Low-Energy ΣN Scattering Parameters: K-Matrix Analysis

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The recently reported new $\Sigma^{\pm}p$ cross-section data have been used to evaluate the low-energy scattering parameters in terms of the reaction K-matrix formalism. In the analysis the Coulomb contributions to the scattering processes were explicitly incorporated into the formalism. Out of the three sets of values obtained in a fit of the six scattering parameters, two are consistent with the observed rise in the forward $\Sigma^{+}p$ differential elastic scattering.

An attempt to estimate the low-energy $\Sigma^{\pm}p$ scattering parameters using the reaction K-matrix formalism has been recently reported.[1] The data used for that analysis (referred to here as paper I) were based on a relatively small number of events which was insufficient to furnish neither information on the Coulomb contributions to the $\Sigma^{\pm}p$ scattering nor its interference with the nuclear part. Consequently, a total of eight sets of values were found for the scattering parameters, all of which described adequately those data. Furthermore, these solutions, which were associated with large errors, could only be obtained by introducing several assumptions into the analysis in order to compensate for the poor data. The scattering parameters were then used to examine the possibility that the observed Λp invariant-mass enhancement at 2128 MeV seen in the $K^-d \to \Lambda p \pi^-$ interactions can be attributed to a virtual ΣN bound state.[2]

Lately the Heidelberg group[3] has concluded a new study of the low-energy $\Sigma^{\pm}p$ elastic scattering cross section based on about six times more events than in their previous work. With these new data it was possible to study in some detail the differential elastic cross sections of these $\Sigma^{\pm}p \to \Sigma^{\pm}p$ channels. Using these new elastic scattering data and the former published inelastic data,[4] we reevaluate in this paper the low-energy $\Sigma^{\pm}p$ scattering parameters, incorporating explicitly the Coulomb contributions. As expected, the number of solutions for the scattering parameters obtained is considerably reduced and it is also possible to relax the additional assumptions that one has to introduce into the analysis.

In the K-matrix formalism with isospin invariance the low-energy s-wave $\Sigma^{\pm}p$ interactions may be expressed in terms of the nuclear complex scattering lengths $A_{J,I} = a_{J,I} + ib_{J,I}$. In paper I explicit expressions were derived for the cross sections of the following channels:

$$\Sigma^+p \to \Sigma^+p\,, \tag{1}$$

$$\Sigma^-p \to \Sigma^-p\,, \tag{2}$$

$$\Sigma^-p \to \Sigma^0 n\,, \tag{3}$$

$$\Sigma^-p \to \Lambda n. \tag{4}$$

In deriving these expressions a constant-scattering-length approximation was used and advantage was taken of the fact that parity and total angular momentum conservations imply that the singlet ($^1S_0$) and triplet ($^3S_1$) contributions can be handled separately. In addition all charge-independence violations have been neglected apart from a phase-space factor which was introduced to compensate for the Σ^-, Σ^0 mass difference.

The charge-independence violations which may well be neglected at high energies are expected to be important at the low-energy region which is of interest in this work. The modifications to the K-matrix formalism due to these violations may be grouped into two classes:

(a) modifications due to the mass differences generated within an isospin multiplet due to electromagnetic interactions, and

(b) modifications due to Coulomb interactions.

The inclusion of these effects in the reaction matrix formalism has been described in detail by Dalitz and Tuan and others.[5] In this method, which is closely followed here, the Schrödinger equations are solved for a charged and noncharged spatial wave function. The use of the modified version of the equivalent-boundary-condition method enables us then to express the nuclear part of the $\Sigma^{\pm}p$ scattering amplitudes in terms of the following parameters: (a) the ΣN isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ complex nuclear scattering lengths $A_{J,I} = a_{J,I} + ib_{J,I}$, (b) the Bohr radius B of the Σp system, and (c) the range R of the nuclear interaction taken to be equal to 0.165 F, the Σ Compton wavelength. The results of the present analysis turned out to be rather insensitive to this choice of R in the range of ≈ 0.1 to ≈ 1.0 F.

With the inclusion of the Coulomb part of the amplitude,[6] we arrive at the following expressions for the $\Sigma^{\pm}p$ differential cross sections:

$$\frac{d\sigma}{d\Omega}(\Sigma^+p\to\Sigma^+p)=\sum_{J=0}^{1}\frac{2J+1}{4}\left|\frac{\exp[(2i/kB)\ln\sin\frac{1}{2}\theta]}{2Bk^2\sin^2(\frac{1}{2}\theta)}+\frac{C_0^2A_{J,3/2}}{1-iC_0^2kA_{J,3/2}(1-i\lambda)}\right|^2, \quad (5)$$

$$\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^-p)=\sum_{J=0}^{1}\frac{2J+1}{4}\left|\frac{\exp[(2i/kB)\ln\sin\frac{1}{2}\theta]}{2Bk^2\sin^2(\frac{1}{2}\theta)}+\frac{C_0^2(A_{J,3/2}+2A_{J,1/2}-3ik_0A_{J,1/2}A_{J,3/2})}{D_J}\right|^2, \quad (6)$$

$$\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^0n)=\frac{k_0}{2k}\sum_{J=0}^{1}(2J+1)\left|\frac{C_0(A_{J,3/2}-A_{J,1/2})}{D_J}\right|^2, \quad (7)$$

$$\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Lambda n)=\frac{1}{k}\sum_{J=0}^{1}(2J+1)\left|\frac{C_0(1-ik_0A_{J,3/2})}{D_J}\right|^2 b_{J,1/2}, \quad (8)$$

where

$$D_J=3-ik_0(A_{J,1/2}+2A_{J,3/2})-kC_0^2(1-i\lambda)[i(A_{J,3/2}+2A_{J,1/2})+3k_0A_{J,1/2}A_{J,3/2}],$$

k stands for the incoming c.m. momentum, and k_0 stands for the c.m. momentum of the $\Sigma^0 n$ system.

$$C_0^2=\frac{2\pi}{kB}\left[1-\exp\left(-\frac{2\pi}{kB}\right)\right]^{-1}$$

and

$$\lambda=\frac{-2[\ln(2kR)+\mathrm{Re}\,\Psi(i/kB)+2\gamma]}{kBC_0^2},$$

where γ is the Euler constant and $\Psi(\xi)=\Gamma'(\xi)/\Gamma(\xi)$.

These expressions contain six parameters, $a_{0,3/2}$, $a_{1,3/2}$, $a_{1,1/2}$, $b_{1,1/2}$, $a_{0,1/2}$, and $b_{0,1/2}$, to be determined by the data, whereas $b_{0,3/2}=b_{1,3/2}=0$ due to the absence of absorption channels in the $I=\frac{3}{2}$ ΣN reactions. For the analysis of the scattering parameters we have used the low-energy ($P_\Sigma<180$ MeV/c) $\Sigma^\pm p$ elastic scattering data reported by the Heidelberg group[3] and the $\Sigma^-p\to\Sigma^0n$ and $\Sigma^-p\to\Lambda n$ data reported earlier by the same group.[4] The total cross-section data reported for the first two reactions have been obtained from the differential cross section, that due to experimental difficulties, was measured in a rather small scattering angle interval around 90°. Thus the finally-quoted experimental results for the cross sections are not expected to be the pure nuclear ones nor the true values that one should expect from scattering data measured over the whole angular range. Consequently we have found it best to follow exactly the experimental procedure in the integration of the expressions (5) and (6) to obtain total cross-section formulas ready to be fitted to the data.

In performing the fits it has been assumed that the data are predominantly pure s-wave processes. This assumption stems from the observation that the highest Σ momentum involved is $k\approx0.4$ F^{-1}. An additional support for this assumption is forthcoming from the final Σ^+ polarization measurement in the Σ^+p elastic scattering process.[3] The $\Sigma^-p\to\Lambda n$ data, on the other hand, indicate the possibility of some p-wave contribution at the upper end of the incident momentum range ($P_{\Sigma^-}>150$ MeV/c) used by the present analysis.[4] The uncertainty introduced into the results of the following analysis due to this possible, small p-wave contribution is estimated to be much smaller than the uncertainties arising from the present cross-section data status. Finally it has been tacitly assumed that the transition ${}^3S_1\to{}^3D_1$ in the $\Sigma^-p\to\Lambda n$ reaction is small and can be neglected. This assumption is in accordance with the findings of Yamamoto *et al.*[7] in their experiment on polarized Σ^-p interactions.

In the systematic search for solutions to the six ΣN scattering parameters we have restricted ourselves to values of $b_{0,1/2}$ in the interval 0 to 1.5 F which corresponds to a singlet contribution to the $\Sigma^-p\to\Lambda n$ cross section up to about 40 mb. This restriction has been motivated by the work of Cline *et al.* and others[2,8] on the low Λp invariant-mass enhancement observed in $K^-d\to\Lambda p\pi^-$ reaction associated with a fast nonspectator proton. From the study of this enhancement, which is predominantly observed in forward emerging π^- events, it has been concluded that the $\Sigma N\to\Lambda p$ transition occurs predominantly in the ${}^3S_1$ state.

The other five scattering parameters were left free to vary in the range of $-13<a_{J,I}<13$ F and $0<b_{1,1/2}<13$ F unless they were bound to a smaller region by the cross-section data. In particular, the value of $a_{1,3/2}$ is restricted by the small $\Sigma^+p\to\Sigma^+p$ cross section to values within the limits -1.0 to 1.4 F, corresponding to a pure triplet scattering [see Fig. 1(a)]. Thus we first studied the solutions for the four parameters $a_{0,3/2}$, $a_{1,1/2}$, $b_{1,1/2}$, and $a_{0,1/2}$ by varying $b_{0,1/2}$ in the region of 0 to 1.5 F and $a_{1,3/2}$ in the region of -1.5 to $+1.5$ F.

The four-parameter fit to the total cross-section data has been carried out in two steps. In the first, a χ^2 mapping for the whole range of values

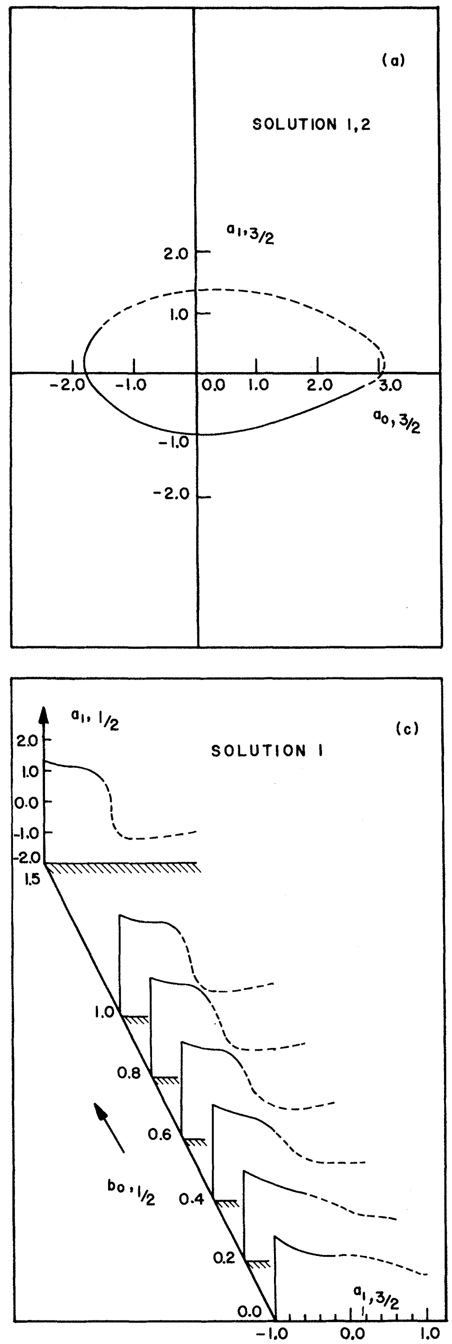

FIG. 1. The four parameters $a_{0,3/2}$, $a_{0,1/2}$, $a_{1,1/2}$, and $b_{1,1/2}$ are shown as functions of $a_{1,3/2}$ at representative values of $b_{0,1/2}$. The solid and dashed lines correspond to constructive and destructive interference with the Coulomb Σ^+p elastic amplitude. (a) The dependence of $a_{0,3/2}$ on $a_{1,3/2}$ for all values of $b_{0,1/2}$ in the range 0 to 1.5 F. (b) The values of $b_{1,1/2}$ (the b lines) and $a_{0,1/2}$ (the a lines) as functions of $a_{1,3/2}$. The indices 1 and 2 correspond to solution 1 with $b_{0,1/2}=0$ and 1 F, respectively. (c) and (d) show the dependence of $a_{1,1/2}$ on $a_{1,3/2}$ and $b_{0,1/2}$ for solutions 1 and 2, respectively.

for the parameters has been undertaken in order to locate the regions of minima. In the second step, a detailed least-squares fit has been undertaken in those minimal χ^2 regions, using the program MINUIT[9] to obtain a final set of values for the parameters. In this way two sets of solutions have been found for each ($b_{0,1/2}$, $a_{1,3/2}$) pair, with an average χ^2 value of less than 20 for 17 degrees of freedom. The dependence of the four parameters on $a_{1,3/2}$ at representative values of $b_{0,1/2}$ is

TABLE I. The three sets of Σp scattering parameters obtained in the six-parameter fit to the experimental data, restricting $b_{0,1/2}$ to the domain 0 to 1.5 F. The solutions sets A and B predict a constructive interference with the Coulomb Σ^+p elastic scattering. All parameters are given in F.

Solution set	$a_{0,1/2}$	$a_{0,3/2}$	$a_{1,1/2}$	$b_{1,1/2}$	$b_{0,1/2}$	$a_{1,3/2}$	χ^2
(A)	−1.3± 0.6	1.4± 1.3	1.1± 0.5	1.7± 0.8	$0.8^{+1.0}_{-0.8}$	−0.7± 0.4	16
(B)	1.3± 1.5	−0.8± 2.0	1.3± 0.5	1.5± 1.4	$1.5^{+4.0}_{-1.5}$	−0.8± 0.7	17
(C)	−1.0± 0.8	−1.2± 1.0	−1.2± 0.3	1.3± 0.6	1.3± 1.2	1.1± 0.6	15

shown in Fig. 1. These solutions were then used to calculate the expected Σ^+p differential elastic cross sections. The scattering parameters values for which a destructive interference with the Coulomb Σ^+p amplitude is expected are given in the same figure by the dashed lines.

The results of these fits may be summarized as follows:

(a) The value of $a_{0,3/2}$ depends strongly on $a_{1,3/2}$, but is insensitive to the choice of $b_{0,1/2}$ and essentially identical for the two solutions. This is not surprising since both $a_{0,3/2}$ and $a_{1,3/2}$ are predominantly determined by the Σ^+p cross section.

(b) The value of $a_{1,1/2}$ is somewhat dependent on the choice of $a_{1,3/2}$ and to a much lesser extent on the choice of $b_{0,1/2}$. While the two solutions yield very close values for $a_{1,1/2}$, they differ by the range of values for which a constructive Coulomb interference is expected for the Σ^+p cross section (see Fig. 1).

(c) The values of $a_{0,1/2}$ have little dependence on either $b_{0,1/2}$ or $a_{1,3/2}$; on the other hand, they are of opposite signs for the two solutions.

(d) The values of $b_{1,1/2}$ vary by not more than 1 F as $b_{0,1/2}$ and $a_{1,3/2}$ are varied in the range under consideration, and a change of about 0.5 F is observed in the transition from one solution to the other.

Since all four parameters exhibit a smooth dependence on $a_{1,3/2}$ and $b_{0,1/2}$, we have continued, and carried out an over-all fit of the six independent scattering parameters to the total cross-section data. The procedure adopted in this fit was similar to the one described earlier, where the search has been conducted in the domain $-5 < a_{J,I} < 5$ F and $0 < b_{J,I} < 5$ F. The three distinct solutions

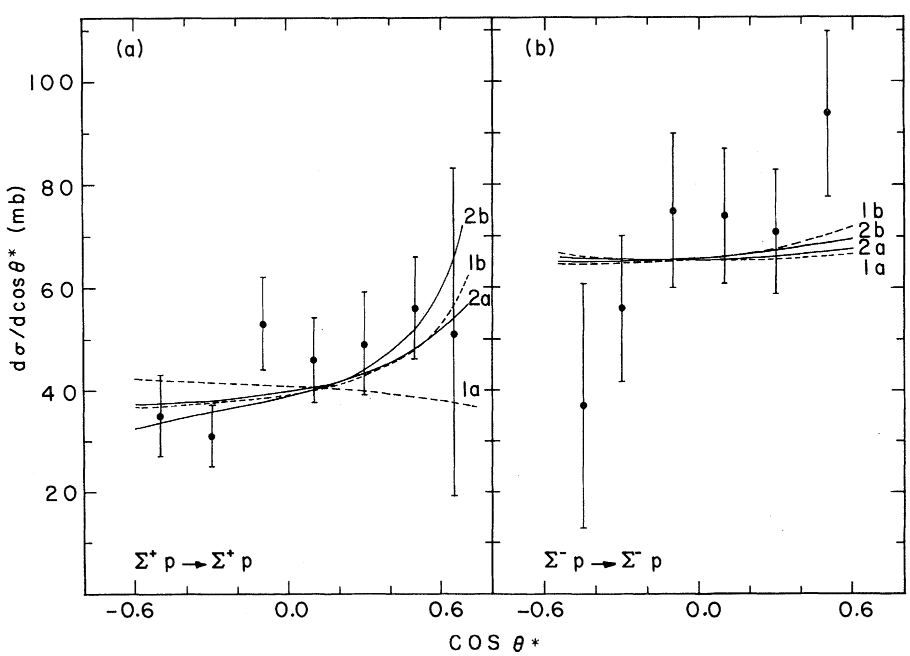

FIG. 2. The $\Sigma^{\pm}p$ elastic differential cross section obtained by the Heidelberg group[3] for the incident momentum range of $150 < P_{\Sigma^-} < 170$ MeV/c and $160 < P_{\Sigma^+} < 180$ MeV/c. The solid lines (2a and 2b) represent the elastic differential cross sections obtained from the accepted six-parameter-fit solutions, sets A and B of Table I. The dashed lines (1a and 1b) are the expected elastic differential cross sections obtained in the four-parameter fit setting $b_{0,1/2} = a_{1,3/2} = 0$ F (see text).

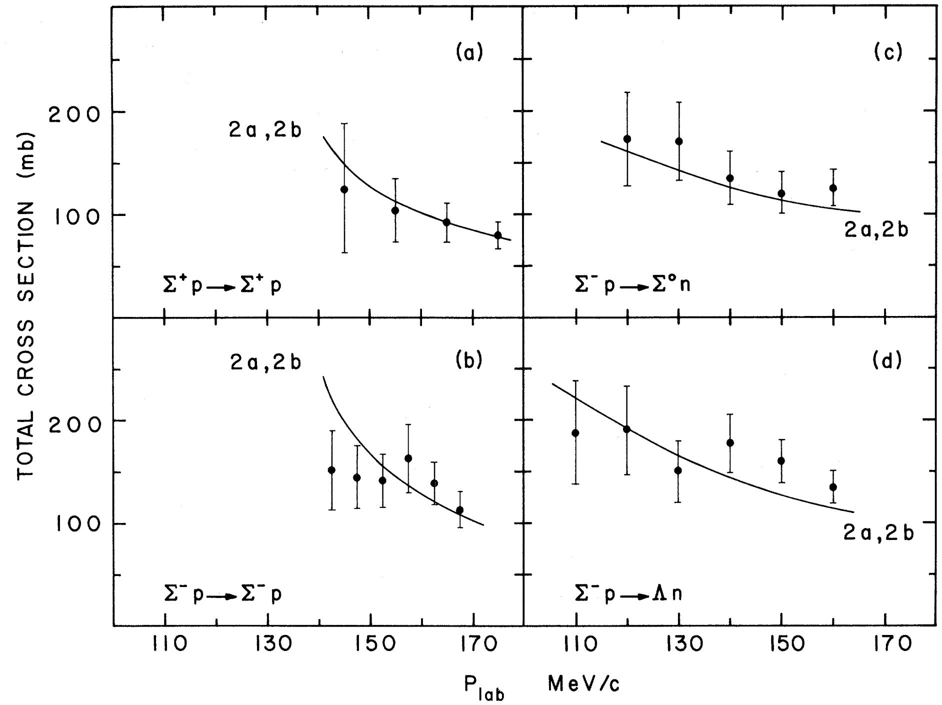

FIG. 3. The $\Sigma^{\pm}p$ cross section values of the Heidelberg group[3,4] shown as a function of the incident Σ momentum. The solid lines represent the cross sections obtained from the accepted six-parameter-fit solutions, sets *A* and *B* of Table I. The curves for the two solutions (labeled as 2a and 2b) coincide one with the other.

which were found to describe the data well are presented in Table I. The differential Σ^+p and Σ^-p elastic scattering cross sections expected from these solutions are shown in Fig. 2, together with the experimental data. Inasmuch as the forward peaking seen in the Σ^+p scattering data can be attributed to the Coulomb interference, one may rule out solution *C*, which clearly prescribes a destructive Coulomb interference. As for the Σ^-p scattering, the three solutions *A*, *B*, and *C* predict a small forward peaking. At the same time the data are not sufficiently accurate to distinguish between these solutions, and thus we are left with the two solutions, *A* and *B*, which differ mainly in the sign of $a_{0,1/2}$ and $a_{0,3/2}$. The $\Sigma^{\pm}p$ cross sections corresponding to these two solutions are compared with the data in Fig. 3.

In both the solutions *A* and *B*, $a_{1,1/2}$ is positive and equal to about 1 F and the associated $b_{1,1/2}$ is of the order of 1.5 F. Hence the interpretation of the experimentally observed Λp enhancement at $\approx$2128 MeV as an ${}^3S_1$ $(\Sigma N)_{I=1/2}$ virtual bound state seems unlikely.[1] This statement, however, is only valid as long as the *K*-matrix element which describes the ΣN close-channel transition can be neglected.[5]

Recently several attempts have been made to evaluate the hyperon-nucleon low-energy scattering parameters using various model-dependent approaches. Eisele *et al.*[3] have used their Σ^+p scattering data to evaluate the $a_{0,3/2}$ and $a_{1,3/2}$ parameters in an effective-range approximation. To reduce the number of parameters, they have taken the relations between the effective ranges and the scattering lengths derived by Dosch and Müller[10] in a dynamical model. In this way they were able to obtain a relation between the two scattering lengths which is described by a close contour in the $a_{0,3/2}$, $a_{1,3/2}$ plane and which closely resembles our results of the four-parameter fit [see Fig. 2(a)]. Since no use was made of all the $\Sigma^{\pm}p$ data, Eisele *et al.* were unable to further narrow down the range of values for these parameters, apart from the restriction imposed by the Coulomb interference.

Nagels *et al.*[11] have estimated the low-energy scattering parameters using potentials derived from pseudoscalar and vector meson exchanges, with coupling constants taken from SU(3) and other well-established assumptions. To these potentials hard cores were added, characterized by four free parameters determined by the data. The value of $a_{1,3/2} = -0.71$ F obtained in that analysis essentially coincides with both values obtained from solutions *A* and *B* of the present work. The value of 2.4 F obtained by Nagels *et al.* for $a_{0,3/2}$ is again close and within one standard deviation of our value of 1.4 ± 1.3 F. On the other hand, the analysis of Nagels *et al.* predicts the existence of an $^3S_1$ $(\Sigma N)_{I=1/2}$ virtual bound state, whereas our analysis finds it unlikely.

A somewhat different approach to the hyperon-nucleon scattering problem has been adopted by Letessier and Tounsi,[12] in which use is made of the N/D method in the framework of a one-boson-exchange model. In that analysis the $\Sigma^+ p$ scattering lengths were estimated to be $a_{1,3/2} = 0.4$ F and $a_{0,3/2} = 4.4$ F. While the value of $a_{1,3/2}$ is within one standard deviation of our values, the value of 4.4 F is definitely much higher than those obtained in the K-matrix formalism.

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Test of Factorization in $p+\pi^- \to p+X$ and $p+p \to p+X$ near the Kinematical Limit*

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We examine new data on $p+\pi^- \to p+X$ at 25 GeV/c and 40 GeV/c and $p+p \to p+X$ at 19 GeV/c and 24 GeV/c. The normalized cross sections of these two reactions agree with each other within 10% near the kinematical limit.

Single-particle inclusive experiments have been a subject of many theoretical and experimental studies.[1] This is a reasonable first step in an attempt to understand the complex multiparticle production. Feynman,[2] and Benecke, Chou, Yang, and Yen,[2] predict that the invariant cross section,

$$E_c \frac{d\sigma}{d^3p_c} \equiv f(x, p_\perp{}^2, s),$$

for $a+b \to c+X$(anything) is a function only of $p_\perp{}^2$ and $x \equiv 2p_c^*/\sqrt{s}$ in the limit of $s \to \infty$, where p_c^* is the center-of-mass longitudinal momentum and s