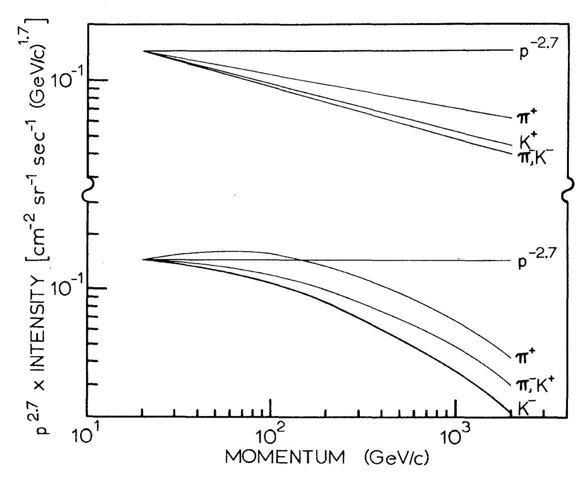

FIG. 1. Comparison of production spectra for $\pi^\pm$ and $K^\pm$ with the $p^{-2.7}$ production spectrum. Upper set of curves is for a constant primary power index and lower set is for variable primary power index.

TABLE II. Muon spectra in arbitrary units for μ^+ and μ^- along with local spectral slopes. $I(p)$ is the intensity and $S(p)$ is the logarithmic slope at a given momentum.

Particle		Momentum (GeV/c)						
		20	40	100	200	400	1000	2000
$\mu^{\pm}$ c,d	$I(p)$	9.44×10^7	1.72×10^7	1.53×10^6	2.17×10^5	2.76×10^4	1.48×10^3	1.41×10^2
	$S(p)$	−2.48	−2.75	−3.04	−3.22	−3.36	−3.48	−3.53
μ^+ a,d	$I(p)$	9.44×10^7	1.34×10^7	7.75×10^5	7.58×10^4	6.62×10^3	2.32×10^2	1.71×10^1
	$S(p)$	−2.66	−2.95	−3.26	−3.45	−3.59	−3.72	−3.77
μ^- a,d	$I(p)$	9.44×10^7	1.28×10^7	6.98×10^5	6.58×10^4	5.58×10^3	1.89×10^2	1.36×10^1
	$S(p)$	−2.73	−3.01	−3.31	−3.50	−3.64	−3.76	−3.80
μ^+ b,d	$I(p)$	9.44×10^7	1.58×10^7	9.87×10^5	9.09×10^4	6.77×10^3	1.66×10^2	8.48×10^0
	$S(p)$	−2.37	−2.78	−3.27	−3.60	−3.88	−4.20	−4.39
μ^- b,d	$I(p)$	9.44×10^7	1.42×10^7	8.05×10^5	7.19×10^4	5.36×10^3	1.36×10^2	7.24×10^0
	$S(p)$	−2.54	−2.91	−3.34	−3.62	−3.87	−4.14	−4.30
$\mu^{\pm}$ c,e	$I(p)$	2.09×10^7	7.12×10^6	1.06×10^6	1.98×10^5	3.22×10^4	2.45×10^3	3.03×10^2
	$S(p)$	−1.35	−1.87	−2.44	−2.78	−3.03	−3.24	−3.30
μ^+ a,e	$I(p)$	2.09×10^7	6.21×10^6	7.17×10^5	1.02×10^5	1.19×10^4	5.49×10^2	4.69×10^1
	$S(p)$	−1.47	−2.02	−2.62	−2.98	−3.25	−3.47	−3.65
μ^- a,e	$I(p)$	2.09×10^7	5.97×10^6	6.49×10^5	8.85×10^4	9.92×10^3	4.38×10^2	3.65×10^1
	$S(p)$	−1.52	−2.09	−2.69	−3.04	−3.31	−3.52	−3.57
μ^+ b,e	$I(p)$	2.09×10^7	6.89×10^6	8.41×10^5	1.14×10^5	1.14×10^4	3.69×10^2	2.14×10^1
	$S(p)$	−1.30	−1.92	−2.64	−3.11	−3.52	−3.96	−4.22
μ^- b,e	$I(p)$	2.09×10^7	6.35×10^6	7.06×10^5	9.16×10^4	9.08×10^3	3.04×10^2	1.87×10^1
	$S(p)$	−1.42	−2.03	−2.72	−3.16	−3.52	−3.89	−4.09

[a] Calculated with constant power index for primary cosmic-ray spectrum using two-temperature model.
[b] Calculated with variable power index for primary cosmic-ray spectrum using two-temperature model.
[c] Calculated using production spectra of the form $p^{-2.7}$.
[d] Vertical intensity (0.0°).
[e] Intensity at a zenith angle of 77.5°.

The logarithmic slope varied from −2.28 at 10 GeV to −3.19 at 10^8 GeV. In the Appendix, we comment on the difference between the slope of the spectrum and the power index, $\alpha(E)$.

RESULTS AND DISCUSSION

Values of the production spectra π^+, π^-, K^+, and K^- along with the local logarithmic slopes of these spectra can be calculated using the parameters a and b of Table I in the function $I(p) \propto p^{a+b\log_{10}p}$. Parameters are given for a primary spectrum with both a constant power of −2.6 and the variable power defined in Eq. (2).

It is interesting to note that the logarithmic slopes of the production spectra vary for different particle species. This is true for both of the above assumptions about the primary spectrum. The difference in the spectral slopes is particularly pronounced in the π^+ and π^- spectra. However, the more important differences are between the slopes of the calculated production spectra and those resulting from the use of a constant slope of −2.7 assumed in earlier models. Even in the case of a primary spectrum with a constant power of −2.6, the nearest resulting production spectrum slope is −2.88 and the slopes are further increased in magnitude when the primary slope is allowed to vary with energy. The differences in production spectra are explicitly displayed in Fig. 1, where we have plotted $p^{2.7}I(p)$ vs p, $I(p)$ being the production spectrum in each case.

The production spectra described above were

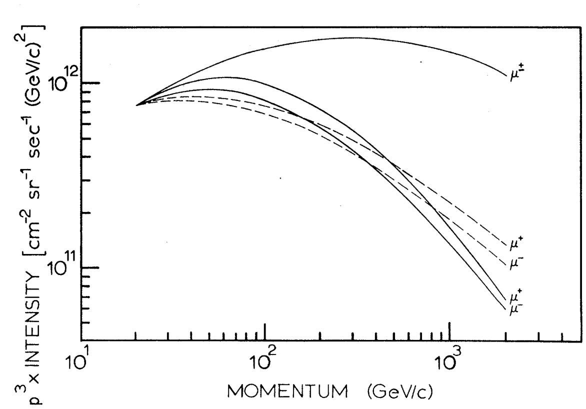

FIG. 2. Comparison of muon intensities with the results assuming a $p^{-2.7}$ production spectrum (Ref. 17). Dashed curves are for a constant primary power index and the lower solid curves are for a variable primary power index.

included in the Maeda-Cantrell model to calculate resulting muon intensities at sea level in the vertical direction and at a zenith angle of 77.5°. The μ^+ and μ^- spectra were calculated independently using a chosen charged K/π ratio of 0.25 in each case.[14,17,20] The results are given in arbitrary units in Table II for three cases: (1) pion and kaon production spectra with a constant logarithmic slope of −2.7; (2) a primary spectrum with a constant power of −2.6; and (3) a primary spectrum with the variable power of Eq. (2). The differences are displayed in Fig. 2, where we have plotted $p^3 I(p)$ versus p. It is clear that significant differences arise between the above three cases. In fact when the intensities are normalized at 20 GeV/c, the resulting intensities at 1 TeV/c differ by a factor of 10 between cases 1 and 3. Also, for both cases 2 and 3 the μ^+ and μ^- spectra differ. Figure 3 compares the μ^+ and μ^- spectra for case 3 with the $\mu^\pm$ spectrum of case 1. Again for convenience, Table III gives the constants A, B, and

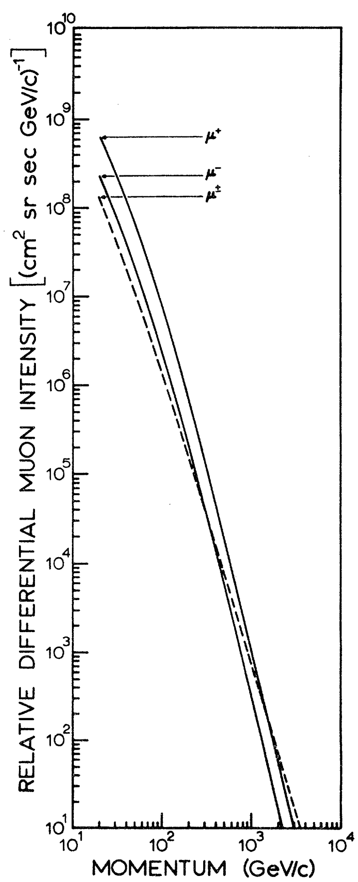

FIG. 3. Muon spectra in arbitrary units assuming a variable primary power index compared with the spectrum resulting from an assumed $p^{-2.7}$ production spectrum (dashed curve).

TABLE III. Constants which provide best fit to muon spectra with the function $I(p) \propto p^{A+B\log_{10}p+C(\log_{10}p)^2}$.

	0.0°			77.5°		
Particle	A	B	C	A	B	C
$\mu^{+\,a}$	−0.941	−0.811	0.0774	1.89	−1.60	0.156
$\mu^{-\,a}$	−1.04	−0.797	0.0767	1.93	−1.65	0.165
$\mu^{+\,b}$	−0.109	−1.01	0.0729	2.11	−1.54	0.117
$\mu^{-\,b}$	−0.526	−0.905	0.0674	2.05	−1.59	0.134
$\mu^{\pm\,c}$	−0.884	−0.754	0.0715	1.89	−1.54	0.152

[a] Calculated with constant power index for primary cosmic-ray spectra using the two-temperature model.
[b] Calculated with variable power index for primary cosmic-ray spectra using the two-temperature model.
[c] Calculated using production spectra of the form $p^{-2.7}$.

C which yield the best fit to the muon spectra with the function $I(p) \propto p^{A+B\log_{10}p+C(\log_{10}p)^2}$.

The differences in spectra lead to predicted charge ratios that are distinctly momentum-dependent. Since the μ^+ and μ^- intensities are calculated in arbitrary units, any charge-ratio predictions require normalization of the ratio at some momentum. We have chosen to normalize this ratio to 1.25 at 20 GeV/c, since the ratio is fairly well defined in that region.[21–26] The charge ratio, assuming a variable primary power index, and the result, assuming a constant primary index, are compared with experimental results[21–26] in Fig. 4. It is clear that significant differences arise in the predicted charge ratios. Curve A appears to represent a better fit to experiment than B. Both curves A and B would show less increase with energy if the primary spectrum were actually somewhat steeper than the two spectra used in the present calculations. Also, in Fig. 1 it is evident that the two-temperature model pre-

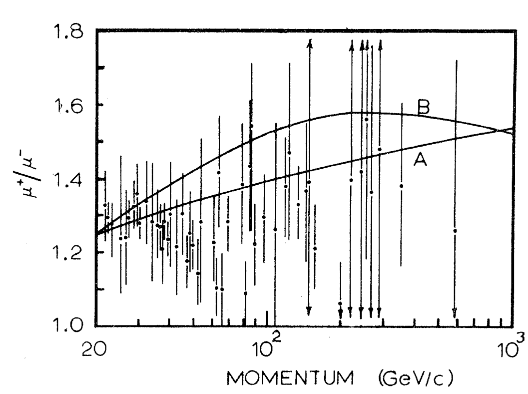

FIG. 4. Muon charge ratios in the vertical direction: curve A – constant primary power index; curve B – variable primary power index. Normalized to 1.25 at 20 GeV/c (Refs. 21–26). Data points from Refs. 21–26.

TABLE IV. Zenithal μ^+ and μ^- intensity enhancements at 77.5°.

Particle	Momentum (GeV/c) 20	40	100	200	400	1000	2000
$\mu^{+\,a}$	0.232	0.487	0.972	1.42	1.89	2.49	2.88
$\mu^{-\,a}$	0.225	0.475	0.949	1.37	1.81	2.36	2.75
$\mu^{+\,b}$	0.259	0.510	0.998	1.47	1.98	2.60	2.96
$\mu^{-\,b}$	0.240	0.486	0.953	1.38	1.84	2.42	2.81
$\mu^{\pm\,c}$	0.247	0.501	0.970	1.38	1.81	2.35	2.74

[a] Calculated with constant power index for primary cosmic-ray spectrum using two-temperature model.

[b] Calculated with constant power index for primary cosmic-ray spectrum using two-temperature model.

[c] Calculated using production spectra of the form $p^{-2.7}$.

dicts K^+ and K^- spectra which show less difference than the π^+ and π^- spectra for both assumed primary spectra. Consequently, an increase in the charged K/π ratio will also tend to decrease the μ^+/μ^- ratio at high momenta. It is interesting to note at this point that the model of limiting fragmentation predicts a constant charge ratio.[27] However, it is difficult to compare these predictions with experimental results reported thus far, since the uncertainties in measurements above 20 GeV/c are quite large and increase with momentum.[21-26]

The ratio of the muon intensity at a large zenith angle to that in the vertical direction is commonly referred to as the *zenithal enhancement*. The dependence of the enhancements on zenith angle and momentum may be sensitive to the assumptions made concerning the production processes for muons. To display the enhancements resulting from the various production models of the present work, we give values of enhancements at a zenith angle of 77.5° in Table IV and compare these with enhancements for the assumption of a production spectrum with constant logarithmic slope −2.7. Differences of about 5% are seen between μ^+ and μ^- enhancements regardless of the assumption about the primary-spectrum power index. At high energies the enhancements from the two-temperature model exceed very slightly those for the constant slope production spectrum case.

CONCLUSIONS

We find that the $\pi^\pm$ and $K^\pm$ production spectra derived using the two-temperature model for cosmic-ray primary interactions in the atmosphere show marked deviations from an assumed power-law production spectrum of index −2.7. These differences persist independent of whether the primary cosmic-ray spectrum is assumed to have a constant power index of −2.6, or, more realistically, a variable power index. The resulting muon intensities differ in that they fall off more rapidly with momentum than the intensities resulting from a pion-kaon production spectrum of power −2.7. An increase in the magnitude of the slope of the muon spectrum at high energies affects the interpretation of deep-underground intensity measurements since the logarithmic slope of the muon spectrum at a given momentum is a significant factor in determining fluctuation corrections. Further differences occur between the μ^+ and μ^- intensities which are the result of the distinctness of the π^+, π^-, K^+, and K^- production spectra. The charge ratio μ^+/μ^- reflects these differences and is clearly sensitive to assumed production models. The intensities calculated for large zenith angles show the same basic variations in spectral character. In fact, the enhancements [e.g., $I(77.5°)/I(0°)$] are insensitive to our assumptions concerning particle production.

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APPENDIX

The muon intensity can be written as

$$I(p) = kp^{\alpha(p)},$$

where k is the normalization constant and α is the power index. The power index can either be a constant or depend upon the momentum. The logarithmic slope is given by

$$\frac{d\log_{10} I}{d\log_{10} p} = \alpha(p) + \frac{p\log_{10} p}{\log_{10} e}\,\frac{d\alpha}{dp}.$$

In the case of a constant power law form, the second term is zero and the logarithmic slope is simply the power index. However, when the power index is a function of momentum, the use of the logarithmic slope as this index can lead to large errors. In fact a fit to a survey of integral muon intensities[5] defines a logarithmic slope near −1.6 at 10 GeV/c, increasing in magnitude to −2.7 at 1000 GeV/c and reaching a value of −3.4 at 7000 GeV/c.

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