the event at A *or* the event at B. Thus, a definite, well-defined, and Lorentz-invariant causal connection between the event at A and that at B will be determined, the tachyon beam being the causal agent. Every observer, in each of the proposed anomalies, can be supplied with such a "causal pointer" making the causal direction unambiguous and Lorentz-invariant.

III. CONCLUSION

In conclusion, the reader should be reminded that tachyons, as physically realizable entities capable of carrying information across spacelike intervals, are the subject of the anomalies. The "virtual" particles or exchanges which arise in series-type solutions of interaction problems as well as any waves obeying a Klein-Gordon equation may have spacelike four-momenta, however they cannot carry *information* across spacelike intervals. Thus, they do not yield any new conclusions about rigid bodies, action at a distance, etc.[1] *Tachyons can exist only if certain experiments cannot be performed, i.e., only if certain of the emitters will not work.*[5] Finally, I wish to remark that only if tachyons (which interact with tardyons) exist may the arrow of causality point in the opposite direction to the arrow of time. Under such circumstances the arrow of causality is well defined by use of a "causal pointer" for tachyon beams. (Naturally occurring events, connected by tachyons, will indeed result in tachyon beams if we take a large enough sample of tachyon-active material, cf. natural radioactivity.)

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Nonlinear Duality Calculation of the Width of the $\pi\pi$ Diffraction Peak*

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We insert Regge-exchange amplitudes into the unitarity relation at intermediate energies. The resulting $\pi\pi$ absorptive part is then required to be dual to one with Pomeranchukon (P), ρ, and f^0 exchange. By assuming ρ and f^0 residue functions consistent with the dual-absorptive model, we can then calculate the P residue near $t = 0$.

In previous papers,[1,2] a form of nonlinear duality, abstracted originally from the multiperipheral model,[3,4] was used to calculate the Pomeranchukon residue at $t=0$. In the present paper, we consider $t\neq 0$. In $\pi\pi$ scattering below the 3ρ threshold, our duality condition takes on the form[2]

$$\int_{s_0}^{s_1} ds\left[\sum_i b_i(t)\nu^{\alpha_i(t)} - \sum_{c=\pi,R} a^c(s,t)\right] = 0\,, \quad (1)$$

where s, t, u are the usual Mandelstam variables, $\nu=\frac{1}{2}(s-u)/m_\pi{}^2$, and $b_i\nu^{\alpha_i}$ is the contribution of the Regge trajectory α_i, while a^π and a^R are the contributions of Figs. 1(a) and 1(b) to the absorptive part A. Here R stands for any accessible $\pi\pi$ resonance, and s_0 and s_1 are taken to lie at channel thresholds. In what follows we take $(s_0, s_1) = (4m_\rho{}^2, 6m_\rho{}^2)$, which coincide with the $\rho\rho$ and $N\bar{N}$ thresholds, respectively.[2]

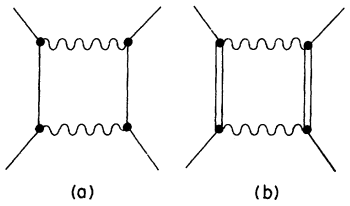

FIG. 1. (a) Unitarity diagram giving a^{π}. Single lines denote pions and wavy lines denote Regge exchanges. Only the $\pi\pi$ intermediate state is retained. (b) Unitarity diagram giving a^{R}. Double lines denote $\pi\pi$ resonances (R). Only the RR intermediate state is retained.

In Ref. 2, the functions b_ρ and b_f, which come into Eq. (1) along with b_P, were extracted from the Lovelace-Veneziano model. Recent investigations of dip structure by Harari[5] seem to show that the resonance distribution in such a model may not be sufficiently peripheral to be consistent with the data, however. One is led, instead, to a model in which A is dominated by the peripheral ($l \sim qr$) resonances. More specifically, we have

$$A \approx C(s)\, e^{B(s)t} J_0(r\sqrt{-t}\,) \,. \tag{2}$$

With $r \approx 1$ F, A has a zero at $|t| \simeq 0.2$ GeV^2. By contrast the Lovelace-Veneziano model gives a zero at $|t| \simeq 0.6$ GeV^2.

To calculate b_ρ, we use a finite-energy sum rule (FESR) in the t-channel isospin $I_t = 1$ state

$$\int_0^N d\nu [b_\rho(t)\nu^{\alpha_\rho(t)} - A_{\text{resonance}}(s,t)] = 0 \,, \tag{3}$$

where N is taken halfway between the f and g resonances. We assume that peripheral resonances continue to dominate even at low energies, so that $A_{\text{resonance}}$ is saturated by the ρ and f^0 (with $m_\rho = 765$ MeV, $\Gamma_\rho = 125$ MeV and $m_f = 1260$ MeV, $\Gamma_f = 140$ MeV), both of which fit in quite well with $l(l+1) \sim (qr)^2$. This means that we drop all daughter resonances such as the ρ' or ϵ, which in any case seem to be either absent or small in the most recent data. If we then take the usual $\alpha_\rho(t) = \alpha(t) \equiv \frac{1}{2}(1 + t/m_\rho{}^2)$, we find that Eq. (3) leads to a zero in b_ρ at $|t| = 0.29$ GeV^2, which would correspond to $r = 0.88$ F in Eq. (2). The corresponding value in Kp scattering is $r = 0.95$ F.[6]

To calculate b_f we could again use an FESR. The zero-moment sum rule might have fixed-pole contributions, however, while the first-moment rule weights the region near $\nu = N$ unduly. We shall therefore assume exchange degeneracy instead, which gives $\alpha_f = \alpha_\rho$ and $b_f = \frac{3}{2} b_\rho$. One gets essentially the same result using the zero-moment sum rule, which means that fixed poles are presumably negligible.

It now remains to evaluate a^{π} and a^{R}. Actually the latter does not have to be calculated explicitly if we assume that the horizontal lines of Fig. 1(b) are dominated by $I = 1$ exchanges, such as the π and A_2 trajectories (see Ref. 2). In that case, all the lines of Fig. 1(b) are $I = 1$ systems, since we are neglecting daughter resonances and can therefore only have $R = \rho$ in the region of interest. Simple isospin considerations then give

$$a^R_{I_t=0} = 4 a^R_{I_t=1} \,. \tag{4}$$

This differs from the corresponding Eq. (20) of Ref. 2, in which, however, R included a rather large ϵ component. If we now apply Eq. (1) in both the $I_t = 1$ state, where $i = \rho$, and in the $I_t = 0$ state, where $i = P, f$, we can use Eq. (4) to eliminate a^R completely. We then have

$$\int_{s_0}^{s_1} ds [b_\rho \nu^{\alpha_\rho} - (4b_\rho - b_f)\nu^{\alpha} + \tilde{a}^{\pi}] = 0 \,, \tag{5}$$

where $\tilde{a}^{\pi} = 5a^{\pi}_{I_s=2} - a^{\pi}_{I_s=1} - a^{\pi}_{I_s=0}$.

Since the horizontal lines of Fig. 1(a) are dominated by the ρ and f^0 Regge exchanges, we can calculate this diagram in terms of the $\pi\pi$ amplitudes $T_i = F(\alpha_i) b_i \nu^{\alpha_i}$, where $F(\alpha)$ is the appropriate signature factor. Now the dominant contribution to Fig. 1(a) comes from T_i near the forward direction, so we shall approximate $F(\alpha)$ by its value at $t = 0$. We also make the exponential approximation $b(t) \simeq b(0)\, e^{\gamma t}$, with $\gamma = (b'/b)_{t=0}$. If we now evaluate Fig. 1(a) for the various isospin states, making the usual asymptotic approximations and inserting a factor of two to take into account the contribution of the backward direction, we obtain, with $\alpha(0) = \frac{1}{2}$,

$$\tilde{a}^{\pi\pi} = 3 m_\pi{}^{-2} b_\rho{}^2(0) \omega_\rho{}^{-1} e^{\omega_\rho t/4} \,, \tag{6}$$

where $\omega_i = 2(\gamma_i + \alpha_i' \ln\nu)$. This turns out to give a rather small contribution to Eq. (5).

Before we can calculate $b_P(t)$ from Eq. (5), we need some kind of model for $\alpha_P(t)$. The simplest is to assume a flat P with $\alpha_P(t) = 1$. We then obtain $b_P(0) = 0.019$ and $\gamma_P \simeq 4.2$ GeV^{-2}. This corresponds to a high-energy cross section $\sigma_{\text{tot}} = 13$ mb and a differential diffraction-peak width $\omega_P = 8.3$ GeV^{-2}. Experimentally, $\sigma_{\text{tot}} = 15$ mb (using factorization) and $\omega_P^{pp} \approx 12.5$ GeV^{-2} at ISR (CERN Intersecting Storage Rings) energies.[7] Our calculated value therefore appears to be somewhat too small, since any simple multiperipheral model would give $\omega_P^{\pi\pi} = \omega_P^{\pi p} = \omega_P^{pp}$. However, factorization only predicts that $\omega^{\pi\pi} = 2\omega^{\pi p} - \omega^{pp}$. Since $\omega^{\pi p}$ is slightly smaller than ω^{pp} at lower energies, it would not be inconsistent to expect $\omega_P^{\pi\pi} < \omega_P^{pp}$ at higher energies. Finally we obtain $R_{\pi\pi} \approx 1.0$, where

$$R_{\pi\pi} = [b_P(0)/b_f(0)](\Lambda/m_\pi{}^2)^{\alpha_P(0) - \alpha_f(0)} \,,$$

with $\Lambda = 1$ GeV^2. This is consistent with the experimental values of $R_{\pi p}$ and R_{pp}, which are predicted to be equal to $R_{\pi\pi}$ in the f-coupled Pomeranchukon hypothesis.[8]

A more sophisticated model for $\alpha_P(t)$ might entail taking a structure from some multiperipheral model. Now for a broad class of such models the $I_t = 0$ projected absorptive part can be written as[9]

$$A(J, t) = \bar{b}(t)/D(J, t) , \tag{7}$$

where D has zeros at $J = \alpha_P$, α_f and a logarithmic branch point at $J = \alpha_c(t) \equiv 2\alpha_P(\frac{1}{4}t) - 1$, whose strength is governed by the triple-Pomeranchukon vertex.[10] We can therefore write a dispersion relation[11] in J with a double subtraction at $J = \alpha_f$:

$$D(J, t) = [J - \alpha(t)]\{c - [J - \alpha(t)]\sigma(J, t)\} , \tag{8}$$

where

$$\sigma(J, t) = \frac{1}{\pi}\int_{-\infty}^{\alpha_c} dJ' \frac{\mathrm{Im}D(J', t)}{[J' - \alpha(t)]^2(J' - J)} \tag{9}$$

and $c \approx 1$ if the effect of the cut is in fact a small perturbation, as we are assuming. In the immediate neighborhood of $J = \alpha_c$, then,

$$\sigma(J, t) \approx -\xi(t)\ln[J - \alpha_c(t)] , \tag{10}$$

where

$$32\pi\alpha_P'(0)\xi(0) \approx [\alpha_c(0) - \alpha(0)]^2 g_P^2(0) , \tag{11}$$

and $g_P(t)$ is the triple-Pomeranchukon vertex function, as defined in Ref. 10.

We have neglected any J dependence in the numerator of Eq. (7), an assumption which was checked explicitly in the case of a specific Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) model and found to be quite reasonable for $\frac{1}{2} \lesssim J \lesssim 1$. Moreover it can be shown that the addition of a certain amount of J dependence will not affect our results very much. From Eqs. (7) and (8) we see immediately that $\bar{b}(t) = b_f(t)$, irrespective of the detailed form of σ. The upper zero of D now gives us $\alpha_P(t)$;

$$D(\alpha_P(t), t) = 0 , \tag{12}$$

with

$$b_P(t) = \bar{b}(t) \Big/ \left[\frac{\partial}{\partial J} D(J, t)\right]_{J=\alpha_P} . \tag{13}$$

Since we are only interested in the properties of the f and P trajectories, we thus see that σ is needed only at $J = \alpha_P(t)$. This is indeed very close to $\alpha_c(t)$ when t is near the forward direction, and means that we can use the approximation (10).

If we now apply Eqs. (5), (6), (12), and (13) and their t derivatives at $t = 0$ we obtain $\alpha_P(0) = 0.985$, $\sigma_{\mathrm{tot}} = 13$ mb, $R_{\pi\pi} \simeq 1.0$, $\alpha_P'(0) = 0.06$ GeV^{-2}, and $\gamma_P = 4.0$ GeV^{-2}. The first three quantities are essentially the same as in the flat-P case discussed already. Our P slope is much smaller than the value $\alpha_P' = 0.5$ GeV^{-2} suggested by a simplistic interpretation of the Serpukhov data, although it is consistent with the ISR results, which suggest a much flatter P trajectory.[7] At the typical ISR energy squared of $s = 2000$ GeV^2 ($p_{\mathrm{lab}} \simeq 1000$ GeV/c), our γ_P gives the diffraction width $\omega_P^{\pi\pi} \simeq 9.0$ GeV^{-2}, which is again somewhat smaller than the experimental $\omega_P^{pp} \simeq 12.5$ GeV^{-2}.

As a by-product of our calculation, we obtain a triple-P coupling $g_P(0) \simeq 0.85$ GeV^{-1}. This is quite close to the value $g_P(0) \simeq 1$ GeV^{-1} calculated within a specific ABFST model in Ref. 10. It is also not too different from the "experimental" value $g_P(0) \approx 0.5$ GeV^{-1} extracted by Rajaraman,[12] although a cleaner measurement which is free of resonance complications will have to await higher-energy experiments.

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