*This work supported in part through funds provided by the Atomic Energy Commission under Contract No. AT(11-1)-3069.

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PHYSICAL REVIEW D VOLUME 6, NUMBER 9 1 NOVEMBER 1972

Decay $\Sigma^{\pm} \to \Lambda e^{\pm}\nu$ *

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(Received 14 June 1972)

The ratio of the vector to axial-vector coupling constant for $\Sigma^{\pm} \to \Lambda e^{\pm}\nu$ decays using 186 events is determined to be -0.37 ± 0.20. The branching ratio for $\Sigma^- \to \Lambda e^-\nu$ is $(0.62 \pm 0.07) \times 10^{-4}$ and for $\Sigma^+ \to \Lambda e^+\nu$ is $(0.21 \pm 0.05) \times 10^{-4}$. An upper limit on the magnitude of the ratio of the axial-magnetism to axial-vector coupling constants is 3.2.

Recently, three comparable experiments were performed[1-3] to study the strangeness-conserving decays $\Sigma^{\pm} \to \Lambda e^{\pm}\nu$. The conserved-vector-current hypothesis (CVC) relates the vector coupling in these decays to the amplitude for $\Sigma^0 \to \Lambda^0\gamma$; it is thus expected that, to zero order in the momentum transfer, the vector coupling should not contribute to such decays.[4]

The sample of events available in each experiment was rather small. We therefore felt it worthwhile to repeat the analysis of the combined sample rather than simply averaging the results presented by each group. The combined sample contains 163 examples of Σ^- decays and 23 examples of Σ^+ decays. The present analysis has also been performed under slightly more general assumptions than before.

The decays $\Sigma^{\pm} \to \Lambda e^{\pm}\nu$ are particularly rich in information because the three-body final state can be completely kinematically analyzed by observing the Λ^0 decay. In addition, the Λ^0 decay is a good analyzer of the Λ^0 polarization, thus allowing one to measure the correlation between momenta and the baryon spin.

Assuming that (1) the interaction is of the form current×current, (2) the leptonic part of the current is the same as in nuclear β decay, (3) the hadronic part of the current is thus a combination only of vector and axial-vector parts, (4) the hadronic current is even under G parity, and (5) the interaction is invariant under time reversal, the matrix element for $\Sigma^- \to \Lambda e^- \nu$ can be written in terms of four real invariant functions of the momentum transfer squared $q^2 = (p_{\Sigma\mu} - p_{\Lambda\mu})^2$ as

$$\begin{aligned} M &= \langle \Lambda | J_\mu | \Sigma \rangle (e\nu)_\mu \\ &= \bar{u}_\Lambda \big(G_V(q^2)\gamma_\mu + G_A(q^2)\gamma_\mu\gamma_5 \\ &\qquad + G_W(q^2)\sigma_{\mu\nu} q_\nu / M_\Sigma + G_{ps}(q^2) q_\mu \gamma_5 / M_\Sigma \big) u_\Sigma (e\nu)_\mu \,, \\ (e\nu)_\mu &= \bar{u}_e \gamma_\mu (1+\gamma_5) u_\nu \,, \end{aligned} \tag{1}$$

where G_V is the vector coupling constant, G_A is the axial-vector coupling constant, G_W is the weak-magnetism coupling constant, and G_{ps} is the induced pseudoscalar coupling constant. In this convention G_A/G_V for neutron β decay is positive.

The conserved-vector-current hypothesis implies that $G_V(q^2=0)$ is zero, and uniquely predicts G_W in terms of the anomalous magnetic moments of the proton and neutron. Contributions from the induced pseudoscalar term can be ignored since they are proportional to the electron mass.

We will in the following ignore the dependence on q^2 of all form factors. This is justified firstly because in the decays of interest, q^2 is extremely small $[0.0034 \leq q^2 \leq 0.0059\ (\mathrm{GeV}/c)^2$ for $\Sigma^- \to \Lambda e^- \nu]$, and secondly because of the limited statistics. In particular, we have checked that the inclusion of a reasonable q^2 dependence (e.g., the same as for the electromagnetic form factors) does not change any of the presented results.

In order to best use all the information available in each decay event, we prefer to obtain a complete distribution function in the 12-dimensional momentum space of the Λ, electron, neutrino, and proton. This distribution will take into account correlations in momentum space as well as Λ-spin correlations in a relativistically correct way. It is obtained by summing over the Λ spin the amplitude for the complete decay chain $\Sigma^- \to \Lambda e^- \nu$, $\Lambda \to p\pi^-$,

$$\begin{aligned} A &= \sum_{\Lambda\ \text{spin}} \bar{u}_p (1+\chi\gamma_5) u_\Lambda \bar{u}_\Lambda O_\mu u_\Sigma (e\nu)_\mu \\ &= \bar{u}_p (1+\chi\gamma_5)(\not{p}_\Lambda + m_\Lambda) O_\mu u_\Sigma (e\nu)_\mu / 2m_\Lambda \,, \end{aligned} \tag{2}$$

and then computing in the usual way,

$$f(\vec{\mathrm{P}}_\Lambda, \vec{\mathrm{P}}_e, \vec{\mathrm{P}}_\nu, \vec{\mathrm{P}}_p) = \sum_{e,\nu,p\ \text{spins}} |A|^2 \,. \tag{3}$$

In the above O_μ stands for the entire operator in brackets between the Λ and Σ spinors in Eq. (1).

χ is related to the α parameter in Λ decay,

$$\alpha = \frac{2P_p \operatorname{Re}(\chi)}{E_p(1+|\chi|^2)} + m_p(1-|\chi|^2) \,.$$

The result is shown in the Appendix.

It should be noted that the matrix element for $\Sigma^+ \to \Lambda e^+ \nu$ is different from the above. In particular, the lepton current is

$$\bar{u}_\nu \gamma_\mu (1+\gamma_5) u_e \,.$$

If one makes the usual assumption that the baryon current is the same for Σ^+ and Σ^-, the only difference between these matrix elements is the order of the leptons. One may thus combine the two types of decays by interchanging the neutrino and electron for the Σ^+ decays.

As a check on the results from the likelihood calculation, it is useful to compare the experimental distributions of the various kinematical parameters with the expected distributions based upon the likelihood calculation. Some of these expected distributions have been calculated assuming the Λ is nonrelativistic and the electron has zero mass.[5] Defining Z as G_V/G_A, the expected distributions of the electron-neutrino angular correlation is

$$W(\cos\theta_{e\nu}) = \tfrac{1}{2}\left(1 + \frac{|Z|^2 - 1}{|Z|^2 + 3}\cos\theta_{e\nu}\right) \,.$$

We define three orthonormal vectors:

$$\hat{\alpha} = \frac{\hat{P}_e + \hat{P}_\nu}{|\hat{P}_e + \hat{P}_\nu|} \,, \quad \hat{\beta} = \frac{\hat{P}_e - \hat{P}_\nu}{|\hat{P}_e - \hat{P}_\nu|} \,, \quad \hat{\gamma} = \frac{\hat{P}_e \times \hat{P}_\nu}{|\hat{P}_e \times \hat{P}_\nu|} \,.$$

The Λ-spin correlations are

$$W(\hat{S}_\Lambda \cdot \hat{\alpha}) = \tfrac{1}{2}\left(1 + \frac{8}{3}\frac{\operatorname{Re}Z}{|Z|^2+3}\hat{S}_\Lambda \cdot \hat{\alpha}\right) \,,$$

$$W(\hat{S}_\Lambda \cdot \hat{\beta}) = \tfrac{1}{2}\left(1 + \frac{8}{3}\frac{1}{|Z|^2+3}\hat{S}_\Lambda \cdot \hat{\beta}\right) \,,$$

$$W(\hat{S}_\Lambda \cdot \hat{\gamma}) = \tfrac{1}{2}\left(1 + \frac{\pi}{2}\frac{\operatorname{Im}Z}{|Z|^2+3}\hat{S}_\Lambda \cdot \hat{\gamma}\right) \,.$$

These nonrelativistic results have been checked by a Monte Carlo integration of the complete matrix element squared. The Monte Carlo result shows excellent agreement with the above expressions except for the coefficient of $\cos\theta_{e\nu}$. Figure 1 shows a comparison of the coefficients. In essence, the coefficient as determined from the Monte Carlo integration is 0.07 smaller than the nonrelativistic result except near large $|Z|$ where the difference approaches 0.13.

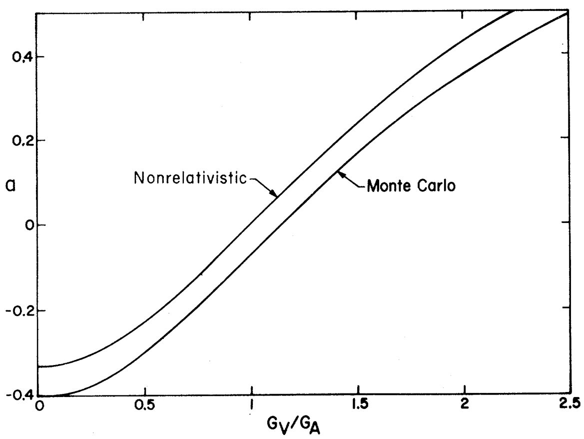

FIG. 1. Value of a in $f(\cos\theta) = 0.5(1 + a\cos\theta_{e\nu})$ comparing nonrelativistic result shown in text with Monte Carlo result.

A likelihood calculation must take into account any biases in the data. In principle, one should determine the detection efficiency as a function of all measured variables and use this to normalize the likelihood function. In practice, with only 186 events, this is impossible. One must then rely on *a priori* arguments to determine any detection biases that exist. We are convinced that the momentum of the Λ, the electron-neutrino correlation angle, and the various spin projections are unbiased, at least to our level of statistics. This follows because the Σ decay with a range of lab momenta and the above quantities are calculated in the Σ rest frame. The Lorentz transformation thus washes out the laboratory detection biases. The same argument cannot, however, be made for the electron momentum. The Lorentz transformation will change the electron momentum only on the order of 6%. We believe that the detection efficiency for the events was independent of the electron momentum, but are unable to prove it. Thus, one should be wary of any determination which depends critically on the electron momentum.

An examination of the expected distributions shows that the ratio G_V/G_A depends most strongly upon the $\cos\theta_{e\nu}$ and the $\hat{S}_\Lambda \cdot \hat{\alpha}$ distributions and is quite insensitive to $\hat{S}_\Lambda \cdot \hat{\beta}$ and the electron momentum distributions. The ratio G_W/G_A depends most strongly on the electron momentum distribution and to a small extent upon the $\hat{S}_\Lambda \cdot \hat{\alpha}$ distribution. Defining Z' as G_W/G_A, the dependence on $\hat{S}_\Lambda \cdot \hat{\alpha}$ is

$$W(\hat{S}_\Lambda \cdot \hat{\alpha}) = \tfrac{1}{2}\left(1 + \frac{2.86\,\mathrm{Re}Z + 0.08\,\mathrm{Re}Z'}{3.07 + 1.02}\,\hat{S}_\Lambda \cdot \hat{\alpha}\right).$$

Figure 2 shows contours at 1 and 2 standard deviations from a two-parameter likelihood calculation for G_V/G_A and G_W/G_A assumed real using only

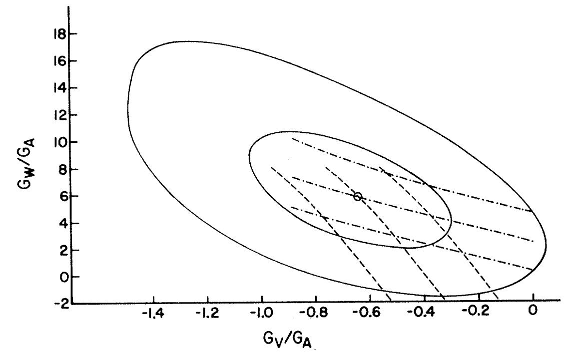

FIG. 2. Contour plot from 2-variable likelihood calculation using 163 Σ^- events. The closed contours are the 1- and 2-standard-deviation contours. The dashed curve shows the trajectory of the most likely value of G_V/G_A and the 1-standard-deviation point as a function of various fixed values of G_W/G_A. The dash-dot curve shows a similar trajectory for G_W/G_A for various fixed values of G_V/G_A.

the 163 Σ^- events. The approximate error matrix derived from the 1-standard-deviation contour is

$$\begin{pmatrix} 0.115 & -0.604 \\ -0.604 & 14.74 \end{pmatrix}.$$

The two parameters are clearly correlated. The result is $G_V/G_A = -0.64^{-0.40}_{+0.34}$, $G_W/G_A = +5.8^{+4.9}_{-3.8}$ where the errors are taken at the extrema of the 1-standard-deviation contour. An alternate procedure is to perform single-parameter likelihood calculations holding the other parameter fixed. Figure 2 shows the trajectories of maximum likelihood as each of the parameters is varied. The error bars are at 1 standard deviation. Assuming the CVC prediction that G_V/G_A is zero, the value of G_W/G_A is 2.4 ± 2.1, consistent with the CVC prediction of 1.9.

Figure 3 shows a plot of log likelihood versus

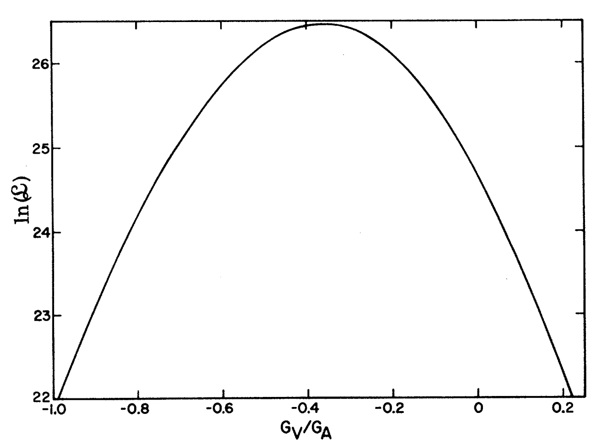

FIG. 3. Likelihood function from 1-variable likelihood calculation using 186 $\Sigma^{\pm}$ events for $G_W = 0$.

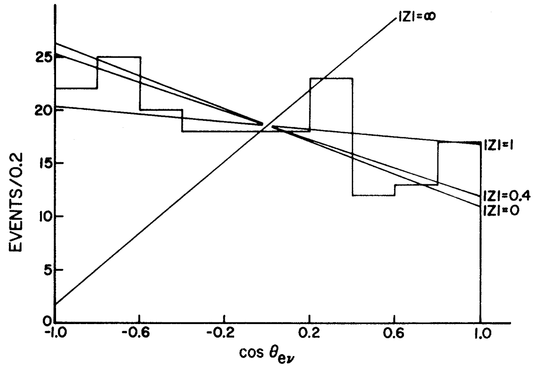

FIG. 4. Distribution of $\cos\theta_{e\nu}$ for 186 $\Sigma^{\pm}$ events. The expected distributions for various values of G_V/G_A (Z) are shown.

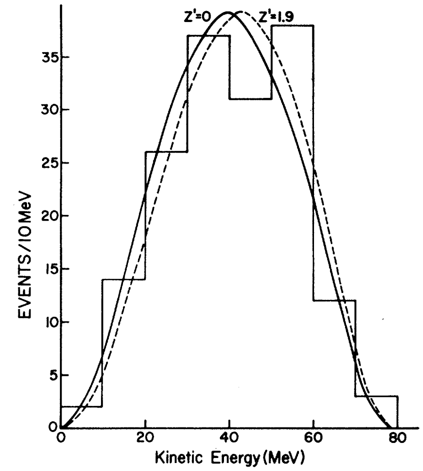

FIG. 6. Distribution of the kinetic energy of the electron in the Σ rest frame for 163 $\Sigma^- \to \Lambda e^- \nu$ events. The expected distributions for two values of G_W/G_A (Z') for $G_V/G_A = 0$ are shown.

G_V/G_A assumed real holding G_W/G_A fixed at zero for 186 Σ^- and Σ^+ events. The standard deviation and central value calculated from the 1-, 2-, and 3-standard-deviation points are virtually identical. The result is $G_V/G_A = -0.37 \pm 0.20$. The results for the separate samples are[6] -0.45 ± 0.21 for the Σ^- sample and -0.06 ± 0.64 for the Σ^+ sample. For $G_W/G_A = 1.9$, the result is $G_V/G_A = -0.45 \pm 0.20$.

Figures 4 through 8 show five experimental distributions of interest. It may be noted that assuming Z real and Z' zero, a fit to the $\cos\theta_{e\nu}$ distribution yields a value for $|Z|$ of $0.65^{+0.27}_{-0.38}$. Similarly a fit to the α projection of the proton momentum (related to the spin of the Λ along the α direction) yields $Z = -0.23 \pm 0.23$. These two values are consistent with the value from the likelihood calculation of -0.37 ± 0.20. A fit to the electron momentum distribution may be done simply by dividing the spectrum at 40 MeV/c. The calculated electron spectrum given by Bender *et al.*[7] may be integrated to yield an expression for δ, the number above 40 minus the number below divided by the total. The expression for $Z = 0$ is

$$\delta = -0.021 + 0.028Z' .$$

The fit yields $Z' = +1.9 \pm 2.8$ which is consistent with the value from the likelihood calculation.

For all the above results, it is assumed that Z, the ratio of the vector to axial-vector coupling constants, is real. The distribution of $\hat{S}_\Lambda \cdot \hat{\gamma}$ is sensitive to the imaginary part of Z. Figure 9 shows a related distribution, $\hat{p}_p \cdot \hat{\gamma}$. A fit to this distribution, assuming Z is pure imaginary, yields Im(Z) $= 0.16^{+0.47}_{-0.41}$, which is consistent with zero. This corresponds to an upper limit on $|\mathrm{Im}Z|$ of 0.63.

Branching Ratios

The branching ratios of $\Sigma^{\pm} \to \Lambda e^{\pm} \nu$ are of interest because they are related to the D to F ratio. The

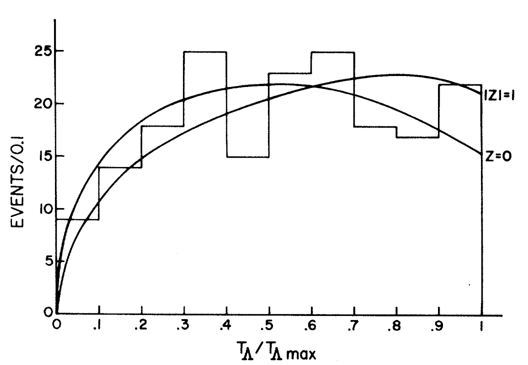

FIG. 5. Distribution of the kinetic energy of the Λ in the Σ rest frame for 186 $\Sigma^{\pm}$ events. The expected distributions for two values of G_V/G_A (Z) are shown.

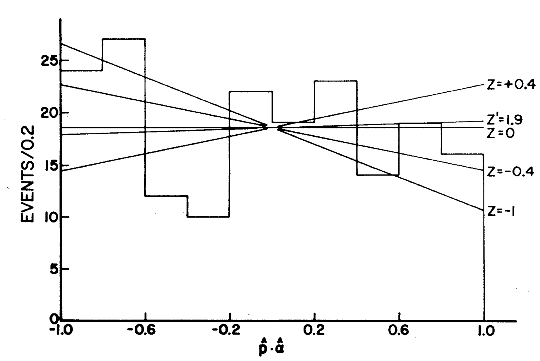

FIG. 7. Distribution of the projection of the proton momentum in the Σ rest frame in the α direction for 186 $\Sigma^{\pm}$ events. The expected distributions for several values of G_V/G_A (Z) as well as $G_W/G_A = 1.9$, $G_V/G_A = 0$ are shown.

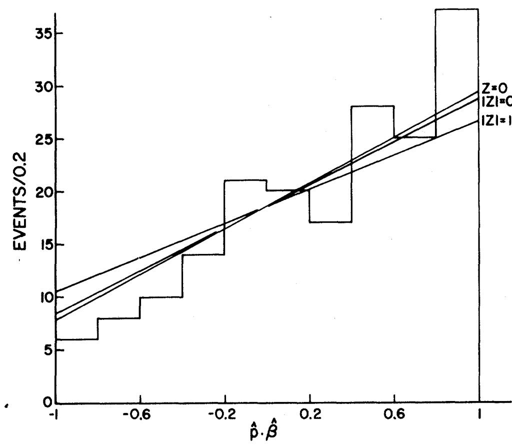

FIG. 8. Distribution of the projection of the proton momentum in the Σ rest frame in the β direction for 186 $\Sigma^{\pm}$ events. The expected distributions for several values of G_V/G_A (Z) are shown.

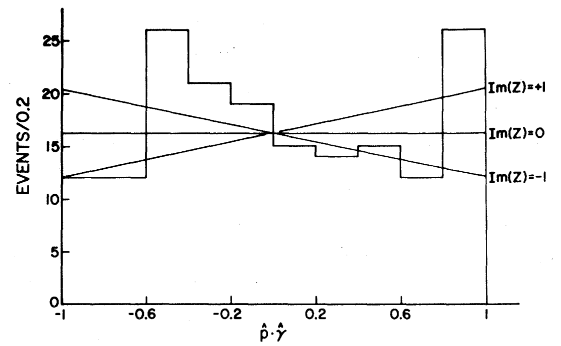

FIG. 9. Distribution of the projection of the proton momentum in the Σ rest frame in the γ direction for 163 $\Sigma^- \rightarrow \Lambda e^-\nu$ events. The expected distributions for several values of $\mathrm{Im}(G_V/G_A)$ (Z) are shown.

ratio of the decay rates is also of interest because a deviation from the phase-space ratio (0.61) is an indication of the occurrence of second-class currents. In order to calculate these numbers, it is essential to know one's detection biases. Each of the groups chose a sample of events appropriate to their detection procedure and made a Monte Carlo estimate of the resulting detection efficiency. Also, in order to calculate the branching ratios it is necessary to know the number of $\Sigma^{\pm}$ produced. Again each group chose a different technique. Table I shows the various numbers needed to calculate these ratios. It should be noted that in all cases the number of Σ^+ and Σ^- produced were determined in ways which yield strongly correlated values. The ratio of the decay rates may be calculated using only the detected number of events, the scan and detection efficiencies, the lifetimes, and the ratio of Σ^+ to Σ^- production by stopping K^-. Thus,[8]

$$\frac{\Gamma(\Sigma^+ \rightarrow \Lambda e^+\nu)}{\Gamma(\Sigma^- \rightarrow \Lambda e^-\nu)} = \frac{35.8 \pm 8.7}{237 \pm 24} \times \frac{1.64 \pm 0.06}{0.810 \pm 0.013} \times 2.2 \pm 0.1$$
$$= 0.67 \pm 0.18\,.$$

The branching ratios are given by the corrected number of events divided by the branching ratio for $\Lambda \rightarrow p\pi$ and the total number of Σ's produced:

$$\frac{\Gamma(\Sigma^+ \rightarrow \Lambda e^+\nu)}{\Gamma(\Sigma^+ \rightarrow \text{all})} = \frac{35.8 \pm 8.9}{(2.66 \pm 0.12) \times 10^6 \times (0.653 \pm 0.012)}$$
$$= (0.21 \pm 0.05) \times 10^{-4}\,,$$

$$\frac{\Gamma(\Sigma^- \rightarrow \Lambda e^-\nu)}{\Gamma(\Sigma^- \rightarrow \text{all})} = \frac{237 \pm 27}{(5.86 \pm 0.20) \times 10^6 \times (0.653 \pm 0.012)}$$
$$= (0.62 \pm 0.07) \times 10^{-4}\,.$$

The errors on the corrected numbers of events are different for the ratio of the decay rates and the branching ratios because the error in the scan

TABLE I. Event rates and efficiencies for branching ratios.[a]

	CU/SB	Maryland	Heidelberg	Total
Σ^- used	31	35	31	97
Scan eff.	0.95± 0.05	0.80± 0.08	0.81± 0.08	
Geom. eff.	0.44	0.44	0.605	
Σ^- corrected	74.2 ± 13.9	99.4 ± 19.5	63.3 ± 13.0	237 ± 27
Σ^- produced ($\times 10^6$)	2.13± 0.11	2.35± 0.12	1.38± 0.11	5.86± 0.20
Σ^+ used	5	6	6	17
Scan eff.	0.95± 0.05	0.80± 0.08	0.81± 0.08	
Geom. eff.	0.49	0.55	0.65	
Σ^+ corrected	10.7 ± 4.8	13.6 ± 5.7	11.4 ± 4.8	35.8 ± 8.9
Σ^+ produced ($\times 10^6$)	0.99± 0.08	1.04± 0.07	0.63± 0.06	2.66± 0.12

[a]The Heidelberg group reports a total of 10 Σ^+ decay events; four events, however, did not have a visible Λ^0 decay. We prefer to use six events to maintain uniformity with the Σ^- decay where events without visible Λ^0 decay are usually due to the decay $\Sigma^- \rightarrow ne^-\bar{\nu}$.

efficiency does not enter into the former.

The measurement of the ratio of the decay rates permits a limit to be set upon the axial-magnetism (AM) coupling constant in the hadron current. This is a second-class current[9] and is of the form

$$G_{AM}\sigma_{\mu\nu}\gamma_5 q_\nu/m_\Sigma .$$

We use the calculated decay rates of Nieto[10] and note that the major effect of axial magnetism is its interference with the axial-vector term. This interference is of opposite sign[11] for Σ^+ and Σ^-. Considering only this interference and the dominant axial-vector contributions, and ignoring the absolute sign of G_{AM}/G_A, one gets

$$\Gamma(\Sigma^+\to\Lambda e^+\nu) = 6.625\times10^5\,G_A{}^2 \pm 0.5487\times10^5\,G_A G_{AM} ,$$

$$\Gamma(\Sigma^-\to\Lambda e^-\nu) = 10.95\times10^5\,G_A{}^2 \mp 0.9979\times10^5\,G_A G_{AM} .$$

This yields, calling $R=\Gamma(\Sigma^+\to\Lambda e^+\nu)/\Gamma(\Sigma^-\to\Lambda e^-\nu)$,

$$\frac{G_{AM}}{G_A} = \pm\frac{R-0.605}{0.0911R+0.0501} = \pm(0.6^{+1.3}_{-1.8}) .$$

This corresponds to a 90% confidence upper limit on G_{AM}/G_A of 3.2.

Assuming that the decay $\Sigma\to\Lambda e\nu$ is pure axial-vector, the branching ratio can be computed by combining the Cabibbo prediction of the decay rate with the lifetime of the Σ; in particular, for the Σ^-,

$$\frac{\Gamma(\Sigma^-\to\Lambda e^-\nu)}{\Gamma(\Sigma^-\to\text{all})} = (1.80\pm0.07)\times10^{-4}\cos^2\theta\,\tfrac{2}{3}D^2 .$$

Using the value for θ at 0.239 ± 0.006 from a one-angle fit,[12] we calculate $D=0.74\pm0.04$.

Conclusions

We find $G_V/G_A=-0.37\pm0.20$, consistent with the CVC prediction of zero. We clearly have very little sensitivity for the quantities G_W/G_A, $\mathrm{Im}(G_V/G_A)$, or G_{AM}/G_A. However, our results are consistent with the various predictions of CVC, time-reversal invariance, and the absence of second-class currents. The branching ratios of Σ^+ and Σ^- to $\Lambda e^\pm\nu$ are determined to be $(0.21\pm0.05)\times10^{-4}$ and $(0.62\pm0.07)\times10^{-4}$, respectively, yielding a value for D of 0.74 ± 0.04.

APPENDIX: MATRIX ELEMENT FOR $\Sigma^-\to\Lambda e^-\bar\nu$ FOLLOWED BY $\Lambda\to p\pi^-$

All coupling constants are assumed to be real and an over-all constant has been deleted. The metric is $(1,1,1,-1)$ and in this convention G_A/G_V for neutron β decay is positive. Four-vectors are abbreviated to the particle name and a dot between two four-vectors indicates an inner product. We define

$$A=\alpha/p^*m_\Lambda, \qquad C=G_V+(m_\Sigma+m_\Lambda)G_W/m_\Sigma, \qquad Q=\Sigma+\Lambda ,$$

where α is the asymmetry parameter for $\Lambda\to p\pi^-$, and p^* is the c.m. momentum in $\Lambda\to p\pi^-$. The matrix element squared is

$$\begin{aligned}
|\mathfrak{M}|^2 ={}& (C^2+G_A{}^2)[\Lambda\cdot e\Sigma\cdot\nu+\Lambda\cdot\nu\Sigma\cdot e+A\Lambda\cdot p(\Lambda\cdot e\Sigma\cdot\nu-\Lambda\cdot\nu\Sigma\cdot e)+Am_\Lambda{}^2(p\cdot e\Sigma\cdot\nu-p\cdot\nu\Sigma\cdot e)]\\
&+(C^2-G_A{}^2)[m_\Sigma m_\Lambda e\cdot\nu-Am_\Sigma m_\Lambda(\Lambda\cdot\nu p\cdot e-\Lambda\cdot ep\cdot\nu)]\\
&+2CG_A[\Lambda\cdot e\Sigma\cdot\nu-\Lambda\cdot\nu\Sigma\cdot e+A\Lambda\cdot p(\Lambda\cdot e\Sigma\cdot\nu+\Lambda\cdot\nu\Sigma\cdot e)+Am_\Lambda{}^2(p\cdot e\Sigma\cdot\nu+p\cdot\nu\Sigma\cdot e)]\\
&+(CG_W/m_\Sigma)[-m_\Lambda(Q\cdot e\Sigma\cdot\nu-Q\cdot\Sigma e\cdot\nu+Q\cdot\nu\Sigma\cdot e)-m_\Sigma(Q\cdot e\Lambda\cdot\nu-Q\cdot\Lambda e\cdot\nu+Q\cdot\nu\Lambda\cdot e)\\
&\qquad+Am_\Lambda Q\cdot\Lambda(p\cdot e\Sigma\cdot\nu-p\cdot\nu\Sigma\cdot e)-Am_\Lambda Q\cdot p(\Lambda\cdot e\Sigma\cdot\nu-\Lambda\cdot\nu\Sigma\cdot e)\\
&\qquad+Am_\Lambda Q\cdot\Sigma(\Lambda\cdot ep\cdot\nu-p\cdot e\Lambda\cdot\nu)]\\
&+(G_AG_W/m_\Sigma)[-Am_\Lambda(m_\Sigma m_\Lambda-\Sigma\cdot\Lambda)(p\cdot eQ\cdot\nu-Q\cdot pe\cdot\nu+p\cdot\nu Q\cdot e)\\
&\qquad-A(m_\Sigma\Lambda\cdot p+m_\Lambda\Sigma\cdot p)(\Lambda\cdot eQ\cdot\nu-Q\cdot\Lambda e\cdot\nu+\Lambda\cdot\nu Q\cdot e)]\\
&+(G_W{}^2/m_\Sigma{}^2)(m_\Sigma m_\Lambda-\Sigma\cdot\Lambda)(Q\cdot eQ\cdot\nu-Q\cdot Qe\cdot\nu/2) .
\end{aligned}$$

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*Research supported in part by the U. S. Atomic Energy Commission.

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PHYSICAL REVIEW D VOLUME 6, NUMBER 9 1 NOVEMBER 1972

Exploration of SU(3) Symmetry of Baryon Regge Residues*

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(Received 15 December 1971)

The SU(3) symmetry of baryon Regge poles is explored by comparing the angular distributions of backward meson-baryon reactions whose Regge exchanges should be SU(3)-related. Several indications of the symmetry are found. In particular, it is discovered that one can generate the observed highly structured angular distribution for $\pi^+p \to p\pi^+$ from the featureless one for $K^+p \to pK^+$ simply by assuming SU(3) symmetry and taking singlet-octet exchange degeneracy in K^+p scattering into account. Using the symmetry, evidence that exchange degeneracy between the Reggeized $\frac{1}{2}^+$ and $\frac{3}{2}^-$ octets is broken is found. Relevant experiments are suggested.

I. INTRODUCTION

We are exploring the question of whether baryon Regge residues show the same kind of SU(3) symmetry that resonance decay rates and stable-particle couplings do. More specifically, we are looking for evidence of such symmetry (and sometimes of exchange degeneracy as well) in the observed features of backward 0^- meson–$\frac{1}{2}^+$ baryon scattering. We would like to report some of the things which have been found.

As is well known, resonance partial widths have proved to obey close-to-exact SU(3) symmetry, once the barrier-penetration–phase-space factor q^{2l+1} is divided out of them.[1] (Here q is the center-of-mass momentum of the outgoing particles, and l is the orbital angular momentum of the resonance.) Thus, the residue of the Breit-Wigner pole in the partial-wave amplitude $(S-1)_{ij}/2i(q_iq_j)^{1/2}$ for formation of a resonance by incoming state i, and its subsequent decay into outgoing state j, is SU(3)-symmetric once the factor $(q_iq_j)^l$ is removed. It is then natural to conjecture that as one moves along the Regge trajectory to which the resonance belongs, this barrier-penetration effect, $(q_iq_j)^l$, continues to be the only thing which breaks the SU(3) symmetry of the residue. Hence, we write the signatured meson-baryon partial wave containing the Regge pole to which some resonance k belongs (for definiteness we treat here the case where k is a member of the δ decuplet [$\Delta_\delta(1236)$, $\Sigma_\delta(1385)$, etc.]) in the form

$$f_{ij}(J,W)=\frac{\beta_{ij}^k(W)/s_0{}^J}{J-\alpha_k(u)}(q_iq_j)^l\left(\frac{E_i+M_i}{W}\,\frac{E_j+M_j}{W}\right)^{1/2}. \tag{1.1}$$

Here J is the total angular momentum, $l=J-\frac{1}{2}$ is the orbital angular momentum, W is the total c.m. energy, $u=W^2$, and E_i is the c.m. energy of the baryon, of mass M_i, in state i. The trajectory on which resonance k lies is denoted by α_k. [We will always take α_k to be the true, somewhat SU(3)-breaking, trajectory.] The amplitude f is defined to coincide with $(S-1)/2i(q_iq_j)^{1/2}$ for physical J, and the slowly varying square root in (1.1) has been included so that $\beta_{ij}^k/s_0{}^J$ will be the residue of the Regge pole in the standard kinematic-singularity-free partial-wave amplitude.[2] The conjec-