1959).

[9]F. J. Belinfante, Physica 6, 887 (1939); 7, 305 (1940).

[10]L. Rosenfeld, Mem. Acad. Roy. Belg. 6, 30 (1940).

[11]G. Wentzel, *Quantum Theory of Fields* (Interscience, New York, 1949).

[12]W. Pauli, Rev. Mod. Phys. 13, 203 (1941).

[13]We have $G_{\nu\lambda\mu} = f_{\mu,\nu\lambda}$, where $G_{\nu\lambda\mu}$ is defined by Eq. (I.17) of Ref. 11 and $f_{\mu,\nu\lambda}$ by Eqs. (I.13) of Ref. 12.

[14]H. Umezawa, *Quantum Field Theory* (North-Holland, Amsterdam, 1956).

[15]We use underlined letters for quantities in the Heisenberg representation and nonunderlined letters for quantities in the interaction representation.

[16]Y. Takahashi and H. Umezawa, Progr. Theoret. Phys. (Kyoto) 9, 14 (1953).

[17]Equation (3.22) is obtained from Eq. (3.11) above and Eq. (28) of Ref. 1, while Eq. (3.23) is easily derived from Eq. (10.30) of Ref. 14.

PHYSICAL REVIEW D VOLUME 6, NUMBER 8 15 OCTOBER 1972

Tensor Approach to Spin-One Mesons. III. Magnetic Dipole Moment and Electric Quadrupole Moment

E. Kyriakopoulos

Nuclear Research Center "Democritos," Aghia, Paraskevi Attikis, Athens, Greece

(Received 14 May 1971)

In previous papers the massive spin-one mesons were described by means of an antisymmetric second-rank tensor field. In the present paper their free Lagrangian is modified in such a way that when the electromagnetic interactions are introduced by the minimal substitution the mesons get an arbitrary magnetic dipole moment. The addition of other terms in the Lagrangian allows the spin-one mesons to also have an arbitrary electric quadrupole moment. The covariance of the S matrix to order e^2 is achieved by the addition of counterterms.

I. INTRODUCTION

In two previous papers[1,2] the massive spin-one mesons were described by means of an antisymmetric second-rank tensor field $T_{\mu\nu}$. In Ref. 2 we considered the Lagrangian

$$\mathcal{L}^0_{\text{free}} = \partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu}\,, \tag{1.1}$$

and we imposed the antisymmetry condition $T_{\mu\nu}(x) = -T_{\mu\nu}(x)$. The above Lagrangian gives the equation of motion

$$\partial_\mu\partial_\lambda T_{\lambda\nu} - \partial_\nu\partial_\lambda T_{\lambda\mu} - m^2 T_{\mu\nu} = 0\,. \tag{1.2}$$

Differentiating Eq. (1.2) we get

$$[(\Box - m^2)\delta_{\nu\mu} - \partial_\nu\partial_\lambda\delta_{\mu\lambda}]V_\mu(x) = 0\,, \tag{1.3}$$

where the field $V_\mu(x)$ is defined by

$$V_\mu(x) = (1/m)\partial_\lambda T_{\lambda\mu}(x). \tag{1.4}$$

The field $V_\mu(x)$ describes the spin-one component of a vector or axial-vector field.

In Ref. 1 we described a way of obtaining the interaction Hamiltonian in the interaction representation $\mathcal{H}_{\text{int}}$, when the field $T_{\mu\nu}$ is involved. The $\mathcal{H}_{\text{int}}$ corresponding to a specific interaction Lagrangian was calculated, and it was shown that the S matrix coming from this $\mathcal{H}_{\text{int}}$ is covariant to any order in perturbation theory. In Ref. 2 the quantization was performed in the free-field case and also in the interacting-field case.

The magnetic dipole moment μ and the electric quadrupole moment Q of the $J^{PC} = 1^{--}$ nonet of vector mesons, except the ρ^0, ω, and φ, are not known. One wants a theory which allows arbitrary values of μ and Q. The Proca theory in which the electromagnetic interactions have been introduced by the minimal substitution describes particles with the "normal" magnetic dipole moment, i.e., with $\mu = e/2m$.[3] An extension was later made by Pauli[4] and by Corben and Schwinger[5] to include particles with arbitrary μ. Further terms can be added, which allow the mesons to also have an arbitrary electric quadrupole moment.[6,7] The values of μ and Q we obtain if we introduce in the Lagrangian of Eq. (1.1) the electromagnetic interactions by the minimal substitution are fixed. It is interesting to see if by proper generalization the tensor formalism can describe particles with arbitrary μ and Q. This is indeed the case, as shown in Sec. II.

In Sec. III the covariance of the S matrix to order e^2 is examined. It is shown that the S matrix can be made covariant to this order, if we add to our Lagrangian some additional terms (counterterms). The same method has been applied in the usual description of spin-one mesons.[7-9] Finally in Sec. IV the Feynman rules are given.

II. EXTENSION TO AN ARBITRARY MAGNETIC DIPOLE MOMENT AND ELECTRIC QUADRUPOLE MOMENT

The electromagnetic interactions are usually introduced by the minimal substitution

$$\partial_\mu V_\nu \to (\partial_\mu - ieA_\mu)V_\nu \equiv D_\mu V_\nu \,, \qquad \partial_\mu V_\nu^\dagger \to (\partial_\mu + ieA_\mu)V_\nu^\dagger \equiv D_\mu^\dagger V_\nu^\dagger \,, \tag{2.1}$$

in the Lagrangian density of the free particle, where e is the charge of the particle, and A_μ is the electromagnetic field. In our case we shall introduce partially the electromagnetic interactions by making the substitutions of Eqs. (2.1) in our free Lagrangian. The Lagrangian $\mathcal{L}^0_{\text{free}}$ of Eq. (1.1) changes as follows:

$$\mathcal{L}^0_{\text{free}} \to \mathcal{L}^0_{\text{free}} + \mathcal{L}^0_{\gamma-V} \,, \tag{2.2}$$

where

$$\mathcal{L}^0_{\gamma-V} = ieA_\mu(T^\dagger_{\mu\lambda}\partial_\nu T_{\nu\lambda} - \partial_\nu T^\dagger_{\nu\lambda}T_{\mu\lambda}) - ieF_{\mu\nu}T^\dagger_{\mu\lambda}T_{\nu\lambda} + e^2A_\mu A_\nu T^\dagger_{\mu\lambda}T_{\nu\lambda} + ie[\partial_\nu(A_\mu T^\dagger_{\nu\lambda}T_{\mu\lambda}) - \partial_\mu(A_\nu T^\dagger_{\nu\lambda}T_{\mu\lambda})]. \tag{2.3}$$

In the above equation $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The four-divergences appearing in Eq. (2.3) do not affect the variation of the action and may be omitted. We find that our Lagrangian describes a particle with zero magnetic moment.

To introduce an arbitrary magnetic moment, we start from the free Lagrangian

$$\mathcal{L}_{\text{free}} = a\partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + (1-a)\partial_\mu T^\dagger_{\mu\lambda}\partial_\nu T_{\nu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu} \,, \tag{2.4}$$

which gives the same equations of motion. We easily see that the Lagrangians of Eqs. (1.1) and (2.4) differ only by four-divergences which do affect the electromagnetic interactions.[10] Making the substitution (2.1) in Eq. (2.4) and dropping unimportant four-divergences, we get

$$\mathcal{L}' = \partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu} + \mathcal{L}'_{\gamma-V} \,, \tag{2.5}$$

where

$$\mathcal{L}'_{\gamma-V} = ieA_\mu(T^\dagger_{\mu\lambda}\partial_\nu T_{\nu\lambda} - \partial_\nu T^\dagger_{\nu\lambda}T_{\mu\lambda}) - ieaF_{\mu\nu}T^\dagger_{\mu\lambda}T_{\nu\lambda} + e^2A_\mu A_\nu T^\dagger_{\mu\lambda}T_{\nu\lambda}. \tag{2.6}$$

The magnetic dipole moment μ is the expectation value of the component μ_3 of the operator

$$\mu_i = \tfrac{1}{2}\epsilon_{ij\kappa}\int d^3x\, x_j J_\kappa(x) \,, \tag{2.7}$$

for a positively charged particle with spin eigenstate $S_z = 1$ at rest. The operator $J_\kappa(x)$ is the electromagnetic-current-density operator. The term $-ieaF_{\mu\nu}T^\dagger_{\mu\lambda}T_{\nu\lambda}$, which contains the arbitrary parameter a, does contribute to μ. Thus the magnetic moment can get arbitrary values in this model.

General group theoretical arguments allow the spin-one mesons to have not only a magnetic dipole moment but also an electric quadrupole moment.[11] The electric quadrupole moment Q is defined as the expectation value of the component Q_{33} of the operator

$$Q_{ij} = \int d^3x(3x_i x_j - x^2\delta_{ij})\rho(x) \tag{2.8}$$

for a positively charged particle with $S_z = 1$ at rest. The operator $\rho(x)$ is the charge-density operator. The $\mathcal{L}'_{\gamma-V}$ of Eq. (2.6) implies that $Q = -e(1+a)/m^2$. Since the moments μ and Q depend on a single parameter a, we cannot assign arbitrary values to both.

Arbitrary values of μ and Q can be obtained if we add to $\mathcal{L}'_{\gamma-V}$ of Eq. (2.6) the term

$$\mathcal{L}''_{\gamma-V} = -(ieb/m^2)F_{\mu\nu}D^\dagger_\rho T^\dagger_{\rho\mu}D_\sigma T_{\sigma\nu}. \tag{2.9}$$

This term cannot be obtained by the substitution $\partial_\mu \to \partial_\mu \mp ieA_\mu$ in a free Lagrangian with no more than two derivatives. It does not seem to be possible to introduce arbitrary μ and Q by the principle of minimal substitution, if our free Lagrangian has at most two derivatives. The same thing happens in the usual description of spin-one mesons.[6]

From our generalized Lagrangian

$$\mathcal{L} = \partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu} + \mathcal{L}'_{\gamma-V} + \mathcal{L}''_{\gamma-V} \,, \tag{2.10}$$

we get the equations of motion[12]

$$\partial_\nu\partial_\rho T_{\rho\lambda} - \partial_\lambda\partial_\rho T_{\rho\nu} - m^2 T_{\nu\lambda} = ie(1-a)(F_{\nu\rho}T_{\rho\lambda} - F_{\lambda\rho}T_{\rho\nu}) + ieA_\rho(\partial_\nu T_{\rho\lambda} - \partial_\lambda T_{\rho\nu}) + ie\partial_\rho(A_\nu T_{\rho\lambda} - A_\lambda T_{\rho\nu}) + e^2A_\rho(A_\nu T_{\rho\lambda} - A_\lambda T_{\rho\nu}) + (ieb/m^2)[\partial_\nu(F_{\lambda\mu}D_\rho T_{\rho\mu}) - \partial_\lambda(F_{\nu\mu}D_\rho T_{\rho\mu}) + ie(F_{\nu\mu}A_\lambda - F_{\lambda\mu}A_\nu)D_\sigma T_{\sigma\mu}]. \tag{2.11}$$

In the notation of Ref. 1 we have the currents

$$J^T_{\nu\lambda} = \frac{\partial(\mathcal{L}'_{\gamma-V} + \mathcal{L}''_{\gamma-V})}{\partial T^\dagger_{\nu\lambda}} = ie(A_\nu\partial_\rho T_{\rho\lambda} - A_\lambda\partial_\rho T_{\rho\nu}) + iea(F_{\nu\rho}T_{\lambda\rho} - F_{\lambda\rho}T_{\nu\rho}) + e^2A_\rho(A_\nu T_{\rho\lambda} - A_\lambda T_{\rho\nu}) + (e^2b/m^2)(F_{\lambda\mu}A_\nu - F_{\nu\mu}A_\lambda)D_\sigma T_{\sigma\mu} \,, \tag{2.12}$$

$$J^T_\nu = \frac{\partial(\mathcal{L}'_{\gamma-V} + \mathcal{L}''_{\gamma-V})}{\partial(\partial_\rho T^\dagger_{\rho\nu})} = -ieA_\rho T_{\rho\nu} - \frac{ieb}{m^2}F_{\nu\lambda}D_\rho T_{\rho\lambda}. \tag{2.13}$$

As $A_\mu \to 0$ Eqs. (2.11) reduce to the free-field equations (1.2).

We may add to the Lagrangian of Eq. (2.10) the free Lagrangian of the electromagnetic field

$$(\mathcal{L}_\gamma)_{\rm free} = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}\,. \tag{2.14}$$

The equation of motion for this field becomes

$$\partial_\mu F_{\mu\nu} = -J_\nu\,, \tag{2.15}$$

where the electromagnetic current J_ν, which is the source of the electromagnetic field, is

$$J_\nu = ie(T^\dagger_{\nu\lambda}\partial_\rho T_{\rho\lambda} - \partial_\rho T^\dagger_{\rho\lambda}T_{\nu\lambda}) + iea\partial_\mu(T^\dagger_{\mu\lambda}T_{\nu\lambda} - T^\dagger_{\nu\lambda}T_{\mu\lambda}) + (ieb/m^2)\partial_\mu(\partial_\rho T^\dagger_{\rho\mu}\partial_\sigma T_{\sigma\nu} - \partial_\sigma T^\dagger_{\sigma\nu}\partial_\rho T_{\rho\mu}) + e^2A_\rho(T^\dagger_{\rho\lambda}T_{\nu\lambda} + T^\dagger_{\nu\lambda}T_{\rho\lambda}) + O(e^2)\,. \tag{2.16}$$

From the variation of the Lagrangian $\mathcal{L} + (\mathcal{L}_\gamma)_{\rm free}$ the total energy-momentum tensor can be calculated and then symmetrized in the same way as before. The explicit expression of the symmetric tensor $\Theta^T_{\lambda\mu}$ is very complicated. We have

$$\partial_\lambda \Theta^T_{\lambda\mu} = 0\,. \tag{2.17}$$

Let us write

$$\Theta^T_{\lambda\mu} = \Theta'_{\lambda\mu} + \Theta^\gamma_{\lambda\mu}\,, \tag{2.18}$$

where $\Theta'_{\lambda\mu}$ comes from the Lagrangian $\mathcal{L}$ and $\Theta^\gamma_{\lambda\mu}$ comes from the Lagrangian $(\mathcal{L}_\gamma)_{\rm free}$. We have

$$\Theta^\gamma_{\lambda\mu} = F_{\lambda\rho}F_{\mu\rho} - \tfrac{1}{4}\delta_{\lambda\mu}F_{\nu\rho}F_{\nu\rho}\,. \tag{2.19}$$

From Eqs. (2.15), (2.17), and (2.19), we get

$$\partial_\lambda \Theta'_{\lambda\mu} = -\partial_\lambda \Theta^\gamma_{\lambda\mu} = -J_\rho F_{\rho\mu}\,. \tag{2.20}$$

From Eqs. (2.7), (2.8), and the explicit expression for the electromagnetic current, the magnetic dipole moment μ and the electric quadrupole moment Q can be calculated. The fields which appear in the electromagnetic current are replaced by their lowest-order approximation, i.e., by free fields. We find

$$\mu = (1 - a + b)e/2m\,, \tag{2.21}$$

$$Q = -(1 + a + b)e/m^2\,. \tag{2.22}$$

Since a and b are arbitrary constants the magnetic dipole moment and the electric quadrupole moment of the mesons can get arbitrary values.

III. COVARIANCE OF THE S MATRIX

To proceed in calculations by perturbation theory we need the interaction Hamiltonian in the interaction representation. Starting from the interaction Lagrangian $\mathcal{L}'_{\rm int} = \mathcal{L}'_{\gamma-V} + \mathcal{L}''_{\gamma-V}$, where $\mathcal{L}'_{\gamma-V}$ and $\mathcal{L}''_{\gamma-V}$ are given by Eqs. (2.6) and (2.9), respectively, and applying the method of Ref. 1, we find to order e^2 the following expression for the interaction Hamiltonian[13]:

$$\mathcal{H}_{\rm int}(x,n) = -\mathcal{L}'_{\rm int}(x) + \frac{1}{2m^2}(J^T_{\mu\nu}J^{\bar T}_{\mu\nu} + 2J^T_{\mu\lambda}J^{\bar T}_{\mu\sigma}n_\lambda n_\sigma) + J^T_\mu J^{\bar T}_\mu + J^T_\lambda J^{\bar T}_\sigma n_\lambda n_\sigma + \tfrac{1}{2}J^A_{\mu\lambda}J^A_{\mu\sigma}n_\lambda n_\sigma\,, \tag{3.1}$$

where

$$J^A_{\mu\lambda} = -\frac{\partial \mathcal{L}'_{\rm int}}{\partial F_{\mu\lambda}} = iea(T^\dagger_{\mu\nu}T_{\lambda\nu} - T^\dagger_{\lambda\nu}T_{\mu\nu}) + (ieb/m^2)(D^\dagger_\rho T^\dagger_{\rho\mu}D_\sigma T_{\sigma\lambda} - D^\dagger_\sigma T^\dagger_{\sigma\lambda}D_\rho T_{\rho\mu})\,, \tag{3.2}$$

and the currents $J^T_{\mu\nu}$ and J^T_μ are given by Eqs. (2.12) and (2.13), respectively.[14]

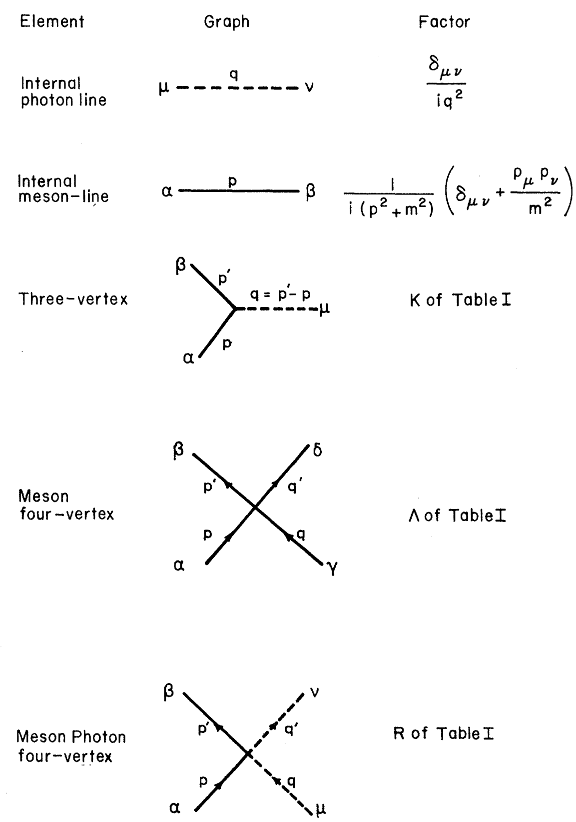

FIG. 1. Feynman rules in the momentum representation for the $(\mathcal{H}_{\rm int})_{\rm eff}$ of Eq. (4.1). Only the values of the three- and four-vertex functions are listed, and in the case of the meson-photon four-vertex the terms proportional to a^2 have been omitted (see Table I).

One of the main problems of the electrodynamics of charged mesons of spin one with arbitrary (anomalous) magnetic moment is the covariance of their S matrix. The usual approach does not give a covariant S matrix even to order e^2. Several methods have been proposed to overcome this difficulty. Lee and Yang[15] define the S matrix without the normal ordering of the fields of $\mathcal{H}_{\text{int}}$. They add to the Lagrangian the terms $-\xi[(\partial_\mu+ieA_\mu)V_\mu^\dagger][(\partial_\nu-ieA_\nu)\times V_\nu]$ and then take the limit $\xi\to 0$ (ξ –limiting formalism). In this theory the free Hamiltonian is not positive definite unless we introduce a negative metric. The introduction of a negative metric destroys the unitarity of the S matrix, but in the limit $\xi\to 0$ the unitarity is restored. The S matrix is covariant. The introduction of the ξ-dependent terms can be avoided by the addition of certain counterterms in the Lagrangian.[8,9] In another method[16] covariance is achieved by generalizing the definition and application of normal products in perturbation theory.

The noncovariant terms of the S matrix come from the noncovariant part of the $\mathcal{H}_{\text{int}}(x,n)$ of Eq. (3.1) and from the noncovariant part of the propagators. We have[1]

$$\left\langle T\left\{\frac{\partial}{\partial x_\mu}T_{\mu\nu}(x)\frac{\partial}{\partial y_\rho}T^\dagger_{\rho\sigma}(y)\right\}\right\rangle_0 = i\left[m^2 G_{F\nu\sigma}(x-y)-n_\nu n_\sigma\delta^4(x-y)\right]\equiv C^V_{\nu\sigma}(x-y)+N_{\nu\sigma}(x-y)\,, \tag{3.3}$$

$$\begin{aligned}\langle T\{T_{\mu\nu}(x)T^\dagger_{\rho\sigma}(y)\}\rangle_0 = \frac{i}{m^2}\Big\{&\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\rho}G_{F\nu\sigma}(x-y)+\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial y_\sigma}G_{F\mu\rho}(x-y)-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\sigma}G_{F\nu\rho}(x-y)-\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial y_\rho}G_{F\mu\sigma}(x-y)\\ &-(\delta_{\mu\rho}n_\nu n_\sigma+\delta_{\nu\sigma}n_\mu n_\rho-\delta_{\mu\sigma}n_\nu n_\rho-\delta_{\nu\rho}n_\mu n_\sigma)\delta^4(x-y)\Big\}\\ &\equiv(1/m^2)[C^H_{\mu\nu\rho\sigma}(x-y)+N_{\mu\nu\rho\sigma}(x-y)]\,,\end{aligned} \tag{3.4}$$

$$\begin{aligned}\langle T\{F_{\mu\nu}(x)F_{\rho\sigma}(y)\}\rangle_0 = i\Big\{&\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\rho}D_{F\nu\sigma}(x-y)+\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial y_\sigma}D_{F\mu\rho}(x-y)-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\sigma}D_{F\nu\rho}(x-y)-\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial y_\rho}D_{F\mu\sigma}(x-y)\\ &-(\delta_{\mu\rho}n_\nu n_\sigma+\delta_{\nu\sigma}n_\mu n_\rho-\delta_{\mu\sigma}n_\nu n_\rho-\delta_{\nu\rho}n_\mu n_\sigma)\delta^4(x-y)\Big\}\\ &\equiv C^F_{\mu\nu\rho\sigma}(x-y)+N_{\mu\nu\rho\sigma}(x-y)\,,\end{aligned} \tag{3.5}$$

where

$$G_{F\nu\rho}(x-y)=-\tfrac{1}{2}i\left(\delta_{\nu\rho}-\frac{1}{m^2}\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x_\rho}\right)\Delta_F(x-y,m)\,, \tag{3.6}$$

$$D_{F\nu\rho}(x-y)=-\tfrac{1}{2}i\delta_{\nu\rho}\Delta_F(x-y,m=0)\,. \tag{3.7}$$

The covariant parts of the propagators are denoted by $C^V_{\nu\sigma}(x-y)$, $(1/m^2)C^H_{\mu\nu\rho\sigma}(x-y)$, and $C^F_{\mu\nu\rho\sigma}(x-y)$, while the noncovariant parts are denoted by $N_{\nu\sigma}(x-y)$, $(1/m^2)N_{\mu\nu\rho\sigma}(x-y)$, and $N_{\mu\nu\rho\sigma}(x-y)$. The noncovariant parts of Eqs. (3.4) and (3.5) are the same apart from the factor $1/m^2$. T in Eqs. (3.3)–(3.5) is the Dyson time-ordered-product operator and $\Delta_F(x-y,m)$ the usual Feynman propagator. The propagators $\langle T\{A_\mu(x)A_\nu(y)\}\rangle_0$, $\langle T\{A_\mu(x)F_{\rho\sigma}(y)\}\rangle_0$, and $\langle T\{(\partial/\partial x_\mu)T_{\mu\nu}(x)T^\dagger_{\rho\sigma}(y)\}\rangle_0$ do not have noncovariant parts.

The S matrix, to order e^2, is given by

$$S=1-i\int d^4x_1\mathcal{H}_{\text{int}}(x_1,n)+\frac{(-i)^2}{2\,!}\int d^4x_1d^4x_2\,T\{\mathcal{H}^{(e)}_{\text{int}}(x_1)\mathcal{H}^{(e)}_{\text{int}}(x_2)\}\,, \tag{3.8}$$

where by $\mathcal{H}^{(e)}_{\text{int}}$ we mean the part of the interaction Hamiltonian which is of order e. We shall not normal order the fields in the interaction Hamiltonian of Eq. (3.8) so that equal-time contractions are allowed. We shall try to find explicitly the noncovariant part of the S-matrix operator of Eq. (3.8) using the $\mathcal{H}_{\text{int}}$ of Eq. (3.1) and the propagators of Eqs. (3.3)–(3.5). If such a part does not exist the S matrix is covariant. The calculations are tedious and will not be reproduced here. Instead we shall indicate how these calculations were done.

One can easily show that the S operator giving rise to processes with four external lines is covariant. This is due to the fact that the noncovariant part of the term $-i\int d^4x_1\mathcal{H}_{\text{int}}(x_1,n)$ which gives rise to such processes is canceled out by the noncovariant part coming from the term

$$\tfrac{1}{2}(-i)^2\int d^4x_1 d^4x_2\, T\{\mathcal{H}^{(e)}_{\rm int}(x_1)\mathcal{H}^{(e)}_{\rm int}(x_2)\}$$

when only two fields are contracted.[17] Observe that the propagators of Eqs. (3.3)–(3.5) contain noncovariant terms. We easily find also that the terms which are proportional to a or b or ab do not create noncovariant parts in the S matrix, because the propagator of at most one of the fields which can be contracted in such terms has a noncovariant part. Then we can show that noncovariant terms independent of a or b do not exist. So if noncovariant terms exist they will be proportional to a^2 or b^2. Such terms indeed exist. We find that the noncovariant and infinite part of the S operator of Eq. (3.8) is given by

$$-\int d^4x\Big[(ea/m^2)^2\,[2H_{\mu\rho}(x)H^\dagger_{\mu\sigma}(x)+2C^H_{\mu\rho\mu\sigma}(0)+F_{\mu\rho}(x)F_{\mu\sigma}(x)+C^F_{\mu\rho\mu\sigma}(0)]\delta^4(0)n_\rho n_\sigma$$
$$+(eb/m^2)^2\{F_{\mu\rho}(x)F_{\mu\sigma}(x)+C^F_{\mu\rho\mu\sigma}(0)+3m^2[V_\rho(x)V^\dagger_\sigma(x)+C^V_{\rho\sigma}(0)]\}\delta^4(0)n_\rho n_\sigma-3i(e/m^2)^2(2a^2+b^2)\delta^4(0)\delta^4(0)\Big]. \tag{3.9}$$

To obtain an S operator without the terms appearing in the expression (3.9), we add to the interaction Lagrangian additional terms of order e^2. If we take

$$\mathcal{L}''_{\rm int}=\gamma(ea)^2T_{\mu\rho}T^\dagger_{\mu\sigma}T_{\nu\sigma}T^\dagger_{\nu\rho}+\delta(ea/m)^2T_{\mu\rho}T^\dagger_{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}+\epsilon(eb/m^2)^2F_{\lambda\mu}F_{\lambda\nu}D^\dagger_\rho T^\dagger_{\rho\mu}D_\sigma T_{\sigma\nu}$$
$$+\zeta(eb/m^2)^2(D_\rho T_{\rho\mu}D^\dagger_\sigma T^\dagger_{\sigma\nu}-D_\sigma T_{\sigma\nu}D^\dagger_\rho T^\dagger_{\rho\mu})(D_\lambda T_{\lambda\mu}D^\dagger_\tau T^\dagger_{\tau\nu}-D_\tau T_{\tau\nu}D^\dagger_\lambda T^\dagger_{\lambda\mu}), \tag{3.10}$$

the values of the constants γ,δ,ϵ, and ζ for which the terms appearing in the expression (3.9) will be eliminated are[18]

$$\gamma=\tfrac{1}{8},\quad \delta=\tfrac{1}{2},\quad \epsilon=1,\quad \zeta=\tfrac{1}{4}. \tag{3.11}$$

For the choice $\zeta=\frac{1}{4}$ the Lagrangian will have a term of the form $-\frac{1}{4}\hat F_{\mu\nu}\hat F_{\mu\nu}$, where $\hat F_{\mu\nu}=F_{\mu\nu}+(ieb/m^2)\times(\partial_\rho T^\dagger_{\rho\mu}\partial_\sigma T_{\sigma\nu}-\partial_\sigma T^\dagger_{\sigma\nu}\partial_\rho T_{\rho\mu})$. The same term appears in the usual description of spin-one mesons. Indeed, it is shown in Refs. 8 and 9 that the electrodynamics of spin-one mesons with an arbitrary magnetic moment becomes fully covariant if in the Lagrangian we make the replacement

$$-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-iebF_{\mu\nu}V^\dagger_\mu V_\nu\to-\tfrac{1}{4}[F_{\mu\nu}+ieb(V^\dagger_\mu V_\nu-V^\dagger_\nu V_\mu)][F_{\mu\nu}+ieb(V^\dagger_\mu V_\nu-V^\dagger_\nu V_\mu)]. \tag{3.12}$$

An S matrix covariant to order e^2 has been constructed by adding certain counterterms. We have no reason to believe that the S matrix will be covariant to order e^3. To achieve covariance we shall add new counterterms.[19] The same procedure will be repeated to the next order and so on. In other words, the covariance of the S matrix seems to require an infinite number of counterterms. Exactly the same thing happens in the usual description of spin-one mesons with abitrary magnetic dipole moment and also arbitrary electric quadrupole moment.[7]

TABLE I. The values of the three- and four-vertex functions for the $(\mathcal{H}_{\rm int})_{\rm eff}$ of Eq. (4.1). The a' is defined by $a'=a/m^2$ (see Fig. 1).

$$K=-e\{[p+p'+a'(p\cdot q)p'-a'(p'\cdot q)p]_\mu\delta_{\alpha\beta}+[-p+bq+a'(p'\cdot q)p-a'(p'\cdot p)q]_\beta\delta_{\alpha\mu}$$
$$+[-p'-bq+a'(p\cdot p')q-a'(p\cdot q)p']_\alpha\delta_{\beta\mu}+a'p_\mu p'_\alpha q_\beta-a'p_\beta p'_\mu q_\alpha\}$$

$$\Lambda=(eb)^2(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}-2\delta_{\alpha\gamma}\delta_{\beta\delta})$$
$$-\tfrac{1}{2}(ea')^2\{(p\cdot p')(q\cdot q')\delta_{\alpha\delta}\delta_{\beta\gamma}+(p\cdot q')(p'\cdot q)\delta_{\alpha\beta}\delta_{\gamma\delta}+[(q\cdot q')p_\delta p'_\gamma-(p\cdot q')p'_\gamma q_\delta-(p'\cdot q)p_\delta q'_\gamma]\delta_{\alpha\beta}$$
$$+[(p'\cdot q)p_\beta q'_\gamma-(p\cdot p')q_\beta q'_\gamma-(q\cdot q')p_\beta p'_\gamma]\delta_{\alpha\delta}+[(p\cdot q')p'_\alpha q_\delta-(p\cdot p')q_\delta q'_\alpha-(q\cdot q')p_\delta p'_\alpha]\delta_{\beta\gamma}$$
$$+[(p\cdot p')q_\beta q'_\alpha-(p'\cdot q)p_\beta q'_\alpha-(p\cdot q')p'_\alpha q_\beta]\delta_{\gamma\delta}+p_\beta p'_\gamma q_\delta q'_\alpha+p_\delta p'_\alpha q_\beta q'_\gamma\}$$

$$R=e^2(2\delta_{\alpha\beta}\delta_{\mu\nu}-\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu})$$
$$-e^2a'\{(p\cdot q'+p'\cdot q)\delta_{\alpha\mu}\delta_{\beta\nu}-(p\cdot q+p'\cdot q')\delta_{\alpha\nu}\delta_{\beta\mu}-[p_\nu(q+q')_\beta+p'_\nu q_\beta]\delta_{\alpha\mu}+[p_\mu(q+q')_\beta+p'_\mu q'_\beta]\delta_{\alpha\nu}$$
$$+[q_\alpha(p+p')_\nu+q'_\alpha p'_\nu]\delta_{\beta\mu}-[q'_\alpha(p+p')_\mu+q_\alpha p'_\mu]\delta_{\beta\nu}+(p-p')\cdot(q-q')\delta_{\mu\nu}\delta_{\alpha\beta}-[(q-q')_\alpha p_\beta-(q-q')_\beta p'_\alpha]\delta_{\mu\nu}$$
$$-[(p-p')_\mu q_\nu-(p-p')_\nu q'_\mu]\delta_{\alpha\beta}+q_\nu(p_\beta\delta_{\alpha\mu}-p'_\alpha\delta_{\beta\mu})-q'_\mu(p_\beta\delta_{\alpha\nu}-p'_\alpha\delta_{\beta\nu})\}.$$

IV. FEYNMAN RULES

The effective interaction Hamiltonian to order e^2, which is obtained from Eq. (3.1) by taking $\mathcal{L}_{\text{int}} = \mathcal{L}'_{\gamma-V} + \mathcal{L}''_{\gamma-V} + \mathcal{L}''_{\text{int}}$ and dropping its noncovariant terms is

$$(\mathcal{H}_{\text{int}})_{\text{eff}} = -ieA_\mu(H^\dagger_{\mu\lambda}V_\lambda - V^\dagger_\lambda H_{\mu\lambda}) + (iea/m^2)F_{\mu\nu}H^\dagger_{\mu\lambda}H_{\nu\lambda} + iebF_{\mu\nu}V^\dagger_\mu V_\nu$$
$$+ e^2[A_\mu V_\nu + (a/m^2)F_{\mu\rho}H_{\nu\rho}][A_\mu V^\dagger_\nu - A_\nu V^\dagger_\mu + (a/m^2)(F_{\mu\sigma}H^\dagger_{\nu\sigma} - F_{\nu\sigma}H^\dagger_{\mu\sigma})] - \tfrac{1}{8}(ea/m^2)^2 H_{\mu\rho}H^\dagger_{\mu\sigma}H_{\nu\sigma}H^\dagger_{\nu\rho}$$
$$- \tfrac{1}{2}(ea/m^2)^2 H_{\mu\rho}H^\dagger_{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \tfrac{1}{4}e^2b^2(V_\mu V^\dagger_\nu - V_\nu V^\dagger_\mu)(V_\mu V^\dagger_\nu - V_\nu V^\dagger_\mu). \tag{4.1}$$

The Feynman diagrams are calculated by using the above interaction Hamiltonian and dropping the noncovariant terms of the propagators of Eqs. (3.3)–(3.5). The vertex functions coming from the $(\mathcal{H}_{\text{int}})_{\text{eff}}$ of Eq. (4.1) are listed in Fig. 1 and Table I.

---

[1]E. Kyriakopoulos, Phys. Rev. 183, 1318 (1969).

[2]E. Kyriakopoulos, preceding paper, Phys. Rev. D 4, 2202 (1971).

[3]G. Wentzel, *Quantum Theory of Fields* (Interscience, New York, 1949).

[4](a) W. Pauli, Solvay Report No. 1939 (unpublished); (b) Rev. Mod. Phys. 13, 203 (1940).

[5]H. C. Corben and J. Schwinger, Phys. Rev. 58, 953 (1940).

[6]J. A. Young and S. A. Bludman, Phys. Rev. 131, 2326 (1963).

[7]Harmon Aronson, Phys. Rev. 186, 1434 (1969).

[8]K. H. Tzou, Nuovo Cimento 33, 286 (1964).

[9]M. Nakamura, Progr. Theoret. Phys. (Kyoto) 33, 279 (1964).

[10]This fact makes the introduction of the electromagnetic interactions by the minimal substitution as expressed by Eqs. (2.1) ambiguous. A better definition of minimality is given by T. D. Lee, Phys. Rev. 140, B967 (1965), who calls the principle of minimal electromagnetic interaction the mathematical requirement that the three-point vertex function for the electromagnetic interaction should have a minimal power dependence on its external momenta.

[11]A particle of spin J does not have moments of order $l > 2J$. Such a particle besides its charge can have a magnetic dipole moment, electric quadrupole moment, magnetic octupole moment, etc., up to a moment of order $2J$. See S. DeBenedetti, *Nuclear Interactions* (Wiley, New York, 1966), p. 34.

[12]Equation (2.11) can be written in the form

$$D_\nu D_\rho T_{\rho\lambda} - D_\lambda D_\rho T_{\rho\nu} - m^2 T_{\nu\lambda}$$
$$= -iea(F_{\nu\rho}T_{\rho\lambda} - F_{\lambda\rho}T_{\rho\nu})$$
$$+ (ieb/m^2)[\partial_\nu(F_{\lambda\mu}D_\rho T_{\rho\mu}) - \partial_\lambda(F_{\nu\mu}D_\rho T_{\rho\mu})$$
$$+ ie(F_{\nu\mu}A_\lambda - F_{\lambda\mu}A_\nu)D_\sigma T_{\sigma\mu}].$$

[13]Equation (3.1) is obtained in the same way with Eq. (6.1) of E. Kyriakopoulos, Phys. Rev. D 1, 1697 (1970). In addition to Eqs. (A12) and (A13), we need the expression for $\partial_\rho \underline{A}_\nu(x)$. We find

$$\partial_\rho \underline{A}_\nu(x) = (\partial_\rho A_\nu[x,\sigma])_{x|\sigma} + \underline{J}^A_{\lambda\nu}(x) n_\rho(x) n_\lambda(x)$$
$$\approx S^{-1}[\sigma][\partial_\rho A_\nu + J^A_{\lambda\nu} n_\rho n_\lambda] S[\sigma].$$

[14]Their terms of order e^2 and e^3 must be omitted since we are interested in obtaining $\mathcal{H}_{\text{int}}$ to order e^2.

[15]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).

[16]F. Salzman and G. Salzman, Nuovo Cimento 37, 924 (1965).

[17]This fact was enough to prove the covariance of the S matrix of Ref. 1, to any order in perturbation theory. The reason is that the propagator of *at most one* of the fields in each term of the $\mathcal{H}_{\text{int}}$ of Ref. 1 has a noncovariant part. This situation does not exist in the present case.

[18]The choice of the counterterms is not unique. The simplest choice to eliminate the undesirable part of the S operator coming from the term $-ieaF_{\mu\nu}T^\dagger_{\mu\lambda}T_{\nu\lambda}$ of the Lagrangian seems to be the first two terms of the expression (3.10).

[19]We must have in mind that the $\mathcal{H}_{\text{int}}(x,n)$ of Eq. (3.1) is correct to order e^2. So in the calculation of the S matrix to order e^3 we must use the $\mathcal{H}_{\text{int}}(x,n)$ which is correct to order e^3.