PHYSICAL REVIEW D VOLUME 6, NUMBER 8 15 OCTOBER 1972

Unified Light-Cone Treatment of Scaling and a Positivity Constraint on Short-Distance Behavior

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(Received 24 April 1972)

We show that scaling in the two processes $e+\Pi(p)\to e+X$ and $e^+ + e^- \to \Pi(p)+X$ (where Π denotes a hadron) is controlled by the behavior near the light cone of the same operator product. This is done by studying the structure of the absorptive part of the forward virtual Compton scattering amplitude. This operator product must exhibit (at least) the singularity structure of the product of two electromagnetic currents. Under the assumption that this indeed is its leading singularity, we conclude that both processes must exhibit the same scaling behavior. It is then shown that positivity leads to a restriction on the short-distance behavior of these products. Under fairly general assumptions this leads to the result that the longitudinal structure function in the process $e^+ + e^- \to \Pi(p)+X$ may not vanish identically. The alternatives are that either local operators contribute on the light cone or that the transverse functions satisfy the lower bound $F_T \geq c\omega$ as ω approaches infinity.

I. INTRODUCTION

The relevance of the singularity on the light cone of the commutator of the electromagnetic current to the Bjorken[1] scaling limit in the process $e+\Pi(p)\to e+X$ (where Π denotes a hadron) is well known.[2] Until recently, however, a similar light-cone treatment for scaling in the annihilation process[3] $e^+ + e^- \to \Pi(p)+X$ was lacking. It was first pointed out by Ellis[4] that scaling in the annihilation process is controlled by the singularity on the light cone of a quadrilocal operator [see Eqs. (28), (29) below], and thus, by assuming the appropriate singularity structure, one obtains the expected scaling behavior. More recently, it was shown by Callan and Gross[5] that if one assumes a strict bilocal expansion for the current commutator one may show that the expected scaling for the annihilation process follows.

In this paper, we show under a fairly general assumption that the scaling behavior in both the scattering and annihilation processes must be identical. Our main tool is a covariant decomposition for the absorptive part of the forward virtual Compton scattering amplitude introduced by Bitar and Khuri.[6] We show from this decomposition, first, that the scattering process in the scaling region is controlled in reality by a quadrilocal operator of the type discussed by Ellis. This operator is the only part of the full absorptive part, namely the full commutator, that survives in the relevant physical region. We show, second, that the annihilation process in its scaling region is controlled by the same quadrilocal operator. If we assume that a single singularity controls the behavior of our operator near the light cone, we then conclude that both processes must exhibit the same scaling behavior. Since the full commutator coincides with our quadrilocal operator in the scattering region, we conclude that both operators have the same singularity, and hence that for both processes the scaling is that suggested by Bjorken[1] and experimentally verified for the scattering process.

This discussion covers Secs. II–IV. In Sec. V, we discuss briefly these quadrilocal operator products and show how one may, under the assumptions of Ref. 5 in particular, "derive" their structure from that of the full commutator.

In Sec. VI, we focus our attention on the positivity properties of our operators and derive a restriction on the short-distance behavior of our products. This is equivalent to the lower bound proved by Bitar and Khuri,[6] but appears here in a different form. It is then shown that in the absence of local contributions in the light-cone expansions either one of two possibilities might occur.

The first is the nonvanishing of the longitudinal structure function in the scaling region of the annihilation process. This would imply that the underlying field structure (partons) of the electromagnetic current is not of pure spin $\frac{1}{2}$. It is interesting to note here that our constraint does not hold for free-field theory and hence results based on such models are not a counterexample.

The second possibility occurs only in the presence of a strong short-distance singularity (beyond that given by the light cone) in the other structure function, for then the longitudinal function may vanish but the transverse function must increase at least linearly with ω as it approaches infinity.

Our analysis does not change the result of a constant multiplicity for the produced hadron in the annihilation process, noted by Callan and Gross.[5] This is so in spite of the possible linear increase in the variable ω of the transverse structure function.

II. STRUCTURE OF THE ABSORPTIVE PART FOR FORWARD VIRTUAL COMPTON SCATTERING

Consider the forward virtual Compton scattering amplitude off a hadronic target $\Pi(p)$ with momentum p and mass M. This is given by

$$M_{\mu\nu}(q,p) = -\int d^4x\, e^{iq\cdot x}\langle \Pi(p)|R(J_\mu(x);J_\nu(0))|\Pi(p)\rangle + \Sigma_{\mu\nu}(q) , \tag{1}$$

where $J_\mu(x)$ is the electromagnetic current, and $R(J_\mu(x);J_\nu(0))$ is the retarded product defined by

$$R(J_\mu(x);J_\nu(0)) = -i\theta(x_0)[J_\mu(x), J_\nu(0)] . \tag{2}$$

We shall consider the target to be spinless for simplicity; for a target with spin, an averaging over spin is to be understood. $\Sigma_{\mu\nu}(q)$ is determined by Schwinger terms and is a polynomial in q which contributes to M_{ij} only.

The absorptive part of $M_{\mu\nu}(q,p)$ may now be calculated using Eq. (2), leading to the familiar expression

$$W_{\mu\nu}(q,p) = 2\,\mathrm{Im}\, M_{\mu\nu} = \int d^4x\, e^{iq\cdot x}\langle \Pi(p)|[J_\mu(x), J_\nu(0)]|\Pi(p)\rangle . \tag{3}$$

One may now decompose this absorptive part into its connected and disconnected parts either by reducing the states $\Pi(p)$ in Eq. (3) directly or by doing the reduction first in Eq. (1) and then calculating $W_{\mu\nu}(q,p)$. We follow this latter prescription and proceed to reduce the states $\Pi(p)$ in Eq. (1). We use a complex scalar field $\Phi(z)$ for simplicity as an extrapolating field for these states. We find

$$\begin{aligned} M_{\mu\nu}(q,p) &= -\int d^4x\, e^{iq\cdot x}\langle 0|A_{\rm in}(p)R(J_\mu(x);J_\nu(0))|\Pi(p)\rangle + \Sigma_{\mu\nu}(q) \\ &= -\int d^4x\, e^{iq\cdot x}\langle 0|[A_{\rm in}(p), R(J_\mu(x);J_\nu(0))]|\Pi(p)\rangle \\ &\quad - \int d^4x\, e^{iq\cdot x}\langle 0|R(J_\mu(x);J_\nu(0))A_{\rm in}(p)|\Pi(p)\rangle + \Sigma_{\mu\nu}(q) . \end{aligned} \tag{4}$$

Following now the usual steps in the reduction formula, the first term leads to a three-operator retarded product.[7] The second term is just the disconnected part, since

$$A_{\rm in}(p')|\Pi(p)\rangle = (2\pi)^3 2p^0\delta^3(\vec{p}-\vec{p}\,') . \tag{5}$$

We obtain therefore

$$\begin{aligned} M_{\mu\nu}(q,p) &= \int d^4x\, e^{iq\cdot x} d^4y\, e^{ip\cdot y}[K_y\langle 0|R(J_\mu(x);J_\nu(0)\Phi(y))|\Pi(p)\rangle] \\ &\quad -\langle p|p\rangle \int d^4x\, e^{iq\cdot x}\langle 0|R(J_\mu(x);J_\nu(0))|0\rangle + \Sigma_{\mu\nu}(q) . \end{aligned} \tag{6}$$

The disconnected part is now distinctly separated. In Eq. (6), we use the notation $K_y = \Box + M^2$. Reducing the second target state, we end up with

$$\begin{aligned} M_{\mu\nu}(q,p) &= -\int d^4x\, d^4y\, d^4z\, e^{iq\cdot x}e^{ip\cdot y}e^{-ip\cdot z}K_yK_z\langle 0|R(J_\mu(x);J_\nu(0)\Phi(y)\Phi^*(z))|0\rangle \\ &\quad -\langle p|p\rangle\int d^4x\, e^{iq\cdot x}\langle 0|R(J_\mu(x);J_\nu(0))|0\rangle + \Sigma_{\mu\nu}(q) . \end{aligned} \tag{7}$$

We may now calculate the imaginary part $W_{\mu\nu}(q,p)$ by using the following expression[6]:

$$\begin{aligned} &R(J_\mu(x);J_\nu(0)\Phi(y)\Phi^*(z)) - R(J_\nu(0);J_\mu(x)\Phi(y)\Phi^*(z)) \\ &\quad = -i[J_\mu(x), R(J_\nu(0);\Phi(y)\Phi^*(z))] - i[R(J_\mu(x);\Phi(y)), R(J_\nu(0);\Phi^*(z))] \\ &\qquad -i[R(J_\mu(x);\Phi^*(z)), R(J_\nu(0);\Phi(y))] - i[R(J_\mu(x);\Phi(y)\Phi^*(z)), J_\nu(0)] . \end{aligned} \tag{8}$$

We thus obtain

$$W_{\mu\nu}(q,p)=\int d^4x\,d^4y\,d^4z\,e^{iq\cdot x}e^{ip\cdot y}e^{-ip\cdot z}$$
$$\times K_y K_z\,[\langle 0|J_\mu(x)R(J_\nu(0);\Phi(y)\Phi^*(z))|0\rangle-\langle 0|R(J_\nu(0);\Phi(y)\Phi^*(z))J_\mu(x)|0\rangle$$
$$+\langle 0|R(J_\mu(x);\Phi(y))R(J_\nu(0);\Phi^*(z))|0\rangle-\langle 0|R(J_\nu(0);\Phi^*(z))R(J_\mu(x);\Phi(y))|0\rangle$$
$$+\langle 0|R(J_\mu(x);\Phi^*(z))R(J_\nu(0);\Phi(y))|0\rangle-\langle 0|R(J_\nu(0);\Phi(y))R(J_\mu(x);\Phi^*(z))|0\rangle$$
$$+\langle 0|R(J_\mu(x);\Phi(y)\Phi^*(z))J_\nu(0)|0\rangle-\langle 0|J_\nu(0)R(J_\mu(x);\Phi(y)\Phi^*(z))|0\rangle]$$
$$-\langle p|p\rangle\int d^4x\,e^{iq\cdot x}\langle 0|[J_\mu(x),J_\nu(0)]|0\rangle\ . \tag{9}$$

The last term is clearly the disconnected-part contribution. The connected-part contribution is represented by the vacuum expectation values of the other eight operator products shown. If we insert a complete set of states in between these operators and use translational invariance, we may cast the contribution of each product to $W_{\mu\nu}(q,p)$ in a form where its support in momentum space is clearly visible. In this manner, we shall be able to identify those operator products relevant to our treatment of any one particular physical process involving high-mass virtual currents. Denoting the connected part by $W^c_{\mu\nu}(q,p)$, we obtain

$$W^c_{\mu\nu}(q,p)=(2\pi)^4\int d^4y\,d^4z\,e^{ip\cdot y}e^{-ip\cdot z}K_yK_z$$
$$\times\sum_N\{\delta^4(q-p_N)\langle 0|J_\mu(0)|N\rangle\langle N|R(J_\nu(0);\Phi(y)\Phi^*(z))|0\rangle$$
$$-\delta^4(q+p_N)\langle 0|R(J_\nu(0);\Phi(y)\Phi^*(z))|N\rangle\langle N|J_\mu(0)|0\rangle$$
$$+\delta^4(q+p-p_N)\langle 0|R(J_\mu(0);\Phi(y))|N\rangle\langle N|R(J_\nu(0),\Phi^*(z))|0\rangle$$
$$-\delta^4(q+p+p_N)\langle 0|R(J_\nu(0);\Phi^*(z))|N\rangle\langle N|R(J_\mu(0);\Phi(y))|0\rangle$$
$$+\delta^4(q-p-p_N)\langle 0|R(J_\mu(0);\Phi^*(z))|N\rangle\langle N|R(J_\nu(0);\Phi(y))|0\rangle$$
$$-\delta^4(q-p+p_N)\langle 0|R(J_\nu(0);\Phi(y))|N\rangle\langle N|R(J_\mu(0);\Phi^*(z))|0\rangle$$
$$+\delta^4(q-p_N)\langle 0|R(J_\mu(0);\Phi(y)\Phi^*(z))|N\rangle\langle N|J_\nu(0)|0\rangle$$
$$-\delta^4(q+p_N)\langle 0|J_\nu(0)|N\rangle\langle N|R(J_\mu(0);\Phi(y)\Phi^*(z))|0\rangle\}\ . \tag{10}$$

In terms of a diagrammatic decomposition, we can see that the first two and the last two terms represent the contribution from class-II states shown in Fig. 1(a) and are nonzero only for $q^2\geq(p_{N,\min})^2$, namely, $q^2>4m_\pi^2$. The fourth and fifth terms represent the contribution of the so-called Z graphs of the class-I states shown in Fig. 1(b). These contribute also only for timelike $q^2>0$ and as we shall see are relevant for the discussion of the process $e^++e^-\rightarrow\Pi(p)+X$ where X denotes anything (Fig. 2). Finally, the third and sixth terms represent the contributions of the so-called direct states of the class-I states shown in Fig. 1(c). These are the only terms which may contribute for spacelike $q^2<0$ and hence are the only terms relevant for the treatment of the scattering process $e+\Pi(p)\rightarrow e+X$ (Fig. 3). We discuss this point now in more detail.

III. DEEP-INELASTIC ELECTRON SCATTERING

As is well known, the process $e+\Pi(p)\rightarrow e+X$ may be described, under the assumption of one-photon exchange, by the structure function $W^c_{\mu\nu}(q,p)$ introduced above, with the restriction that q^2 is spacelike.

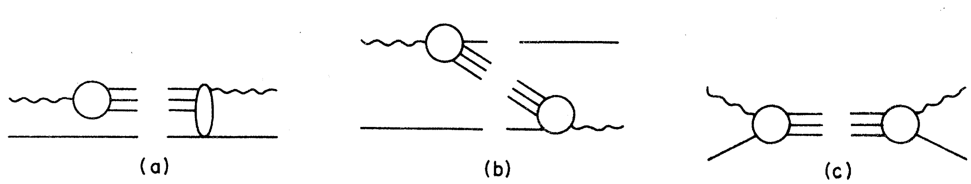

FIG. 1. Schematic representation of contributions to the absorptive part of the forward virtual Compton scattering amplitude. (a) Class-II states; (b), (c) Class-I states.

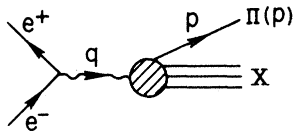

FIG. 2. One-photon-exchange approximation for the process $e^+ + e^- \to \Pi(p) + X$.

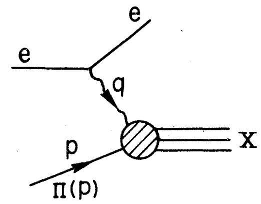

FIG. 3. One-photon-exchange approximation for the process $e + \Pi(p) \to e + X$.

Here q_μ is the momentum of the exchanged photon (see Fig. 3).

From the decomposition explicit in Eq. (10), we see that for spacelike $q^2<0$ only the third and sixth terms contribute. Thus we have

$$W^c_{\mu\nu(ep)}(q,p)=(2\pi)^4\sum_N\int d^4y\,d^4z\,e^{ip\cdot y}e^{-ip\cdot z}K_yK_z\,[\delta^4(q+p-p_N)\langle 0|R(J_\mu(0);\Phi(y))|N\rangle\langle N|R(J_\nu(0);\Phi^*(z))|0\rangle - \delta^4(q-p+p_N)\langle 0|R(J_\nu(0);\Phi(y))|N\rangle\langle N|R(J_\mu(0);\Phi^*(z))|0\rangle]\ . \tag{11}$$

Also, since $\nu>0$ ($q^0>0$), only the first term in Eq. (11) contributes. Upon inspecting Eq. (9), we may rewrite Eq. (11) as

$$W^c_{\mu\nu(ep)}(q,p)=\int d^4x\,d^4y\,d^4z\,e^{iq\cdot x}e^{ip\cdot y}e^{-ip\cdot z}\times K_yK_z\{\langle 0|R(J_\mu(x);\Phi(y))R(J_\nu(0);\Phi^*(z))|0\rangle-\langle 0|R(J_\nu(0);\Phi(y))R(J_\mu(x);\Phi^*(z))|0\rangle\}\ . \tag{12}$$

Thus it is evident from this equation that for $q^2<0$, $\nu>0$ the behavior of $W^c_{\mu\nu(ep)}(q,p)$ in the Bjorken scaling limit (B) ($q^2\to-\infty$, $\nu\to+\infty$, and $\omega=-q^2/2\nu$ fixed) is controlled by the singularity on the light cone of the operator product:

$$A^{(1)}_{\mu\nu}(x,p)=\int d^4y\,d^4z\,e^{ip\cdot y}e^{-ip\cdot z}K_yK_z\langle 0|R(J_\mu(x);\Phi(y))R(J_\nu(0);\Phi^*(z))\,|0\rangle\ . \tag{13}$$

Referring to Fig. 4, we note that the physical region for the process $e+\Pi(p)\to e+X$ is region I′, bounded by the lines $q^2=-2\nu$ and $q^2=0$ for $\nu>0$. The operator product $A^{(1)}_{\mu\nu}(x,p)$ contributes however also in regions II and III. Therefore, if we assume that a single light-cone singularity determines the behavior in the scaling limit irrespective of whether q^2 is timelike or spacelike, the scaling behavior of the contribution of $A^{(1)}_{\mu\nu}(x,p)$ is the same in all these regions. Thus its support in the variable ω is $-\infty<\omega<+1$ for $\nu>0$. The physical region is of course limited to $0<\omega<1$.

The second product in Eq. (12) contributes in regions I′, II′, and III′. This product is obtained from $A^{(1)}_{\mu\nu}(x,p)$ by the exchange $\mu\leftrightarrow\nu$, $0\leftrightarrow x$ (i.e., $x\to-x$). Denote this term by $A^{(2)}_{\mu\nu}(x,p)$.[8] We have then

$$A^{(2)}_{\mu\nu}(x,p)\equiv-A^{(1)}_{\nu\mu}(-x,p)\ . \tag{14}$$

Therefore, $A^{(2)}_{\mu\nu}(x,p)$ has the *same* singularity on the light cone as $A^{(1)}_{\mu\nu}(x,p)$. We conclude then that $W^c_{\mu\nu(ep)}(q,p)$ displays in all these three unphysical regions the same scaling behavior. Here the support in ω is for $\nu<0$; $-1<\omega<\infty$.

The structure of $A^{(1)}_{\mu\nu}(x,p)$ near the light cone ($x^2\to0$), namely the current commutator matrix element for $\nu>0$, $q^2<0$, in relation to the observed scaling behavior, has been discussed by many authors.[2]

Let us refer to $W^c_{\mu\nu(ep)}$ simply by $W_{\mu\nu}(q,p)$ and decompose it in momentum space as follows:

$$W_{\mu\nu}(q,p)=\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)W_1(q^2,\nu)+\left(p_\mu-\nu\frac{q_\mu}{q^2}\right)\left(p_\nu-\nu\frac{q_\nu}{q^2}\right)W_2(q^2,\nu)\ . \tag{15}$$

In the Bjorken scaling limit (B), the following behavior, first suggested by Bjorken, seems to be consistent with experimental observations:

$$\lim_B W_1(\nu,q^2)=F_1(\omega)\ ,\qquad \lim_B \nu W_2(\nu,q^2)=F_2(\omega)\ . \tag{16}$$

Defining the longitudinal structure function $W_L(q^2,\nu)$ by

$$W_L(q^2,\nu)=W_1+\frac{\nu^2}{q^2}W_2\ , \tag{17}$$

one then has

$$\lim_{B} W_L(q^2,\nu)=F_L(\omega)=F_1(\omega)-\frac{1}{2\omega}F_2(\omega)\ . \tag{18}$$

Let us correspondingly decompose $A_{\mu\nu}^{(1)}(x,p)$ in position space as follows:

$$A_{\mu\nu}^{(1)}(x,p)=(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)A_1^{(1)}(x^2,x\cdot p)+[p_\mu p_\nu-p\cdot\partial(p_\mu\partial_\nu+p_\nu\partial_\mu)+(p\cdot\partial)^2 g_{\mu\nu}]A_2^{(1)}(x^2,x\cdot p)\ . \tag{19}$$

It has been shown by various authors[2] that the scaling behavior of Eq. (16) is consistent with the following structure for the functions $A_1^{(1)}(x^2,x\cdot p)$ and $A_2^{(2)}(x^2,x\cdot p)$ near the light cone:

$$\begin{aligned}A_1^{(1)}(x^2,x\cdot p)&=\left(\frac{1}{-x^2+i\epsilon x_0}\right)f_1(x^2,x\cdot p)+\text{less singular terms},\\ A_2^{(1)}(x^2,x\cdot p)&=\ln(-x^2+i\epsilon x_0)f_2(x^2,x\cdot p)+\text{l.s.t.}\end{aligned} \tag{20}$$

$f_1(x^2,x\cdot p)$ and $f_2(x^2,x\cdot p)$ are assumed to be nonsingular as x^2 approaches zero. They determine the functions $F_1(\omega),F_2(\omega)$ and consequently $F_L(\omega)$. In particular, if we define $g_i(\alpha)$ as

$$g_i(\alpha)=\frac{1}{2\pi}\int d(x\cdot p)e^{-i(x\cdot p)\alpha}f_i(0,x\cdot p),\qquad i=1,2 \tag{21}$$

then

$$\frac{1}{\omega}F_L(\omega)=4\pi^2 i\int d\alpha g_1(\alpha)\frac{1}{\omega-\alpha+i\epsilon}=4\pi^2\int d\eta e^{-i\eta\omega}f_1(0,\eta)\theta(-\eta)$$

and (22)

$$\frac{1}{\omega}F_2(\omega)=36\pi^2 i\int d\alpha g_2(\alpha)\frac{1}{(\omega-\alpha+i\epsilon)^2}\ ,$$

where $\nu<0$; $-\infty<\omega<1$.

$A_{\mu\nu}^{(2)}(x,p)$ has an expansion similar to Eq. (19), where one replaces $A_1^{(1)}(x^2,x\cdot p)$ and $A_2^{(1)}(x^2,x\cdot p)$ by

$$\begin{aligned}A_1^{(2)}(x^2,x\cdot p)&=\frac{-1}{-x^2-i\epsilon x_0}f_1(x^2,-x\cdot p)+\text{l.s.t.}\ ,\\ A_2^{(2)}(x^2,x\cdot p)&=-\ln(-x^2-i\epsilon x_0)f_2(x^2,-x\cdot p)+\text{l.s.t.}\end{aligned} \tag{23}$$

One consequently has a scaling behavior in the unphysical regions I′, II′, and III′ similar to that of Eqs. (16) and (18).

In particular, one also has

$$\frac{1}{\omega}F_L'(\omega)=-4\pi^2 i\int d\alpha g_1(-\alpha)\frac{1}{\omega-\alpha-i\epsilon}=4\pi^2\int d\eta e^{i\omega\eta}f_1(0,\eta)\theta(-\eta)$$

and (24)

$$\frac{1}{\omega}F_2'(\omega)=-36\pi^2 i\int d\alpha g_2(-\alpha)\frac{1}{(\omega-\alpha-i\epsilon)^2}\ ,$$

where $\nu<0$; $\infty>\omega>-1$.

Comparing Eq. (24) with Eq. (22) we find

$$\begin{aligned}F_L'(-\omega)&=-F_L(\omega)\ ,\\ F_2'(-\omega)&=F_2(\omega)\ .\end{aligned} \tag{25}$$

Thus, referring to Fig. 4, we see that the functions are equal and opposite in sign at points that are mirror reflections of each other about the q^2 axis, i.e., $\nu=0$.

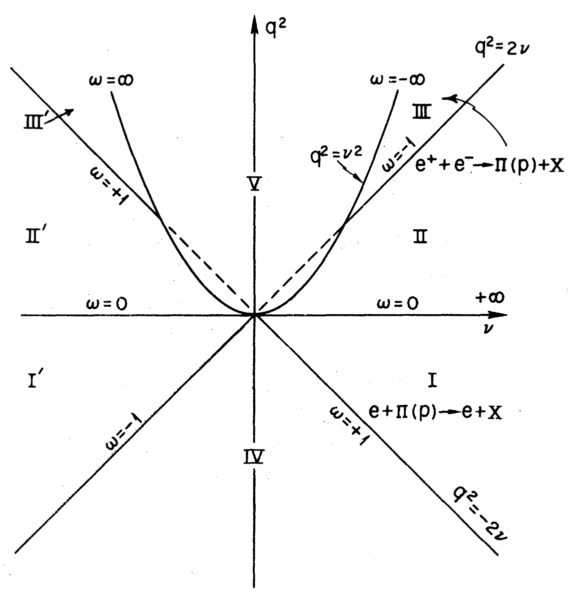

FIG. 4. The (ν, q^2) plane.

IV. THE ANNIHILATION PROCESS $e^+ + e^- \to \Pi(p) + X$

To the order that this process is mediated by a single timelike photon one may summarize the hadron structure being probed by the structure functions defined by

$$\overline{W}_{\mu\nu}(q,p) = \sum_N (2\pi)^4 \{\delta^4(q-p-p_N)\langle 0|J_\mu(0)|\Pi(p)N\rangle\langle\Pi(p)N|J_\nu(0)|0\rangle - \delta^4(q+p+p_N)\langle 0|J_\nu(0)|\Pi(p)N\rangle\langle\Pi(p)N|J_\mu(0)|0\rangle\} \tag{26}$$

and

$$\overline{W}_{\mu\nu}(q,p) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\overline{W}_1(q^2,\nu) + \left(p_\mu - \nu\frac{q_\mu}{q^2}\right)\left(p_\nu - \nu\frac{q_\nu}{q^2}\right)\overline{W}_2(q^2,\nu) . \tag{27}$$

The physical region for the process is denoted by III in Fig. 4. Only the first term in Eq. (26) contributes to it; the second term contributes for $\nu < 0$ only and is restricted to the unphysical region III′ of Fig. 4.

Reducing the hadron states in Eq. (26), one obtains

$$\overline{W}_{\mu\nu}(q,p) = \int d^4x\, d^4y\, d^4z\, e^{iq\cdot x} e^{ip\cdot y} e^{-ip\cdot z} K_y K_z \times\{\langle 0|R(J_\mu(x);\Phi^*(z))R(J_\nu(0);\Phi(y))|0\rangle - \langle 0|R(J_\nu(0);\Phi^*(z))R(J_\mu(x);\Phi(y))|0\rangle\} . \tag{28}$$

These are just the fourth and fifth terms in Eq. (9) and are graphically represented in Fig. 1(b). It is clear from Eq. (28) that the behavior of $\overline{W}_{\mu\nu}(q,p)$ in the scaling limit $(B')\, q^2 \to +\infty$ (timelike), $\nu \to +\infty$, and $\omega = -q^2/2\nu$ fixed is controlled by the singularity on the light cone of the operator product[4]:

$$B^{(1)}_{\mu\nu}(x,p) = \int d^4y \int d^4z\, e^{ip\cdot y} e^{-ip\cdot z} K_y K_z \langle 0|R(J_\mu(x);\Phi^*(z))R(J_\nu(0);\Phi(y))|0\rangle . \tag{29}$$

Using charge conjugation, we then find

$$B^{(1)}_{\mu\nu}(x,p) = \int d^4y\, d^4z\, e^{ip\cdot y} e^{-ip\cdot z} K_y K_z \langle 0|R(J_\mu(x);\Phi(z))R(J_\nu(0);\Phi^*(y))|0\rangle = A^{(1)}_{\mu\nu}(x,-p) . \tag{30}$$

Moreover, we have, by using the combined operations of parity and time reversal,[9]

$$B^{(1)}_{\mu\nu}(x,p) = A^{(1)}_{\mu\nu}(-x,p) . \tag{31}$$

It follows then that

$$W_{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} A^{(1)}_{\mu\nu}(x,p)$$

and (32)

$$\overline{W}_{\mu\nu}(q,p) = \int d^4x\, e^{-iq\cdot x} A^{(1)}_{\mu\nu}(x,p) .$$

Thus, the same operator controls the scaling behavior of both structure functions. If one now assumes that there is one single leading light-cone singularity in $A^{(1)}_{\mu\nu}(x,p)$, namely that observed for spacelike q^2 and given in Eqs. (19) and (20), then one may conclude that both structure functions must exhibit the same scaling behavior. The assumption above is necessary in spite of the apparent simplicity of Eq. (32). This is so because $W_{\mu\nu}(q,p)$ and $\overline{W}_{\mu\nu}(q,p)$ have, as we have seen, different support properties in momentum space. Thus, if $A^{(1)}_{\mu\nu}(x,p)$ has a singularity stronger than that of Eq. (20) but whose Fourier transform has support for timelike $q^2 > 0$ only, $\overline{W}_{\mu\nu}$ may scale differently from $W_{\mu\nu}$. We shall here assume that the leading light-cone singularity of $A^{(1)}_{\mu\nu}(x,p)$ is that observed in the scattering process and displayed in Eq. (20). Let us recall that this singularity is the maximum allowed from considerations of scale invariance and canonical dimensionality, so that the presence of more singular terms contributing exclusively for timelike $q^2 > 0$ could lead to a violation of either of these principles.

We may write from Eqs. (19) and (20) and Eq. (23)

$$B^{(1)}_{\mu\nu}(x,p) = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\frac{1}{-x^2 + i\epsilon x_0} f_1(x^2, -x\cdot p) + \text{l.s.t.} + [p_\mu p_\nu \Box - p\cdot\partial(p_\mu\partial_\nu + p_\nu\partial_\mu) + (p\cdot\partial)^2 g_{\mu\nu}]\ln(-x^2 + i\epsilon x_0) f_2(x^2, -x\cdot p) + \text{l.s.t.} \tag{33}$$

In the scaling limit, we necessarily must have then

$$\lim_{B'} \overline{W}_1(\nu, q^2) = \overline{F}_1(\omega) , \qquad \lim_{B'} \nu \overline{W}_2(\nu, q^2) = \overline{F}_2(\omega) , \tag{34}$$

and

$$\lim_{B'} \overline{F}_L(\nu, q^2) = \overline{W}_1 + \frac{\nu^2}{q^2} \overline{W}_2 = \overline{F}_L(\omega) . \tag{35}$$

In the above, ω spans the physical region $-\infty < \omega < -1$ (region III in Fig. 4). One also has

$$\begin{aligned} \frac{1}{\omega} \overline{F}_L(\omega) &= 4\pi^2 i \int d\alpha\, g_1(-\alpha) \frac{1}{\omega - \alpha + i\epsilon} = 4\pi^2 \int d\eta\, e^{i\omega\eta} f_1(0,\eta)\theta(\eta) , \\ \frac{1}{\omega} \overline{F}_2(\omega) &= 36\pi^2 i \int d\alpha\, g_2(-\alpha) \frac{1}{(\omega - \alpha + i\epsilon)^2} . \end{aligned} \tag{36}$$

The second term in Eq. (28) may be obtained from the first by the interchange $\mu \leftrightarrow \nu$ and $x \leftrightarrow 0$ $(x \to -x)$. Thus, it also has the same singularity on the light cone. Therefore, $\overline{W}_{\mu\nu}(q,p)$ displays the same behavior in the scaling limit (B') in the unphysical region III′ of Fig. 4. Denoting this term by $B^{(2)}_{\mu\nu}(x,p)$ we obtain an expression similar to Eq. (29), namely,

$$\begin{aligned} B^{(2)}_{\mu\nu}(x,p) &= (\partial_\mu \partial_\nu - g_{\mu\nu}\Box) \frac{-1}{-x^2 - i\epsilon x_0} f_1(x^2, x\cdot p) \\ &\quad + [p_\mu p_\nu \Box - p\cdot\partial(p_\mu\partial_\nu + p_\nu\partial_\mu) + (p\cdot\partial)^2 g_{\mu\nu}](-)\ln(-x^2 - i\epsilon x_0) f_2(x^2, x\cdot p) . \end{aligned} \tag{37}$$

One also has in region III′

$$\frac{1}{\omega}\overline{F}'_L(\omega) = -4\pi^2 i \int d\alpha\, g_1(\alpha) \frac{1}{\omega - \alpha - i\epsilon} = 4\pi^2 \int d\eta\, e^{-i\omega\eta} f_1(0,\eta)\theta(\eta)$$

and (38)

$$\frac{1}{\omega} \overline{F}'_2(\omega) = -36\pi^2 i \int d\alpha\, g_2(\alpha) \frac{1}{(\omega - \alpha - i\epsilon)^2} .$$

Thus again comparing Eq. (38) with Eq. (36), we obtain the symmetry about the q^2 axis:

$$\overline{F}'_L(-\omega) = -\overline{F}_L(\omega) , \qquad \overline{F}'_2(-\omega) = \overline{F}_2(\omega) . \tag{39}$$

V. OPERATOR-PRODUCT EXPANSIONS ON THE LIGHT CONE

The central operator in our discussion is $A^{(1)}_{\mu\nu}(x,p)$. Its behavior near the light cone was introduced "phenomenologically" in the sense that the singularities in Eq. (20) are consistent with the observed scaling in the deep-inelastic region of $e + \Pi(p) \to e + X$. These singularities are identical with those of the product of two electromagnetic currents and, as we remarked above, are consistent with general considerations of scale invariance and canonical dimensionality. As we have also seen above, if this is the leading singularity in $A^{(1)}_{\mu\nu}(x,p)$ the process $e^+ + e^- \to \Pi(p) + X$ displays an identical scaling behavior to the scattering process.

It is of interest therefore to study the singularity structure of $A^{(1)}_{\mu\nu}(x,p)$ in models. This may be done, of course, by using the explicit form in Eq. (13), but is not a simple matter. An indirect method is that of Callan and Gross,[5] which is applicable to theories where the current commutator has a strict bilocal expansion. For if we have

$$[J_\mu(x), J_\nu(0)] = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\epsilon(x^0)\delta(x^2)[\psi(x), \psi(0)] + \text{l.s.t.} , \tag{40}$$

we may then, by using an expression similar to Eq. (8), calculate the commutator $[R(J_\mu(x); \Phi^*(z)), R(J_\nu(0); \Phi(y))]$, for example. One obtains

$$\begin{aligned} &\int d^4y\, d^4z\, e^{ipy} e^{-ipz} K_y K_z \langle 0 | [R(J_\mu(x); \Phi^*(z)), R(J_\nu(0); \Phi(y))] | 0\rangle \\ &\quad = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\epsilon(x^0)\delta(x^2) \left\{ \int d^4y\, d^4z\, e^{ipy} e^{-ipz} K_y K_z \langle 0 | [R(\psi(x); \Phi^*(z)), R(\psi(0); \Phi(y))] | 0\rangle \right\} + \text{l.s.t.} \end{aligned} \tag{41}$$

Thus, assuming that no further singularity is obtained from the integration on the right-hand side, we find that the singularity of the full commutator on the light cone is the leading singularity in our commutator as well. In other words, $A^{(1)}_{\mu\nu}(x,p)$, in this general class of theories, has only a single leading light-cone singularity, namely, that of the full commutator measured in the scattering process for spacelike $q^2<0$.

VI. POSITIVITY CONSTRAINT ON SHORT-DISTANCE BEHAVIOR AND NONVANISHING OF LONGITUDINAL SCALING FUNCTION

Consider the contribution of class-I states shown in Figs. 1(a) and 1(b), namely the contribution of the third through sixth terms in Eq. (9). We have

$$W^{I}_{\mu\nu}(q,p)=W_{\mu\nu}(q,p)+\overline{W}_{\mu\nu}(q,p)$$
$$=\int d^4x\,d^4y\,e^{iq\cdot x}e^{ip\cdot y}e^{-ipz}K_yK_z$$
$$\times\{\langle 0|[R(J_\mu(x);\Phi(y)),R(J_\nu(0);\Phi^*(z))]|0\rangle+\langle 0|[R(J_\mu(x);\Phi^*(z)),R(J_\nu(0);\Phi(y))]|0\rangle\}\ . \tag{42}$$

Thus

$$W^{\rm I}_{\mu\nu}(q,p)=\int d^4x\,e^{iq\cdot x}C_{\mu\nu}(x^2,x\cdot p)\ . \tag{43}$$

$C_{\mu\nu}(x^2,x\cdot p)$ may be constructed from the expression for $A^{(1)}_{\mu\nu}(x,p)$ by adding the various terms $A^{(2)}_{\mu\nu}$, $B^{(1)}_{\mu\nu}$, and $B^{(2)}_{\mu\nu}$ introduced above. We obtain

$$C_{\mu\nu}(x^2,x\cdot p)=(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)\pi\epsilon(x^0)\delta(x^2)F_1(x^2,x\cdot p)+\text{l.s.t.}$$
$$+[p_\mu p_\nu\Box+(p\cdot\partial)(p_\mu\partial_\nu+p_\nu\partial_\mu)+(p\cdot\partial)^2g_{\mu\nu}]\,\pi\epsilon(x^0)\theta(x^2)F_2(x^2,x\cdot p)+\text{l.s.t.}\ , \tag{44}$$

where

$$F_{1,2}(x^2,x\cdot p)=f_{1,2}(x^2,x\cdot p)+f_{1,2}(x^2,-x\cdot p)\ . \tag{45}$$

Let us consider $W^{\rm I}_{00}(q,p)$. From Eq. (10) we find

$$W^{\rm I}_{00}(q,p)=(2\pi)^4\sum_N\Big[\delta(q^0+p^0-p^0_N)\Big|\int e^{ipy}\,d^4y\,K_y\langle 0|R(J_0(0);\Phi(y))|N,\vec{q}+\vec{p}\rangle\Big|^2$$
$$+\delta(q^0-p^0-p^0_N)\Big|\int e^{-ipz}\,d^4zK_z\langle 0|R(J_0(0);\Phi^*(z))|N,\vec{q}-\vec{p}\rangle\Big|^2$$
$$-\delta(q^0+p^0+p^0_N)\Big|\int e^{-ipz}\,d^4z\,K_z\langle 0|R(J_0(0);\Phi^*(z))|N,-\vec{q}-\vec{p}\rangle\Big|^2$$
$$-\delta(q^0-p^0+p^0_N)\Big|\int e^{ipy}\,d^4y\,K_y\langle 0|R(J_0(0);\Phi(y))|N,\vec{q}+\vec{p}\rangle\Big|^2\Big]\ . \tag{46}$$

We thus see that at any value of $\vec{q}$ and $\vec{p}$, $W^{\rm I}_{00}$ satisfies the following positivity condition:

$$W^{\rm I}_{00}>0,\quad q^0>0\ , \tag{47}$$
$$W^{\rm I}_{00}<0,\quad q^0<0\ . \tag{48}$$

Let us for convenience decompose $W^{\rm I}_{\mu\nu}$ as follows:

$$W^{\rm I}_{\mu\nu}(q,p)=(q_\mu q_\nu-g_{\mu\nu}q^2)M_1(q^2,\nu)+[q^2p_\mu p_\nu-(q\cdot p)(q_\mu p_\nu+q_\nu p_\mu)+(q\cdot p)^2g_{\mu\nu}]M_2(q^2,\nu)\ . \tag{49}$$

We then have

$$M_1(q^2,\nu)=-\int d^4x\,e^{iq\cdot x}\pi\epsilon(x^0)\delta(x^2)F_1(x^2,x\cdot p)\ ,$$
$$M_2(q^2,\nu)=-\int d^4x\,e^{iq\cdot x}\pi\epsilon(x^0)\theta(x^2)F_2(x^2,x\cdot p)\ . \tag{50}$$

Using Eq. (49) for $\vec{p}=0$, $p^0=m$, we find from Eqs. (47) and (48) the positivity condition valid for all $\vec{q}$:

$$(M_1-m^2M_2)>0,\quad q^0>0$$
$$(M_1-m^2M_2)<0,\quad q^0<0\ . \tag{51}$$

Define

$$C(x^2,x\cdot p)=-\pi[\epsilon(x^0)\delta(x^2)F_1(x^2,x\cdot p)-m^2\epsilon(x^0)\theta(x^2)F_2(x^2,x\cdot p)]\ , \tag{52}$$

then

$$C(x^2, x\cdot p) = \frac{1}{(2\pi)^4}\int e^{-iq\cdot x}(M_1 - m^2 M_2) . \quad (53)$$

Consider then

$$\partial_0 C(x^2, x\cdot p)|_{x_\mu=0} = \frac{-i}{(2\pi)^4}\int dq^0 q^0 \int d^3q(M_1 - m^2 M_2) . \quad (54)$$

From Eq. (51), we then find

$$\partial_0 C(x^2, x\cdot p)|_{x_\mu=0} = -ir, \quad r>0 . \quad (55)$$

r may vanish only if $W^{\rm I}_{00}$ is identically zero, which is not the case for a theory with interactions.

From Eq. (52), we find that if $F_2(x^2, x\cdot p)$ is nonsingular as $x_\mu \to 0$ we must have (see Appendix A)

$$F_1(0,0) = ir/2\pi^2, \quad r \neq 0 \quad (56)$$

which then implies

$$f_1(0,0) = ir/4\pi^2, \quad r\neq 0 . \quad (57)$$

This result would have followed directly had we applied the above analysis to either the first or second commutator in Eq. (42) since these obey the positivity condition separately.

The important consequence of Eq. (57) follows immediately from Eq. (36), as then we have

$$\frac{1}{\omega}\overline{F}_L(\omega) \propto if_1(0,0)/\omega \quad \text{as } \omega\to\infty , \quad (58)$$

so that

$$\lim_{\omega\to\infty}\overline{F}_L(\omega) = i4\pi^2 f_1(0,0) \neq 0 . \quad (59)$$

Therefore the longitudinal scaling structure function for the process $e^+ + e^- \to \Pi(p) + X$ may *not* vanish identically. The significance of this result is of course that the underlying field (parton) constituents of the electromagnetic current are not pure spin-$\frac{1}{2}$.

Of course, similar statements may be made for the rest of the structure functions, in particular $F_L(\omega)$. But, in this case, the limit $\omega\to\infty$ takes us outside the physical region.

A slightly weaker version of the result of Eq. (59) is obtained by considering for example the sum of Eq. (36) and Eq. (24). We obtain

$$\frac{1}{\omega}[\overline{F}_L(\omega) + F'_L(\omega)] = 4\pi^2\int e^{i\omega\eta} f_1(0,\eta)d\eta$$
$$= (2\pi)^3 g_1(-\omega) . \quad (60)$$

Inverting Eq. (60) and using the support properties of $\overline{F}_L(\omega)$ and $F_L(\omega)$ and the symmetry relationship of Eq. (25) we find

$$f(0,0) = \frac{1}{8\pi^3}\left[\int_{-\infty}^{-1}\frac{d\omega}{\omega}\overline{F}_L(\omega) + \int_{-\infty}^{+1}\frac{d\omega}{\omega}F_L(\omega)\right] \neq 0 . \quad (61)$$

Thus $\overline{F}_L(\omega)$ or $F_L(\omega)$ must be nonzero over a finite range in ω. This may of course be satisfied by $F_L(\omega)$ nonzero outside its physical region, but in view of Eq. (59) and a similar equation for $F_L(\omega)$, it is satisfied by $\overline{F}_L(\omega)$ and/or $F_L(\omega)$ nonzero for regions of finite ω.

The nonvanishing of $f_1(0,0)$, and hence of $\overline{F}_L(\omega)$, is not the only solution to the positivity restriction of Eq. (55). Two other possibilities exist.

The first possibility is the existence of a local contribution in $A^{(1)}_{\mu\nu}(x,p)$ of the form

$$A^{(1)}_{\mu\nu}(x,p) = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\frac{c(0)}{-x^2 + i\epsilon x_0} + (\text{Rest}) . \quad (62)$$

Equation (55) eventually leads to the statement that $c(0)\neq 0$. Such a local contribution shows up as a contribution of the form $\delta(1/\omega)$ in the longitudinal structure functions and therefore escapes detection. In this case, $f_1(0,0)$ may vanish, and so could $\overline{F}_L(\omega)$ and the other structure functions. However, the presence of such local contributions in light-cone expansions is not characteristic of currents bilinear in fermion fields. They do show up, however, in theories where scalar fields are present (σ model, for example), and so could again be an indication of the presence of partons (underlying fields) that are not of spin $\frac{1}{2}$.

The second possibility is that, even with a structure as in Eq. (45), $F_2(x^2, x\cdot p)$ may contribute to Eq. (55). This is possible only if $F_2(0,\eta)$ is singular as $\eta\to 0$. This, of course, is only a short-distance singularity. It would not alter the light-cone singularity nor the resulting scaling behavior. For such a contribution to occur, and hence allow $f_1(0,0)$ to vanish, one must have

$$f_2(0,\eta) \underset{\eta\to 0}{\sim} \delta''(\eta) + \text{l.s.t.} \quad (63)$$

Such a behavior leads to the following lower bound for the behavior of the structure function $F_{1,2}(\omega)$ and $\overline{F}_{1,2}(\omega)$ in the limit $\omega\to\infty$. We have from Eqs. (22) and (36)

$$\left.\begin{array}{l} F_2(\omega),\ \overline{F}_2(\omega) \geq c\omega^2 \\ F_1(\omega),\ \overline{F}_1(\omega) \geq c\omega \end{array}\right\} \text{for large } \omega . \quad (64)$$

This behavior for $\overline{F}_{1,2}(\omega)$ is easily checked experimentally and is in contrast to the expectation that $\overline{F}_1(\omega) = O(1/\omega)$, $F_2(\omega) = \text{constant}$ as $\omega\to\infty$ if $f_2(0,\eta)$ is regular for $\eta = 0$. We must, at this point, remark that such a singular behavior for $f_2(0,\eta)$, which is present in the full current commutator, is not in accord with general considerations of scale invariance and canonical dimensionality which would imply a regular behavior. Thus, this last possibility would imply the breakdown of either one of these principles along with the behavior of Eq. (64).

Finally, from the analysis of Callan and Gross we conclude, as they do, that in spite of Eq. (64),

$$\sigma_{\rm tot}(e^+e^-)\times \text{multiplicity } (\Pi(p)) \geq \text{constant}/q^2 \quad \text{for large } q^2 .$$

Thus, if $\sigma_{\rm tot}(e^+e^-)\propto 1/q^2$ for large q^2, then the multiplicity of the hadron $\Pi(p)$ is bounded below by a constant.

VII. CONCLUSION AND DISCUSSION

We have shown that scaling in the two processes $e+\Pi(p)\to e+X$ and $e^++e^-\to\Pi(p)+X$ is controlled by the behavior near the light cone of the same operator, namely $A^{(1)}_{\mu\nu}(x,p)$ of Eq. (13). We have also shown that this product must exhibit, at least, the singularity structure of the product of two electromagnetic currents. We saw then that if we assume that this is the leading singularity in $A^{(1)}_{\mu\nu}(x,p)$ both processes must exhibit the same scaling behavior.

It was shown then that for a fairly general form for the behavior of $A^{(1)}_{\mu\nu}(x,p)$ near the light cone, a positivity constraint on the short-distance behavior implies the nonvanishing of the longitudinal scaling structure function in the process $e^++e^-\to\Pi(p)+X$. Free field theory results, e.g., Ref. 5, are not a counterexample. They indeed violate this constraint and should not in this case be taken seriously, for, in this case, $W^{\rm I}_{00}$ is zero except for the residues of the poles corresponding to the Born poles. If these residues are calculated using Eq. (46), it is found that the result is not gauge-invariant. Upon adding contact terms to ensure gauge invariance, the positivity property is lost. This loss of positivity may be verified directly by calculating the absorptive part of the gauge-invariant contribution of the sum of the direct and crossed Born terms for a free Dirac field (see Appendix B).

Our short-distance constraint is the same as that discussed by Bitar and Khuri in Ref. 6. It was shown there that the Jin-Martin lower bound applied to the contribution of Class-I states to the forward virtual Compton amplitude leads to the presence of a Schwinger-like term in $C_{\mu\nu}(x,p)$. This Schwinger term will be absent in the current commutator only if there is a canceling term from the Class-II contributions. In the present discussion, the Class-II states do not interfere with our analysis; we are only concerned with the Class-I states. Thus, we must have

$$C_{0i}(x,p)|_{x^0=0}=ir\partial_i\delta^3(\vec{x}) . \tag{65}$$

From Eq. (44) we see that this implies our result Eq. (56).

Consider now the possibility that $C_{\mu\nu}(x,p)$ has singularities stronger than those of Eq. (44) but contributing only for timelike $q^2>0$. In this case, scaling in the annihilation process $e^++e^-\to\Pi(p)+X$ will be different from that of the scattering process and determined by the strength of the singularity. Also, in this case, one should definitely expect the longitudinal structure function not to vanish. In this case, $\nu^\alpha\overline{W}_2$ and $\nu^{\alpha-1}\overline{W}_1$ will scale, where $\alpha>1$ is the strength of a singularity of the form

$$A^{(1)}_{\mu\nu}(x,p)=(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)(-x^2+i\epsilon x_0)^{-\alpha}h_1(x^2,x\cdot p) + \text{l.s.t.}$$

In other words, in this case the picture of free parton constituents interacting with the photon must be abandoned completely.

It is worth remarking here that such terms are not present in the full commutator. Therefore these terms, if present in $C_{\mu\nu}(x,p)$, must be canceled by a similar term from the Class-II contributions.

We point out also that the pieces of the commutator we studied lead to structure functions with support in ω that extends beyond the range $-1<\omega<+1$ known to be true for the full commutator. What happens for the full commutator is that all these pieces combine with the Class-II contributions to give zero outside this range.

The functions $f_i(x^2,x\cdot p)$ appearing in our analysis are general and model-dependent. Clearly, however, if these are known functions, one would be able to derive relations between the structure functions of the scattering process and those of the annihilation process using the representations given in Secs. III and IV. Conversely, relations between the structure functions may be phenomenologically described by special choices for the functions $f_i(0,\eta)$.

In general, and in view of the support properties of $\overline{F}_L(\omega)$ and $F'_L(\omega)=-F_L(-\omega)$, one obtains from Eq. (60) for example

$$\frac{1}{\omega}F_L(\omega)=(2\pi)^3g_1(-\omega), \quad -\infty<\omega<-1$$

and (66)

$$\frac{1}{\omega}F_L{}'(\omega)=(2\pi)^3g_1(-\omega), \quad -1<\omega<\infty .$$

Thus, a *single* function $g_1(\omega)$ determines the scaling structure functions in both the scattering and annihilation process. However, unless $g_1(\omega)$ has some special property connecting its values, in these two regions of ω, $F_L(-\omega)=-F_L(\omega)$ and $F_L(\omega)$ are completely independent. One, of course, may investigate this question by studying, in models, the dynamical content of the operator product $A^{(1)}_{\mu\nu}(x,p)$. This however takes us beyond the scope of this paper.

Furthermore, our method may be used to relate more complicated processes such as, for example, the process $e+\Pi(p)\to e+\Pi(p')+X$ and the process $e^+ + e^- \to \Pi(p)+\Pi(p')+X$. These are described by hadron matrix elements of similar quadrilocal operators. Thus, scaling in the appropriate variables controlled by the light-cone structure of these quadrilocal operators must be identical for both reactions.

These and other questions are left for a separate publication.

VIII. ACKNOWLEDGMENT

I wish to thank Dr. J. T. Manassah and Professor K. Wilson for useful discussions and Dr. Carl Kaysen for the hospitality of The Institute for Advanced Study.

APPENDIX A

Starting with the short-distance restriction of Eq. (55) we present here a derivation of Eq. (56) using the expression given in Eq. (52) for $C(x^2,x\cdot p)$, namely,

$$C(x^2,x\cdot p) = -\pi[\epsilon(x^0)\delta(x^2)F_1(x^2,x\cdot p) - m^2\epsilon(x^0)\theta(x^2)F_2(x^2,x\cdot p)]\,. \quad \text{(A1)}$$

We first point out that

$$\epsilon(x^0)\delta(x^2)|_{x_\mu=0}=0\,,\qquad \epsilon(x^0)\theta(x^2)|_{x_\mu=0}=0\,, \quad \text{(A2)}$$

and furthermore that

$$\partial_0[\epsilon(x^0)\delta(x^2)]|_{x^0=0}=2\pi\delta^3(\vec{x})\,,\qquad \partial_0[\epsilon(x^0)\theta(x^2)]|_{x^0=0}=0\,. \quad \text{(A3)}$$

One may easily convince oneself of the validity of Eqs. (A2) and (A3) by recalling the properties of $\Delta(x,m^2)$ given as

$$\Delta(x,m^2) = -\frac{1}{2\pi}\epsilon(x^0)\left[\delta(x^2)-\tfrac{1}{2}m^2\theta(x^2)\frac{J_1(m(x^2)^{1/2})}{m(x^2)^{1/2}}\right] \quad \text{(A4)}$$

where

$$J_1(y)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,(n+1)!}(\tfrac{1}{2}y)^{2n+1}\,, \quad \text{(A5)}$$

for one has

$$\Delta(0,m^2)=0$$

and (A6)

$$\partial_0\Delta(0,m^2)=-\delta^3(\vec{x})\,.$$

By considering Eq. (A6) for $m=0$ and then arbitrary m one obtains Eqs. (A2) and (A3).

Clearly now if $F_1(x^2,x\cdot p)$ and $F_2(x^2,x\cdot p)$ are regular at $x_\mu=0$ we have

$$\begin{aligned}\partial_0 C(x^2,x\cdot p) = {}& -2\pi^2\delta^3(\vec{x})F_1(x^2,x\cdot p) \\ & -\pi\epsilon(x^0)\delta(x^2)\partial_0F_1(x^2,x\cdot p) \\ & +\pi m^2\partial_0(\epsilon(x^0)\theta(x^2))F_2(x^2,x\cdot p) \\ & +\pi m^2\epsilon(x^0)\theta(x^2)\partial_0F_2(x^2,x\cdot p)\,.\end{aligned} \quad \text{(A7)}$$

Evaluating Eq. (A7) at $x_\mu=0$ we clearly obtain

$$\partial_0 C(x^2,x\cdot p)|_{x_\mu=0} = -2\pi^2\delta^3(\vec{x})F_1(0,0)\,. \quad \text{(A8)}$$

Therefore the positivity restriction on the distribution $C(x^2,x\cdot p)$,

$$\partial_0 C(x^2,x\cdot p)|_{x_\mu=0}=-ir\,,\quad r>0 \quad \text{(A9)}$$

implies

$$F_1(0,0)=i\frac{r}{2\pi^2}\,,\quad r>0. \quad \text{(A10)}$$

This result may also be obtained by studying the distribution $C(x^2,x\cdot p)$ of Eq. (A1) using the representations

$$\pi\epsilon(x_0)\delta(x^2)=\frac{1}{2i}\left(\frac{1}{-x^2-i\epsilon x_0}-\frac{1}{-x^2+i\epsilon x_0}\right)\,, \quad \text{(A11)}$$

$$\pi\epsilon(x_0)\theta(x^2)=\frac{1}{2i}[\ln(-x^2-i\epsilon x_0)-\ln(-x^2+i\epsilon x_0)]$$

and the Wilson[10] definition of the equal-time limit. By using Eq. (A11) we also may see that only if $F_2(x^2,x\cdot p)$ is singular as $x_\mu\to 0$ could there be a contribution from it. The singularity must be of the form

$$F_2(0,\eta)=\delta''(\eta)G(x^2,\eta)\,, \quad \text{(A12)}$$

where $G(0)$ is regular, for then for $\vec{p}=0, \eta=mx^0$ and one has from (A1)

$$C(x^2,x\cdot p)=-\pi[\epsilon(x^0)\delta(x^2)F_1(x^2,x\cdot p) - \partial_0{}^2(\epsilon(x^0)\theta(x^2))G(x^2,\eta)]\,. \quad \text{(A13)}$$

Thus by using Eq. (A11) and Ref. 10 we obtain

$$\partial_0 C(x^2,x\cdot p)=-2\pi^2\delta^3(\vec{x})[F_1(0,0)+G(0,0)]\,. \quad \text{(A14)}$$

Thus in this case $F_1(0)$ may vanish if $G(0,0)$ does not. The behavior of Eq. (A12) then leads to the lower bounds of Eq. (64) in the text.

The point is that only terms as singular as $\epsilon(x^0)\delta(x^2)$ contribute to the $x_\mu=0$ limit of the restriction of Eq. (55).

APPENDIX B

We discuss in this appendix the behavior of $W_{\mu\nu}(q^0,\vec{q})$ in the free-field case. Calculating the absorptive part of the Born terms for a free Dirac field, one obtains

$$W_{\mu\nu}=(e^2/M^2)\{\delta(q^2+2q\cdot p)[2p_\mu p_\nu - q\cdot p g_{\mu\nu}+(p_\mu q_\nu+p_\nu q_\mu)]+\delta(q^2-2q\cdot p)[2p_\mu p_\nu+q\cdot p g_{\mu\nu}-(p_\mu q_\nu+p_\nu q_\mu)]\}\ . \tag{B1}$$

These two terms are separately gauge-invariant on account of the δ functions they carry. The corresponding Compton amplitude $M_{\mu\nu}$ is given by

$$M_{\mu\nu}=C\left\{\frac{1}{q^2+2q\cdot p-i\epsilon}[2p_\mu p_\nu-q\cdot p g_{\mu\nu}+(p_\mu q_\nu+p_\nu q_\mu)]+\frac{1}{q^2-2q\cdot p-i\epsilon}[2p_\mu p_\nu+q\cdot p g_{\mu\nu}-(p_\mu q_\nu+p_\nu q_\mu)]\right\}\ . \tag{B2}$$

$M_{\mu\nu}$ is gauge-invariant due to a cancellation between the two terms appearing above.

Let us consider W_{00} as a function of q^0 and $\vec{q}$. Considering $\vec{p}=0$ we obtain

$$W_{00}=(e^2/M)[\delta((q^0)^2-\vec{q}^{\,2}+2q^0M)(q^0+2M)-\delta((q^0)^2-\vec{q}^{\,2}-2q^0M)(q^0-2M)]\ . \tag{B3}$$

Now

$$\delta((q^0)^2-\vec{q}^{\,2}+2q^0M)=\frac{1}{2|a+M|}\delta(q^0-a)+\frac{1}{2|b+M|}\delta(q^0-b)\ ,$$

$$\delta((q^0)^2-\vec{q}^{\,2}-2q^0M)=\frac{1}{2|a+M|}\delta(q^0+a)+\frac{1}{2|b+M|}\delta(q^0+b)\ ,$$

where

$$a=-M+(M^2+\vec{q}^{\,2})^{1/2}>0\ ,\qquad b=-M-(M^2+\vec{q}^{\,2})^{1/2}<-2M\ .$$

Thus from Eq. (B3) we see that W_{00} is zero everywhere except at the points a, b, $-a$, and $-b$. However, it is positive at $q^0=a>0$ but negative at $q^0=-b>0$, while it is negative at $q^0=b<0$ and positive at $q^0=-a<0$. Therefore the free-field gauge-invariant expression does not satisfy the positivity requirement.

Calculating $W_{\mu\nu}$ using Eq. (47) amounts to taking the difference instead of the sum of the two terms in Eq. (B1). In this case we find, by construction, that the positivity property is obeyed. However, the corresponding Compton amplitude $M_{\mu\nu}$ is *not* gauge-invariant, as it becomes the difference of the two terms in Eq. (B2) instead of the sum, which is crucial to its gauge invariance. The structure function νW_2 does scale but is zero everywhere except at $\omega=-q^2/2q\cdot p=\pm 1$, and the longitudinal structure function is zero. Free-field theory therefore is not the proper laboratory to test the consequences of positivity and gauge invariance presented in the text.

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[8]$A^{(2)}_{\mu\nu}(x,p)$ is also the Hermitian adjoint of $A^{(1)}_{\mu\nu}(x,p)$. Thus, we have $f^*_{1,2}(x^2,x\cdot p)=f_{1,2}(x^2,-x\cdot p)$.

[9]Due to the complex conjugation operation of c numbers associated with time reversal the $i\epsilon$ prescription in Eq. (31) for $A^{(1)}_{\mu\nu}(-x,p)$ is the same as $A^{(1)}_{\mu\nu}(x,p)$.

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