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Exploration of SU(3) Symmetry of Baryon Regge Residues*

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The SU(3) symmetry of baryon Regge poles is explored by comparing the angular distributions of backward meson-baryon reactions whose Regge exchanges should be SU(3)-related. Several indications of the symmetry are found. In particular, it is discovered that one can generate the observed highly structured angular distribution for $\pi^+ p \to p \pi^+$ from the featureless one for $K^+ p \to p K^+$ simply by assuming SU(3) symmetry and taking singlet-octet exchange degeneracy in $K^+ p$ scattering into account. Using the symmetry, evidence that exchange degeneracy between the Reggeized $\frac{1}{2}^+$ and $\frac{3}{2}^-$ octets is broken is found. Relevant experiments are suggested.

I. INTRODUCTION

We are exploring the question of whether baryon Regge residues show the same kind of SU(3) symmetry that resonance decay rates and stable-particle couplings do. More specifically, we are looking for evidence of such symmetry (and sometimes of exchange degeneracy as well) in the observed features of backward 0^- meson$-\frac{1}{2}^+$ baryon scattering. We would like to report some of the things which have been found.

As is well known, resonance partial widths have proved to obey close-to-exact SU(3) symmetry, once the barrier-penetration–phase-space factor q^{2l+1} is divided out of them.[1] (Here q is the center-of-mass momentum of the outgoing particles, and l is the orbital angular momentum of the resonance.) Thus, the residue of the Breit-Wigner pole in the partial-wave amplitude $(S-1)_{ij}/2i(q_iq_j)^{1/2}$ for formation of a resonance by incoming state i, and its subsequent decay into outgoing state j, is SU(3)-symmetric once the factor $(q_iq_j)^l$ is removed. It is then natural to conjecture that as one moves along the Regge trajectory to which the resonance belongs, this barrier-penetration effect, $(q_iq_j)^l$, continues to be the only thing which breaks the SU(3) symmetry of the residue. Hence, we write the signatured meson-baryon partial wave containing the Regge pole to which some resonance k belongs (for definiteness we treat here the case where k is a member of the δ decuplet [$\Delta_\delta(1236)$, $\Sigma_\delta(1385)$, etc.]) in the form

$$f_{ij}(J,W)=\frac{\beta_{ij}^k(W)/s_0{}^J}{J-\alpha_k(u)}(q_iq_j)^l\left(\frac{E_i+M_i}{W}\,\frac{E_j+M_j}{W}\right)^{1/2}. \tag{1.1}$$

Here J is the total angular momentum, $l=J-\frac{1}{2}$ is the orbital angular momentum, W is the total c.m. energy, $u=W^2$, and E_i is the c.m. energy of the baryon, of mass M_i, in state i. The trajectory on which resonance k lies is denoted by α_k. [We will always take α_k to be the true, somewhat SU(3)-breaking, trajectory.] The amplitude f is defined to coincide with $(S-1)/2i(q_iq_j)^{1/2}$ for physical J, and the slowly varying square root in (1.1) has been included so that $\beta_{ij}^k/s_0{}^J$ will be the residue of the Regge pole in the standard kinematic-singularity-free partial-wave amplitude.[2] The conjectured symmetry is imposed by taking β to have the form[3]

$$\Gamma(\alpha_k+1)\beta^k_{ij}(W)=[gC^k_i\gamma(W)][gC^k_j\gamma(W)]. \qquad (1.2)$$

Each bracket in (1.2) expresses a Regge vertex in terms of an over-all coupling strength g, SU(3) Clebsch-Gordan coefficient C, and SU(3)-invariant energy dependence $\gamma(W)$. Note that in order for $[\beta/s_0{}^l]$ to obey SU(3), the scale constant s_0, which will appear in the characteristic Regge energy dependence $(s/s_0)^{\alpha-(1/2)}$, must be SU(3)-*invariant*. Given this invariance and the behavior (1.2), the complete residue of the pole (1.1) will obey exact SU(3), except for the same factor $(q_iq_j)^l$ as appears in the Breit-Wigner residue, and apart from small additional kinematical effects.

The residues of the other Regge poles (α octet, etc.) are handled in a similar way, the square root in (1.1) being modified as necessary to remove the kinematical singularities from β. For an *octet* of Regge poles, the vertex $[gC\gamma]$ is replaced by

$$[g(C^k_{id}d+C^k_{if}f)\gamma(W)], \qquad (1.3)$$

to accommodate the two independent octet-octet-octet couplings. The C_d and C_f are Clebsch-Gordan coefficients, and we normalize $d+f\equiv 1$. We assume that $d(W)/[d(W)+f(W)]$ is a constant, independent of the energy W carried by the trajectory, but it must be stressed that this assumption is *not* required by SU(3), and is to be justified by confrontation with the data.

The assumed SU(3) symmetry has strong implications for backward angular distributions. Consider the trajectories to be exchanged in the u channel (with the "energy" $W=\sqrt{u}$) of reactions whose c.m. energy squared is s. Suppose several different reactions are all dominated by exchange of a member (not necessarily the same member) of some SU(3) multiplet of trajectories. At a given s, the u dependence of each reaction is determined by the u dependence of $\beta(\sqrt{u})$, the factor $(s/s_0)^{\alpha(u)}=(s/s_0)^{\alpha_0+\alpha' u}$, and other factors which depend only on the exchanged trajectory $\alpha_k(u)$ and on the external masses, and which, therefore, involve only small, kinematical SU(3) breakings. From (1.2) or (1.3), all the β's have the same $\sqrt{u}$ dependence. Further, if s_0 is SU(3)-invariant, then, since all trajectories are observed to have about the same slope α', the factor $(s/s_0)^\alpha$ will contribute exactly the same exponential u dependence to the various reactions. Thus, we expect these reactions to have almost identical backward angular distributions at a given s.

Of course, when one is comparing two reactions, either of which involves *more* than one Regge pole, then identical angular distributions are in general not expected. However, even in this case SU(3) can make it possible to predict the shape of the one angular distribution from a knowledge of the shape of the other.

In Sec. II we illustrate the SU(3) generation of one angular distribution from an experimental knowledge of a completely dissimilar one. In Sec. III we test the predicted SU(3) relations between reactions whose features are similar. Section IV uses SU(3) symmetry to obtain evidence against strong α-octet–γ-octet exchange degeneracy from the observed polarization in $\pi^-p\to\Lambda K^0$. In Sec. V we summarize our results and suggest experiments which would make possible further testing of SU(3) and of exchange degeneracy.

In most of the analysis, we will not consider the contributions from possible Regge cuts. This is not because we are convinced that Regge cuts are always small; on the contrary, we suspect that, in reactions such as $K^+p\to pK^+$ which have no dips and involve amplitudes which are largely real, they are sizeable. Rather, it is because the explicit inclusion of Regge cuts would greatly complicate the picture and lead to a lot of arbitrariness. Therefore, at this stage we explore the SU(3) symmetry of Regge poles by testing the SU(3) relations between experimental quantities which follow when cuts are neglected, hoping that these relations are not qualitatively affected by any cuts which may be present. The results suggest that this is indeed the case.

II. GENERATION OF π^+p FROM K^+p

Consider the reactions $K^+p\to pK^+$ and $\pi^+p\to p\pi^+$. The first of these has an undistinguished, rather flat backward peak,[4,5] while the second has a steep backward peak, deep dip, and secondary maximum.[5] Is it possible, by constructing a very simple picture of each reaction and applying SU(3), to generate the highly structured π^+p angular distribution from the featureless one for K^+p?

To begin with, one expects the Regge poles in the exotic reaction $K^+p\to pK^+$ to contribute in strongly exchange-degenerate pairs, the predominant pair being $\Lambda_\alpha(1115)$ and $\Lambda_\gamma(1520)$.[6] Given the small Σ_α-Λ_α trajectory splitting and SU(3) for residues, any contribution from a Σ_α-Σ_γ pair will have about the same u and s dependence as that from Λ_α-Λ_γ, and will be in phase with it. Thus it will simply change the apparent size of the Λ_α-Λ_γ contribution, so let us pretend, for the moment, that d has the value $d=\frac{1}{2}$, for which only the Λ pair couples. Let us also neglect the small Δ_δ contribution to π^+p, and the related Σ_δ-Σ_β contribution, which from SU(3) is also small, to K^+p. Then the invariant amplitude A for K^+p has the form

$$A = h_A^\Lambda(u)[(1+e^{-i\pi\bar\alpha_\Lambda}) + (1-e^{-i\pi\bar\alpha_\Lambda})] \times\Gamma(-\bar\alpha_\Lambda)(s/s_0{}^\Lambda)^{\bar\alpha_\Lambda(u)} = h_A^\Lambda(u) 2\Gamma(-\bar\alpha_\Lambda)(s/s_0{}^\Lambda)^{\bar\alpha_\Lambda(u)}. \quad (2.1)$$

Here h_A^Λ is related to the common residue of the Λ pair, and $\bar\alpha_\Lambda \equiv \alpha_\Lambda - \frac{1}{2}$, where α_Λ is the common trajectory. The Λ_α and Λ_γ contributions are, respectively, the $(1+e^{-i\pi\bar\alpha_\Lambda})$ and $(1-e^{-i\pi\bar\alpha_\Lambda})$ pieces of (2.1). The invariant amplitude B has exactly the same form, with h_A^Λ replaced by some other coefficient h_B^Λ.

From the observed properties of the $\frac{3}{2}^-$ Λ resonance at 1520 MeV, the Λ_γ trajectory is mostly an SU(3) singlet.[1] Thus it has no analog in π^+p. The Λ_α, on the other hand, is part of an octet of trajectories, represented by the N_α in π^+p. Thus, the application of SU(3) to (2.1) yields for π^+p the amplitude

$$A = h_A^N(u)(1+e^{-i\pi\bar\alpha_N})\Gamma(-\bar\alpha_N)(s/s_0{}^N)^{\bar\alpha_N(u)}, \quad (2.2)$$

and a similar expression for B, with $h_A^N \to h_B^N$. Here $\bar\alpha_N \equiv \alpha_N - \frac{1}{2}$. If the SU(3) residue symmetry we have discussed holds, we must have $s_0{}^N = s_0{}^\Lambda \equiv s_0$. Furthermore, $h_A^N(u)$ and $h_B^N(u)$ (which are proportional to the N_α residue) will be equal, respectively, to $h_A^\Lambda(u)$ and $h_B^\Lambda(u)$, apart from a common SU(3) Clebsch-Gordan coefficient.

Apart from small kinematical SU(3) breakings in the relations between A and B and the differential cross section, we may compare the predicted π^+p and K^+p angular distributions at a given energy s simply by comparing the squares of A, or of B. Thus, neglecting the relatively unimportant factor $[\Gamma(-\bar\alpha_N)/\Gamma(-\bar\alpha_\Lambda)]^2$, we expect that

$$\left(\frac{d\sigma}{du}\right)_{\pi^+p} \Big/ \left(\frac{d\sigma}{du}\right)_{K^+p} \cong \left(\frac{h_B^N}{h_B^\Lambda}\right)^2 |\tfrac{1}{2}(1+e^{-i\pi\bar\alpha_N(u)})|^2 \left(\frac{s}{s_0}\right)^{2(\alpha_N-\alpha_\Lambda)}. \quad (2.3)$$

Since α_N and α_Λ are parallel, $(s/s_0)^{2(\alpha_N-\alpha_\Lambda)}$ does not depend on u, so the entire difference in the u dependences of π^+p and K^+p is being ascribed to the fact that in the latter reaction one has the u-independent sum of two signature factors $(1+e^{-i\pi\bar\alpha})$ and $(1-e^{i\pi\bar\alpha})$, while in the former only the first of these factors survives.

The new data on the K^+p backward peak at 5 GeV/c from CERN[4] may be described by[4]

$$\left(\frac{d\sigma}{du}\right)_{K^+p} = 17.5\, e^{3.6u}\ \mu\text{b/GeV}^2, \quad (2.4)$$

with u in GeV^2. Thus, from (2.3), one predicts that at the same energy

$$\left(\frac{d\sigma}{du}\right)_{\pi^+p} = C\, e^{3.6u} \cos^2\frac{\pi}{2}\bar\alpha_N(u), \quad (2.5)$$

with the constant C given by

$$C = \left(\frac{h_B^N}{h_B^\Lambda}\right)^2 \left(\frac{s}{s_0}\right)^{2(\alpha_N-\alpha_\Lambda)} (17.5\ \mu\text{b/GeV}^2). \quad (2.6)$$

To evaluate C, we take $s_0 = 1$ GeV^2, a round number close to the 1.3 GeV^2 suggested by the slope of the K^+p data, and $\alpha_N - \alpha_\Lambda = 0.36$, as indicated by fits to backward reactions.[6,7] Then at 5 GeV/c, $(s/s_0)^{2(\alpha_N-\alpha_\Lambda)} = 5.5$ [a large SU(3)-breaking effect]. The remaining factor, $(h_B^N/h_B^\Lambda)^2$, depends on d, so rather than predict C, we see what value the π^+p data suggest, and determine the corresponding value of d.

The prediction (2.5), with $C = 250\ \mu$b/GeV^2, is compared with the data[5] for $\pi^+p \to p\pi^+$ at 5.2 GeV/c in Fig. 1. The observed u dependence is very well reproduced. The steepness of the backward peak, the general shape of the secondary maximum, and the approximate height of the backward peak relative to that of the secondary maximum are all correctly given. The value $C = 250\ \mu$b/GeV^2 was chosen simply by visual fitting, and corresponds to $d = 0.54$, but the important thing is the π^+p *u dependence,* which does turn out to be just $\cos^2\frac{1}{2}\pi\bar\alpha_N(u)$ times that for K^+p.[8]

Of course, (2.5) undershoots the data in the bottom of the dip, because the N_α vanishes at this point and we have omitted the Δ_δ background. However, even this detail can be taken care of by eliminating the $I_u = \frac{3}{2}$ background from the πN data (I_u being the isospin in the u channel). The latter has been accomplished by Barger and Olsson, who compute from data at 5.9 GeV/c (Ref. 9) the quantity[10]

$$\left(\frac{d\sigma}{du}\right)_N \equiv \frac{1}{2}\left\{3\left[\left(\frac{d\sigma}{du}\right)_{\pi^+p\to p\pi^+} + \left(\frac{d\sigma}{du}\right)_{\pi^-p\to n\pi^0}\right] - \left(\frac{d\sigma}{du}\right)_{\pi^-p\to p\pi^-}\right\}. \quad (2.7)$$

This quantity is the square of the $I_u = \frac{1}{2}$ amplitude, and if that amplitude is indeed almost entirely an N_α pole it should have a zero, and not just a dip, when $\alpha_{N_\alpha} = -\frac{1}{2}$ ($u = -0.14$ GeV^2). Barger and Olsson[10] find that, to within errors, the $I_u = \frac{1}{2}$ amplitude does indeed have this zero (see Fig. 2). Consequently, the SU(3) prediction (2.5), with C and the slope of 3.6 suitably corrected to correspond to $(d\sigma/du)_N$ at 5.9 GeV/c, should give a good description of the latter, even in the vicinity of $\alpha_{N_\alpha} = -\frac{1}{2}$.

There is one, essentially experimental, difficulty with this. Namely, if one presumes that π^+p is N_α-dominated, then the 5.2- and 5.9-GeV/c π^+p data are not entirely compatible. If one folds into

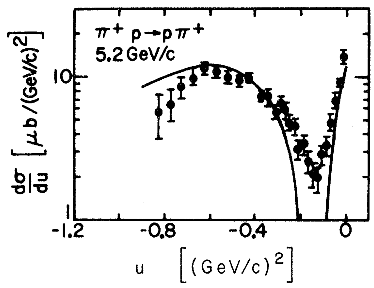

FIG. 1. Prediction of the angular distribution for $\pi^+ p \to p\pi^+$ at 5.2 GeV/c, obtained by applying SU(3) symmetry to the angular distribution for $K^+p \to pK^+$ at the same energy. Apart from the overall normalization, no parameters have been adjusted. Data for π^+p from Ref. 5.

the slope of 3.6 in (2.5) the Regge shrinkage that an N_α pole would produce, one expects $(d\sigma/du)_{\pi^+p}$ to have a slope of 3.8 at 5.9 GeV/c. And the data at this energy[9] [hence also $(d\sigma/du)_N$, of which $(d\sigma/du)_{\pi^+p}$ is a large part] are much steeper than this. Thus, to describe the 5.9-GeV/c $(d\sigma/du)_N$, one is forced to treat the normalization and slope in (2.5) as free parameters. Doing this, we find that the best fit to $(d\sigma/du)_N$ is achieved with $C = 566$ μb/GeV^2 and the slope of 3.6 replaced by 5.3. The agreement between the corresponding curve, shown in Fig. 2, and experiment is impressive.

If we assume that it is the 5.9-GeV/c πN data which are correct, we may now turn the SU(3) analysis around and ask what K^+p cross section they imply. Correcting the slope of 5.3 back down to the energy of the CERN K^+p measurements, we expect the latter to display a slope of 5.05. Visually fitting an exponential with this slope to the data, we find reasonable agreement (see Fig. 3) when one takes

$$\left(\frac{d\sigma}{du}\right)_{K^+p} = 28\, e^{5.05u}\ \mu\text{b/GeV}^2\,. \tag{2.8}$$

(Note that the K data show curvature, presumably reflecting the increasing importance of nonperipheral backgrounds, as one moves down the backward peak. Thus, for our purposes, attention should be focused on the backwardmost points.) Given that $C = 566$ μb/GeV^2 in $(d\sigma/du)_N$, the normalization in (2.8) implies that $d = 0.51$. Since (2.8) does not agree with the data quite as well as the exponential with a slope of 3.6, we see that, depending on which πN data are correct, SU(3) is more or less exact. But it is obvious from Figs. 1–3 that SU(3) is quite successful at relating the completely different angular distributions for π^+p and K^+p in any case.

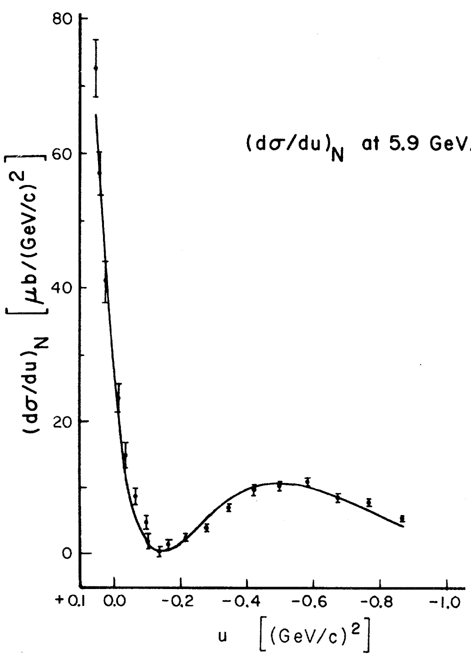

FIG. 2. Comparison between the $I_u = \frac{1}{2}$ backward πN cross section and the form (2.5) expected from SU(3), but with the slope of the exponential and the overall normalization adjusted as required by the 5.9-GeV/c data of Ref. 9.

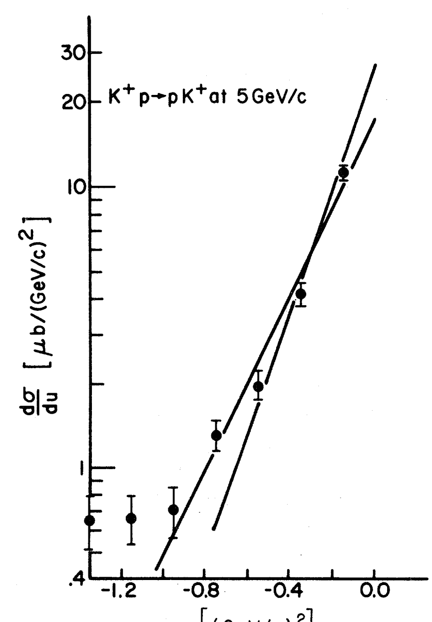

FIG. 3. Angular distribution for $K^+p \to pK^+$ at 5 GeV/c. The shallower straight line is the description (2.4) of the data given by the experimentalists. The steeper one has the slope of 5.05 required by SU(3) applied to the πN data at 5.9 GeV/c. Data from Ref. 4.

In our analysis, we have, of course, attributed the dip in $\pi^+ p \to p\pi^+$ at $u = -0.14$ GeV^2 to a wrong-signature nonsense (WSN) zero in the N_α contribution when $\alpha_{N_\alpha} = -\frac{1}{2}$. We remark that if Regge residues do have the SU(3) symmetry we are looking for, this is a particularly natural explanation of the dip. To begin with, the symmetry implies that the N_α contribution to any of the 0^- meson$-\frac{1}{2}^+$ baryon reactions should indeed vanish at the WSN point. All that is required for this vanishing is that the N_α residue β not blow up and cancel the zero. Now the α octet residue will not blow up in any reaction whose s channel is exotic.[11] Consideration of two such reactions then suffices to conclude that neither the d- nor f-type α octet coupling is singular. Thus, the N_α residue will always be well behaved. Since the resulting zero in the N_α contribution is predicted to occur at $u = -0.14$ GeV^2, it is then natural to suppose that the dips seen at this u value in various reactions involving the N_α are *due* to this zero.

III. SU(3) SYMMETRY OF N_α EXCHANGE

We turn now to an illustration of the operation of the symmetry when several different reactions involve exchange of exactly the same Regge pole and differ only in the external particles involved. The processes $\pi^+ p \to p\pi^+$, $K^- n \to \Lambda\pi^-$, and $K^- n \to \Sigma^0\pi^-$ are all generally thought to be dominated by N_α exchange. Thus, SU(3) predicts their u dependences to be rather similar. That they are indeed similar is evident in Figs. 1, 4, and 5. However, the most dramatic feature of these reactions, the dip, is expected in the latter two just from its presence in the first, even without SU(3). Once it is assumed that the N_α dominates $\pi^+ p \to p\pi^+$, the observed dip in this process tells us that the N_α contribution to it vanishes at $\alpha_{N_\alpha} = -\frac{1}{2}$. This implies that the $N_\alpha \pi N$ vertex is not singular at this point. Then the N_α must also vanish at $\alpha_{N_\alpha} = -\frac{1}{2}$ in $K^- n \to \Lambda\pi^-$, because if it did not, that would mean that the $N_\alpha K\Lambda$ vertex is sufficiently singular to cancel the WSN zero. This, in turn, would imply that the N_α contribution to the reaction $K^-\Lambda \to \Lambda K^-$, which involves the $N_\alpha K\Lambda$ vertex squared, becomes infinite when $\alpha_{N_\alpha} = -\frac{1}{2}$. Thus we expect a dip in $K^- n \to \Lambda\pi^-$, and similarly in $K^- n \to \Sigma^0\pi^-$, and it becomes of interest to test SU(3) more quantitatively in these reactions.

To make such a quantitative test, we have used a simple model based on N_α and Δ_δ poles (plus weak cuts which are unimportant for the present considerations) to describe $\pi^- p \to p\pi^-$, $\pi^+ p \to p\pi^+$, $K^- n \to \Lambda\pi^-$, and $K^- n \to \Sigma^0\pi^-$. Except for the d/f ratio of the α octet, all parameters of the model are determined by fitting $\pi^\pm p \to p\pi^\pm$.[7] Assuming SU(3), one can then predict $d\sigma/du$ for $K^- n \to \Lambda\pi^-$, apart from the over-all normalization, which depends on d. A fit to the data at 3.9 GeV/c (Ref. 12) yields the value $d \simeq 0.8$. The corresponding theoretical curve, which compares quite favorably with the data, is shown in Fig. 4.[13] With d now fixed, one may make a parameter-free prediction for the reaction $K^- n \to \Sigma^0\pi^-$. The predicted angular distribution for 3.0 GeV/c is compared with the data[14] in Fig. 5. The agreement is quite reasonable.[15]

The successes represented by Figs. 4 and 5, besides suggesting that the N_α and Δ_δ residues do indeed possess SU(3) symmetry, give some evidence for factorization of the octet u-channel amplitude. Apart from $\pi^- p \to p\pi^-$, to which octet exchange cannot contribute, all the reactions under discussion are dominated by this exchange in the u channel. Now, *a priori*, there are three independent octet

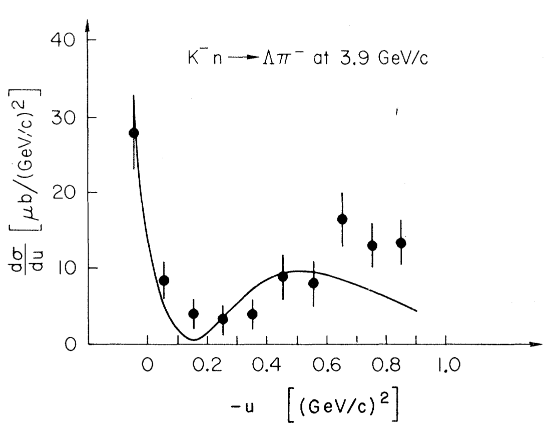

FIG. 4. Differential cross section for $K^- n \to \Lambda\pi^-$ at 3.9 GeV/c. The shape of the theoretical curve follows from SU(3), with no parameters. The overall normalization has been adjusted to give the best fit to the data, and corresponds to $d = 0.8$. Data from Ref. 12.

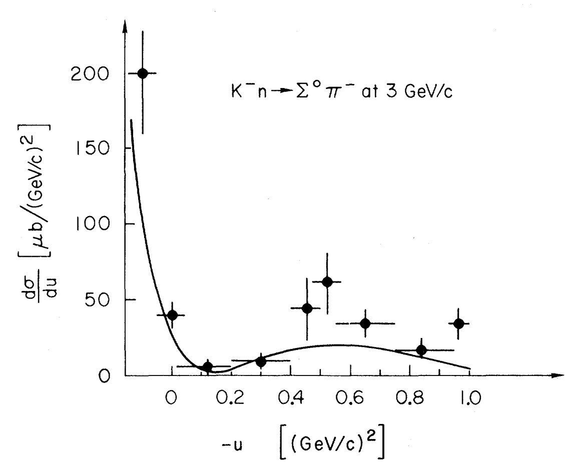

FIG. 5. Parameter-free SU(3) prediction for the $K^- n \to \Sigma^0\pi^-$ cross section at 3.0 GeV/c. Data from Ref. 14.

amplitudes, connecting the symmetric and antisymmetric octet meson-baryon states to themselves and to each other. It would take three reactions to measure these amplitudes; then one could predict a fourth. On the other hand, if the octet amplitude factors into a product of vertices, then there are only two independent amplitudes, both determined by the two independent couplings, f-type and d-type. In the model which we used, the small decuplet exchange was fixed by the reaction $\pi^- p \to p\pi^-$, then the *two* reactions $\pi^+ p \to p\pi^+$ and $K^- n \to \Lambda\pi^-$ were used to fix the octet amplitude, and then a *third* octet-dominated reaction, $K^- n \to \Sigma^0 \pi^-$, was successfully predicted.

IV. POLARIZATION AND EXCHANGE DEGENERACY

Let us now assume the SU(3) symmetry of Regge residues, for which we have seen evidence in the preceding sections, to be valid, and use it as a tool in the testing of *exchange degeneracy* (EXD) of the residues. [Given an even- and an odd-signature Regge pole whose trajectories are degenerate, exchange degeneracy of the residues means equality of the partial-wave residues β, or, equivalently, equality of the coefficients of the respective signature factors in the invariant amplitudes, as in (2.1).] One of the first places one thinks to look for such exchange degeneracy is in polarization. For example, if in the s-exotic reaction $K^+ p \to pK^+$ all trajectories contribute in EXD pairs analogous to the dominant Λ_α-Λ_γ pair of (2.1), the amplitudes will be real and the polarization will vanish. This polarization is thus an interesting quantity to measure at high energy, but if we use SU(3), we can already say something about exchange degeneracy in $K^+ p \to pK^+$ on the basis of the polarization observed in another process.

Consider the reaction $\pi^- p \to \Lambda K^0$. The backward polarization in this process is known to be of the order of 100% and positive (the cross section shows a fairly smooth backward peak).[16] In the u channel this reaction has $I = 1$, and so it can involve exchange of $\Sigma_\alpha(1190)$, $\Sigma_\gamma(1670)$, $\Sigma_\delta(1385)$, and $\Sigma_\beta(1765)$. Applying SU(3) to the Δ_δ contribution measured in $\pi^- p \to p\pi^-$, and taking into account the major kinematical SU(3)-breaking effects as in the discussion of $\pi^+ p$ and $K^+ p$, one estimates that the Σ_δ contribution to $\pi^- p \to \Lambda K^0$ at 6 GeV/c and $u = 0$ is 6% of the measured rate.[16] From rough EXD requirements and SU(3), one can easily argue that the Σ_β contribution is quite a bit smaller still.[17] This leaves Σ_α and Σ_γ to supply most of the observed cross section.

Now consider the polarization, and assume that Σ_α and Σ_γ are strongly exchange degenerate in $K^+ p \to pK^+$. Then, if SU(3) holds, the Σ_α-Σ_γ pair cannot by itself do any polarizing in $\pi^- p \to \Lambda K^0$. When one goes from the first reaction to the second, the Σ_α contributions to the two invariant amplitudes A and B both change by just some common Clebsch-Gordan coefficient x. Similarly, the Σ_γ contributions change by some other coefficient y. If the Σ_α-Σ_γ pair entered A and B for $K^+ p \to pK^+$ in the EXD form

$$(1 + e^{-i\pi\bar{\alpha}_\Sigma}) + (1 - e^{-i\pi\bar{\alpha}_\Sigma}), \qquad (4.1)$$

then it will enter both A and B for $\pi^- p \to \Lambda K^0$ with the same phase factor

$$x(1 + e^{-i\pi\bar{\alpha}_\Sigma}) + y(1 - e^{-i\pi\bar{\alpha}_\Sigma}). \qquad (4.2)$$

(Here $\bar{\alpha}_\Sigma = \alpha_\Sigma - \frac{1}{2}$, where α_Σ is the common Σ_α-Σ_γ trajectory.) Since the polarization can be expressed in terms of $\mathrm{Im}[(A + \bar{M}B)B^*]$, where $\bar{M}$ is the average of the external baryon masses, the Σ_α-Σ_γ pair evidently does not by itself cause any polarization.[18] [Kinematical SU(3) breaking has no effect on this situation.] Thus, the polarization in $\pi^- p \to \Lambda K^0$ must be coming from interference between the contribution from Σ_α-Σ_γ, and that from Σ_δ-Σ_β. But polarizations of the order of 100% require an interference between two comparably sized contributions that are both important in the cross section. In view of our conclusions about Σ_δ-Σ_β, there seems to be something wrong.

This conclusion is reinforced by consideration of the *sign* of the polarization. As stated earlier, the Σ_δ-Σ_β pair is mostly Σ_δ (for $u \approx -0.3$ GeV^2, where $\alpha_{\Sigma\beta} = -\frac{1}{2}$, it is *all* Σ_δ). Given the known signature factors, if Σ_α and Σ_γ are strongly exchange degenerate in $K^+ p \to pK^+$, and $d \geq \frac{1}{2}$ for the α octet, while $d \leq \frac{1}{2}$ for the γ octet,[1] then the Σ_α, Σ_δ interference and the Σ_γ, Σ_δ interference contribute to the $\pi^- p \to \Lambda K^0$ polarization with the same sign. This sign then depends on those of $h^{\Sigma\delta}_{A+\bar{M}B} h^{\Sigma\alpha}_B$ and $h^{\Sigma\alpha}_{A+\bar{M}B} h^{\Sigma\delta}_B$ [in the notation of (2.1)], and possibly also on their relative size. Now these products of h's are related by SU(3) to the corresponding ones involving Δ_δ and N_α in elastic πN scattering. And in πN scattering, Berger and Fox have determined what amounts to the sign of $h^{\Delta\delta}_{A+\bar{M}B} h^{N\alpha}_{A+\bar{M}B}$ at 180°.[19] To obtain from this the signs that we require, we recall that the form of the N_α and Δ_δ residues which is suggested by the absence of parity doublet partners to the nucleon and delta, and which has proved successful in fits to data,[20] is

$$\beta_N(\sqrt{u}) = (\sqrt{u} + \mathcal{a}_N) f_N(u), \qquad (4.3)$$

$$\beta_\Delta(\sqrt{u}) = (\sqrt{u} + \mathcal{a}_\Delta) f_\Delta(u), \qquad (4.4)$$

where the constants $\mathcal{a}_N$ and $\mathcal{a}_\Delta$ are approximately equal, respectively, to the nucleon and delta masses, and f_N and f_Δ are arbitrary functions of u. Any residues of this form, with $\mathcal{a}_N$ and $\mathcal{a}_\Delta$ any positive numbers, imply that $h_B^{N\alpha} h_{A+\overline{M}B}^{N\alpha} > 0$ and $h_B^{\Delta\delta} h_{A+\overline{M}B}^{\Delta\delta} < 0$.[21] Hence, from Ref. 19 one can infer the signs of $h_{A+\overline{M}B}^{\Delta\delta} h_B^{N\alpha}$ and $h_{A+\overline{M}B}^{N\alpha} h_B^{\Delta\delta}$ at 180°, and, assuming no random zeroes of the Regge contributions in the backward region, at other values of u as well. Then SU(3) leads immediately to the signs of the corresponding products of h's in $\pi^- p \to \Lambda K^0$; since the two products turn out to produce like-signed contributions to the polarization, their relative size does not matter. Thus, a clear prediction of the sign of the polarization in $\pi^- p \to \Lambda K^0$ is obtained, and unfortunately the predicted sign is wrong.

We see that if SU(3) holds, the assumption of Σ_α-Σ_γ exchange degeneracy in $K^+ p \to pK^+$ implies a polarization in $\pi^- p \to \Lambda K^0$ which is wrong in both size and sign. Regarding the size, even if our estimates of SU(3) breaking are not precisely correct, it does seem likely that the Σ_δ will be small compared to Σ_α in $\pi^- p \to \Lambda K^0$, given that the Δ_δ is small compared to N_α in $\pi^+ p \to p\pi^+$. As for the sign, the argument relied on reasonable assumptions about β_N and β_Δ, and on the kind of SU(3) sign predictions which have proved very successful elsewhere.[1] Thus, the evidence suggests that the Σ_α and Σ_γ residues are not in fact exchange-degenerate in $K^+ p \to pK^+$. In particular, these residues must have different $\sqrt{u}$ dependences, so that when one transforms to $\pi^- p \to \Lambda K^0$, they will differ by more than a ratio y/x, and will allow the Σ_α-Σ_γ pair to produce some polarization by itself. There is, to be sure, motivation and evidence for some degree of Σ_α-Σ_γ EXD in $\pi^- p \to \Lambda K^0$, apart from a factor $y/x = -1$,[22] but Field and Jackson have found that situations are possible where in certain respects a fair amount of exchange degeneracy is present, and still a large polarization is generated by the degeneracy breaking.[22,23]

If the Σ_α and Σ_γ residues have different $\sqrt{u}$ dependences in $K^+ p \to pK^+$, then, of course, SU(3) implies that EXD between the α and γ octets is quite generally broken. However, one still expects the exotic $K^+ p$ amplitudes to be approximately real; consequently, one still expects the Λ_α-Λ_γ pair which dominates $K^+ p$ to enter approximately in the EXD form (2.1) (so that, in particular, the analysis of Sec. II remains valid). Since the Λ_γ is mostly singlet, the Λ pair will contribute in the desired manner if EXD between the α octet and γ *singlet* is relatively unbroken. We conjecture that this is indeed the case.

As always, one must be cautious in drawing conclusions from polarizations. The process $\pi^- p \to p\pi^-$ is generally thought to involve just Δ_δ, while $K^- n \to \Lambda\pi^-$ and $\pi^+ p \to p\pi^+$ are thought to be strongly dominated by N_α. Since a single Regge pole whose trajectory depends only on u causes no polarization, one would then expect no polarization in the first reaction, and small polarizations in the other two. In fact, however, the polarization in the first process is of order 20%,[24] that in the second of order 50% (with large error bars),[12] and that in the last also of order 50%.[24] Mindful of the various pieces of evidence that the $I_u = \frac{1}{2}$ πN amplitude *is* largely an N_α pole, Barger and Olsson[10] have suggested that the polarization observed in $\pi^+ p \to p\pi^+$ is due to a $\sqrt{u}$ term in the N_α trajectory.[25] If their plausible suggestion is correct, then the Σ_α trajectory may very well also contain a $\sqrt{u}$ term, which could contribute to the polarization in $\pi^- p \to \Lambda K^0$ and affect some of the conclusions just drawn.

V. SUMMARY AND SUGGESTED EXPERIMENTS

We have found several pieces of evidence for the SU(3) symmetry of baryon Regge residues, and, using this symmetry, one piece of evidence against their exchange degeneracy. Additional experimental information would be very valuable for the further testing of both the symmetry and the degeneracy. There is a need for angular distributions for the many reactions about which little is known at relevant energies (4 GeV/c and beyond); for example, $\pi^- p \to \Sigma^0 K^0$ and $K^- p \to \Xi^0 K^0$. [Data at 3.9, 4.6, and 5.0 GeV/c have just appeared for the related reaction $K^- p \to \Xi^- K^+$.[26] Since this process involves the infrequently seen coupling of a Ξ particle, it is interesting that its angular distribution has been found[26] to be related to that for $K^+ p \to pK^+$ as SU(3) would require.]

There is also a need for data at several higher energies for reactions which have so far been studied only at 3 or 4 GeV/c ($K^- n \to \Lambda\pi^-$, $\pi^- p \to \Sigma^- K^+$, etc.). Such data will permit testing of the energy dependence predicted by the Regge poles which underlie the SU(3)-symmetric picture.

Additional observations of the $\pi^+ p \to p\pi^+$ and $K^+ p \to pK^+$ cross sections in the 5–6-GeV/c range will facilitate a more stringent test of the SU(3) relation represented by Figs. 1–3. Cross section measurements for $K^+ p$ at 10 GeV/c, to complement the existing measurements of backward πN scattering at the same energy,[9] will allow verification of this relation at a higher energy. Fitting $(d\sigma/du)_N$ at 10 GeV/c (Ref. 10) as we did at the lower energy, we predict from SU(3) that at 10 GeV/c

$$\left(\frac{d\sigma}{du}\right)_{K^+ p} = 3\, e^{6.6u}\, \mu\text{b/GeV}^2 . \quad (5.1)$$

(The normalization here has been computed assuming $d \simeq 0.5$, as found in comparing πN and $K^+ p$ at the lower energy.)

With regard to the question of Σ EXD breaking, it would be useful to know the cross section for the process $K^+n \to pK^0$ at about 5 GeV/c. This process and $K^+p \to pK^+$ are controlled by different linear combinations of the same Λ_α-Λ_γ and Σ_α-Σ_γ pairs. According to SU(3), the Σ_α and Λ_α residues are proportional, so that if both pairs obey EXD, their u dependences, hence those of the two reactions, will be identical. But if only the Λ pair respects EXD, the two cross sections could differ. Now at 2 GeV/c they differ dramatically.[27] It will be interesting to learn their behavior at energies in the Regge regime.

Given the expected reality of the Λ pair in KN scattering, the polarization in $K^+p \to pK^+$, though interesting, is not expected to be large. However, in the reactions $K_L^0 p \to pK_s^0$ and $K^+n \to nK^+$, only the Σ pair can contribute, so the suspected Σ EXD breaking should result in large polarizations.[28]

Further work on the SU(3)-symmetry and exchange-degeneracy properties of the baryon Regge residues is in progress. We look forward to the experimental information which will help confirm the conclusions pointed to by the existing evidence.

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[1]E. Flaminio *et al.*, BNL Report No. BNL 14572, 1970 (unpublished). Some authors refine the factor q^{2l+1} somewhat; see, for example, R. Tripp, in *Proceedings of the Third Hawaii Topical Conference on Particle Physics*, edited by S. F. Tuan (Western Periodicals, Hollywood, California, 1970).

[2]V. Singh, Phys. Rev. 129, 1889 (1963).

[3]$\Gamma(\alpha_k+1)\beta^k$ is the combination which actually contributes to the cross sections; to avoid infinities one must obviously take this combination to be the SU(3)-symmetric quantity.

[4]V. Chabaud *et al.*, Phys. Letters 38B, 445 (1972).

[5]W. Baker *et al.*, Phys. Letters 28B, 291 (1968).

[6]A successful Λ_α-Λ_γ model for K^+p has been presented by V. Barger [Phys. Rev. 179, 1371 (1969)].

[7]F. Hayot and B. Kayser, Nuovo Cimento 3A, 45 (1971).

[8]F. Halzen, in Proceedings of the Sixth Rencontre de Moriond, 1971 (unpublished), has calculated the N_α and Λ_α residues from the cross sections for $K^-n \to \Lambda\pi^-$ and $K^+p \to pK^+$, respectively, and found them to be related according to SU(3). His analysis, unlike ours, includes a non-negligible N_γ in $K^-n \to \Lambda\pi^-$, but this should not make too much difference. Thus his result and ours are confirmations of the same basic SU(3) picture.

[9]D. Owen *et al.*, Phys. Rev. 181, 1794 (1969); J. Boright *et al.*, Phys. Rev. Letters 24, 964 (1970); Phys. Letters 33B, 615 (1970).

[10]V. Barger and M. Olsson, Univ. of Wisconsin report (unpublished). We thank Professor Olsson for making the numerical values of $(d\sigma/du)_N$ at 5.9 and 10 GeV/c available to us.

[11]B. Kayser, Phys. Rev. D 3, 747 (1971).

[12]D. Crennell *et al.*, Phys. Rev. Letters 23, 1347 (1969).

[13]This favorable result is confirmed by a residue comparison made by J. Frøyland, F. Halzen, and B. Petersson, CERN Report No. TH-1255 (unpublished).

[14]R. Barloutaud *et al.* (SABRE collaboration), Nucl. Phys. B26, 557 (1971).

[15]Exploratory pole-model fits to the $\bar{K}N \to \Sigma\pi$ reactions, incorporating SU(3) constraints, have been given by V. Barger, in *Coral Gables Conference on Fundamental Interactions at High Energy, I*, edited by T. Gudehus, G. Kaiser, and A. Perlmutter (Gordon and Breach, New York, 1969).

[16]W. Beusch *et al.*, Nucl. Phys. B19, 546 (1970).

[17]Duality diagrams for $\pi^-p \to \Lambda K^0$ suggest that the coefficients of the Σ_β and Σ_δ signature factors are of comparable size, and for $-0.5\ \text{GeV}^2 < u < 0\ \text{GeV}^2$ the ratio between the squares of the Σ_β and Σ_δ signature factors is everywhere less than 0.17.

[18]V. Barger, D. Cline, and J. Matos [Phys. Letters 29B, 121 (1969)] have constructed a model for $\pi^-p \to \Lambda K^0$ (including the polarization) based on Σ_α and Σ_γ poles. However, the residues of their poles are not related by a ratio y/x, but differ greatly in their $\sqrt{u}$ dependences.

[19]E. Berger and G. Fox, Nucl. Phys. B26, 1 (1971). Their sign, which is confirmed by πN charge-exchange data postdating their analysis, corresponds to the *absence* of the extra sign-reversing factor $(\alpha - \frac{1}{2})$, suggested by EXD, in the δ decuplet residue.

[20]See, for example, V. Barger and D. Cline, Phys. Rev. Letters 19, 1504 (1967).

[21]D. Roy *et al.* [Phys. Rev. D 6, 1317 (1972)] have re-

cently worked with more sophisticated residues which can be required to give exactly the right πNN coupling constant and Δ width, as well as to fit the backward data. For the signs of interest to us, their residues imply the same results as (4.3) and (4.4).

[22]The motivation is from duality diagrams, whose prediction of a relative minus sign between the Breit-Wigner residues of the resonances belonging to Σ_α and those belonging to Σ_γ in the reaction $\bar{K}N \to \pi\Lambda$ is experimentally correct. Evidence for exchange-degenerate behavior of the Y^* contributions to the imaginary parts of invariant amplitudes has been given by C. Schmid and J. Storrow [Nucl. Phys. B29, 219 (1971)] and by R. Field and J. D. Jackson [Phys. Rev. D 4, 693 (1971)].

[23]C. Schmid and J. Storrow, Nucl. Phys. B44, 269 (1972), have recently constructed a model for the $\pi^- p \to \Lambda K^0$ polarization based on Σ_α-Σ_γ interfering with Σ_δ-Σ_β. To produce the correct polarization, such a model must, of course, conflict with the SU(3) estimate that Σ_δ-Σ_β is small compared to the observed cross section, and must involve a Σ_δ-Σ_α sign relationship differing from that suggested, via SU(3), by πN elastic scattering.

[24]Private communications from V. Barger and M. Olsson.

[25]This conjecture leads to a correct prediction for the sign of the polarization in $K^- n \to \Lambda\pi^-$.

[26]R. Eisner, R. Field, S. Chung, and M. Aguilar-Benitez, Phys. Rev. D (to be published).

[27]D. Davies *et al.*, Phys. Rev. D 5, 1 (1972).

[28]These two reactions are also exotic, as $K^+p \to pK^+$ is. What the evidence from Sec. IV for Σ EXD breaking suggests is that *small* exotic amplitudes, such as those for these two reactions, are *not* approximately real.

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Effects of Proton–Atmospheric-Nuclei Interaction Processes on Predicted Muon Intensities*

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In numerous calculations of the muon intensity deep in the atmosphere, the details of the interactions which produce pions and kaons have been avoided by assuming a production spectrum proportional to $E^{-\alpha}$, where α is a constant. In contrast to this approach, we have used a realistic form for the primary cosmic-ray spectrum in which the spectral index varies with energy, and we have described the cosmic-ray proton–atmospheric-nuclei interactions using a statistical model for pion and kaon particle production. The resulting relative muon intensities at sea level have been calculated and are found to fall off more rapidly with momentum than intensities predicted by previous calculations. The muon charge ratio as a function of momentum (conveniently normalized to agree with experiment at low energies) is found to be sensitive to the spectral character of the primary cosmic-ray spectrum. Similar effects were found for muon intensities at large zenith angles resulting in no significant effects on predicted zenithal enhancements.

INTRODUCTION

During the past twenty years, numerous papers have been published[1-5] concerning muon differential intensities. In the earliest work by Barrett *et al.*[6] and in a relatively recent model developed by Maeda[2] and extended by Cantrell[3] the complexity of the interaction of the primary cosmic rays with the atmosphere is avoided by the use of production spectra for the muon parents, namely, pions and kaons. These spectra[6-12] are assumed to be proportional to $E^{-\alpha}$, where α is the logarithmic slope of the primary cosmic-ray spectrum and has been assumed to be a constant 2.7. Although the details of the propagation of muon parents and muons through the atmosphere are treated differently by various authors, the assumption of the above power-law production spectrum is common to most models. In this paper, pion and kaon production spectra are calculated in arbitrary units by using the two-temperature model developed by Wayland[13,14,15] to account for the interaction of the primary cosmic rays with nuclei in the atmosphere. These spectra are then used in place of the power-law production spectra in the Maeda-Cantrell formulation. The spectral character of the relative muon intensities resulting from these different production spectra are compared. In addition, the effect on muon intensities of a primary spectrum whose slope varies realistically with energy is examined.

PROCEDURE

In the two-temperature model, the transverse-momentum distribution of secondary particles is