ever, Roy and Pestieau[2] have pointed out that the behavior of the structure functions may be quite different for positive and negative t.

At a formal level, the structure functions have to scale trivially if the theory is to be acceptable.

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[1]S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1969).

[2]J. D. Bjorken, Phys. Rev. 179, 1547 (1969); P. Roy and J. Pestieau, Phys. Letters 30B, 483 (1969).

[3]H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 2, 425 (1955).

[4]S. D. Drell and F. Zachariasen, Phys. Rev. 119, 463 (1960).

[5]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[6]T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969).

[7]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[8]A. M. Gleeson and E. C. G. Sudarshan, Phys. Rev. D 1, 474 (1970).

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Tensor Approach to Spin-One Mesons. II. Quantization

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The spin-one mesons were described previously by means of an antisymmetric second-rank tensor field. In the present paper the quantization is performed in the free-field case, and also in the interacting-field case.

I. INTRODUCTION

The massive spin-one mesons are usually described by a vector or axial-vector field. In a previous paper[1] we described them starting from an antisymmetric second-rank tensor field $T_{\mu\nu}$. The antisymmetry was imposed as a separate condition and it was preserved in the presence of interactions.

Another tensor approach to a massive spin-one field has been proposed by Takahashi and Palmer.[2] The antisymmetry condition is not imposed separately but it is derived from the Lagrangian, which of course is more complicated than our Lagrangian.

We shall generalize slightly the formalism of Ref. 1 to charged fields. We consider the free Lagrangian,

$$\mathcal{L}^0_{\text{free}} = \partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu}\,, \tag{1.1}$$

where $T_{\mu\nu}(x) = -T_{\nu\mu}(x)$. The above Lagrangian gives the equation of motion

$$\partial_\mu\partial_\rho T_{\rho\nu} - \partial_\nu\partial_\rho T_{\rho\mu} - m^2 T_{\mu\nu} \equiv \Lambda_{\mu\nu\rho\sigma}(\partial)T_{\rho\sigma}(x) = 0\,, \tag{1.2}$$

where

$$\Lambda_{\mu\nu\rho\sigma}(\partial) = \tfrac{1}{2}[\partial_\mu\partial_\rho\delta_{\nu\sigma} - \partial_\mu\partial_\sigma\delta_{\nu\rho} - \partial_\nu\partial_\rho\delta_{\mu\sigma} + \partial_\nu\partial_\sigma\delta_{\mu\rho} - m^2(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})]\,. \tag{1.3}$$

Differentiating Eq. (1.2), we get

$$[(\Box - m^2)\delta_{\nu\mu} - \partial_\nu\partial_\rho\delta_{\mu\rho}]V_\mu(x) \equiv \Lambda_{\nu\mu}(\partial)V_\mu(x) = 0\,, \tag{1.4}$$

where $V(x)$ is defined by[3]

$$V_\mu(x) = (1/m)\partial_\lambda T_{\lambda\mu}(x)\,. \tag{1.5}$$

Equation (1.4) is the equation of motion of a spin-one field.

As we see from Eq. (1.5) the spin-one field is proportional to the divergence of the antisymmetric tensor. In Ref. 1 it was assumed that the fields $V_\mu(x)$ obey the usual commutation relations of the spin-one fields, and the interaction Hamiltonian in the interaction representation $\mathcal{H}_{\text{int}}(x, n)$ was calculated. It was found that in S-matrix calculations the $(\mathcal{H}_{\text{int}})_{\text{eff}}$ does not reduce to $-\mathcal{L}_{\text{int}}$ (Matthews's rule).[4-7]

The present paper deals with the quantization of the fields $V_\mu(x)=(1/m)\partial_\lambda T_{\lambda\mu}(x)$. In Sec. II the quantization of free fields is considered. It is shown that these fields are quantized as the usual spin-one fields. More specifically, we find that the expressions for the energy-momentum four-vector, the angular momentum tensor, and the charge are the same in both descriptions of spin-one mesons. In Sec. III the quantization of interacting fields is discussed. Also the commutation relations and the energy-momentum four-vector are given for a specific interaction Lagrangian.

II. QUANTIZATION OF FREE FIELDS

The free fields are usually quantized by the method of canonical quantization. According to this method, to each field which appears in the Lagrangian a canonical conjugate variable is defined, and certain commutation relations of the field variables and the independent canonical conjugate variables are postulated. We shall not apply the method of canonical quantization and we shall not quantize the field $T_{\mu\nu}$ directly.

Let $\Theta_{\mu\nu}$ be the symmetric energy-momentum tensor calculated from the Lagrangian under the assumption that all fields are classical fields. The energy-momentum four-vector P_μ and the angular momentum tensor $M_{\lambda\mu}$ are given by

$$P_\mu=-i\int\Theta_{\mu4}(x)d^3x, \tag{2.1}$$

$$\begin{aligned}M_{\lambda\mu}&=-i\int M_{\lambda\mu,4}(x)d^3x\\&=-i\int[x_\lambda\Theta_{\mu4}(x)-x_\mu\Theta_{\lambda4}(x)]\,d^3x,\\&\qquad\lambda,\mu=1,2,\ldots,4.\end{aligned} \tag{2.2}$$

In the quantum-field case we shall assume that P_μ and $M_{\lambda\mu}$ are again expressed by Eqs. (2.1) and (2.2), where all fields appearing in $\Theta_{\lambda\mu}$ are replaced by operator field functions, and these operators have been arranged in an appropriate order. This assumption, which is an application of the correspondence principle, and from which the commutation relations of the field operators can be derived, is sometimes taken as the fundamental postulate for the quantization of the wave fields.[8]

From the Lagrangian of Eq. (1.1) we obtain the following expression for the energy-momentum tensor $R_{\lambda\mu}$:

$$\begin{aligned}R_{\lambda\mu}=&-\partial_\kappa T^\dagger_{\lambda\nu}\partial_\mu T_{\kappa\nu}-\partial_\mu T^\dagger_{\kappa\nu}\partial_\kappa T_{\lambda\nu}\\&+\delta_{\lambda\mu}(\partial_\rho T^\dagger_{\nu\sigma}\partial_\nu T_{\rho\sigma}+\tfrac{1}{2}m^2T^\dagger_{\rho\sigma}T_{\rho\sigma}).\end{aligned} \tag{2.3}$$

The above expression is not symmetric in λ and μ. The symmetric energy-momentum tensor $\Theta_{\lambda\mu}$ is given by[9-11]

$$\Theta_{\lambda\mu}=R_{\lambda\mu}-\partial_\nu G_{\nu\lambda\mu}, \tag{2.4}$$

where

$$\begin{aligned}G_{\nu\lambda\mu}&=-G_{\lambda\nu\mu}\\&=\tfrac{1}{2}[(\partial_\lambda T^\dagger_{\nu\kappa}-\partial_\nu T^\dagger_{\lambda\kappa}+2\partial_\kappa T^\dagger_{\lambda\nu})T_{\mu\kappa}\\&\quad+(\partial_\lambda T^\dagger_{\mu\kappa}+\partial_\mu T^\dagger_{\lambda\kappa})T_{\nu\kappa}\\&\quad-(\partial_\mu T^\dagger_{\nu\kappa}+\partial_\nu T^\dagger_{\mu\kappa})T_{\lambda\kappa}]+\text{H.c.}\end{aligned} \tag{2.5}$$

Using Eqs. (1.2) and (1.5) we can write the expression $\Theta_{\lambda\mu}$ as follows:

$$\Theta_{\lambda\mu}=\Theta^P_{\lambda\mu}+\partial_\rho d_{\lambda\rho,\mu}+g_{\lambda\mu}+h_{\lambda\mu}, \tag{2.6}$$

where $\Theta^P_{\lambda\mu}$ is the usual symmetric form of the energy-momentum tensor of the spin-one field, given for instance by Pauli,[12]

$$\begin{aligned}\Theta^P_{\lambda\mu}=&H^\dagger_{\lambda\rho}H_{\mu\rho}+H^\dagger_{\mu\rho}H_{\lambda\rho}-\tfrac{1}{2}\delta_{\lambda\mu}H^\dagger_{\rho\sigma}H_{\rho\sigma}\\&+m^2(V^\dagger_\lambda V_\mu+V^\dagger_\mu V_\lambda-\delta_{\lambda\mu}V^\dagger_\rho V_\rho).\end{aligned} \tag{2.7}$$

The quantity $H_{\lambda\rho}$ of the previous equation is defined by

$$H_{\lambda\rho}\equiv\partial_\lambda V_\rho-\partial_\rho V_\lambda. \tag{2.8}$$

The tensor $d_{\lambda\rho,\mu}$ is given by

$$\begin{aligned}d_{\lambda\rho,\mu}=&\tfrac{1}{2}\partial_\nu[\delta_{\lambda\mu}(T^\dagger_{\nu\kappa}T_{\rho\kappa}+T^\dagger_{\rho\kappa}T_{\nu\kappa})\\&-\delta_{\rho\mu}(T^\dagger_{\nu\kappa}T_{\lambda\kappa}+T^\dagger_{\lambda\kappa}T_{\nu\kappa})+2(T^\dagger_{\rho\lambda}T_{\mu\nu}+T^\dagger_{\mu\nu}T_{\rho\lambda})]\\&+\tfrac{1}{2}[\partial_\rho(T^\dagger_{\lambda\kappa}T_{\mu\kappa}+T^\dagger_{\mu\kappa}T_{\lambda\kappa})-\partial_\lambda(T^\dagger_{\rho\kappa}T_{\mu\kappa}+T^\dagger_{\mu\kappa}T_{\rho\kappa})],\end{aligned} \tag{2.9}$$

while the tensors $g_{\lambda\mu}$ and $h_{\lambda\mu}$ are given by

$$\begin{aligned}g_{\lambda\mu}=&-V^\dagger_\mu\partial_\lambda\partial_\sigma V_\sigma-\partial_\lambda\partial_\sigma V^\dagger_\sigma V_\mu\\&+\tfrac{1}{2}\delta_{\lambda\mu}(V^\dagger_\rho\partial_\rho\partial_\sigma V_\sigma+\partial_\rho\partial_\sigma V^\dagger_\sigma V_\rho),\end{aligned} \tag{2.10}$$

$$\begin{aligned}h_{\lambda\mu}=&V^\dagger_\mu(\Box-m^2)V_\lambda+[(\Box-m^2)V^\dagger_\lambda]V_\mu\\&-\tfrac{1}{2}\delta_{\lambda\mu}\{V^\dagger_\sigma(\Box-m^2)V_\sigma+[(\Box-m^2)V^\dagger_\sigma]\,V_\sigma\}.\end{aligned} \tag{2.11}$$

Since $\partial_\sigma V_\sigma=0$, the expression $g_{\lambda\mu}$ vanishes identically. Also the expression $h_{\lambda\mu}$ vanishes in view of the field equations (1.4) and their Hermitian conjugates. The tensor $d_{\lambda\rho,\mu}$ is antisymmetric in λ and ρ. So $\partial_\rho d_{4\rho,\mu}=\partial_i d_{4i,\mu}$, $i=1,2,3$ and this term does not contribute to the integral of Eq. (2.1). We get

$$P_\mu=-i\int\Theta^P_{\mu4}(x)d^3x, \tag{2.12}$$

i.e., the P_μ can be calculated from the usual expression of the energy-momentum tensor of a spin-one particle.

The angular momentum tensor density $M_{\lambda\mu,\kappa}$ is given by[12, 13]

$$M_{\lambda\mu,\kappa}=x_\lambda\Theta_{\mu\kappa}-x_\mu\Theta_{\lambda\kappa}+\partial_\rho(x_\lambda G_{\rho\kappa\mu}-x_\mu G_{\rho\kappa\lambda}), \tag{2.13}$$

where the quantity $G_{\rho\kappa\mu}$ is given by Eq. (2.5). Dropping the vanishing terms $g_{\lambda\mu}$ and $h_{\lambda\mu}$ of Eq. (2.6) and substituting the resulting expression in Eq. (2.13), we get

$$M_{\lambda\mu,\kappa} = x_\lambda \Theta^P_{\mu\kappa} - x_\mu \Theta^P_{\lambda\kappa} + \partial_\rho [x_\lambda (G_{\rho\kappa\mu} + d_{\kappa\rho,\mu}) - x_\mu (G_{\rho\kappa\lambda} + d_{\kappa\rho,\lambda})] + d_{\kappa\mu,\lambda} - d_{\kappa\lambda,\mu} \,. \tag{2.14}$$

Since $G_{\rho\kappa\mu} = -G_{\kappa\rho\mu}$ and $d_{\kappa\rho,\mu} = -d_{\rho\kappa,\mu}$, the four-divergence of the above equation does not contribute to the angular momentum $M_{\lambda\mu}$ of Eq. (2.2), while the contribution of the last two terms of Eq. (2.14) becomes for $\lambda = i \neq 4$, $\mu = j \neq 4$ $(i \neq j)$

$$-i\int (d_{4j,i} - d_{4i,j}) d^3x = -i\int [\partial_l (T^\dagger_{j4} T_{il} + T^\dagger_{il} T_{j4} - T^\dagger_{i4} T_{jl} - T^\dagger_{jl} T_{i4}) + \tfrac{1}{2}\partial_j (T^\dagger_{4l} T_{il} + T^\dagger_{il} T_{4l}) - \tfrac{1}{2}\partial_i (T^\dagger_{4l} T_{jl} + T^\dagger_{jl} T_{4l})]\, d^3x\,, \qquad l = 1,\, 2,\, 3\,. \tag{2.15}$$

The three-divergence term of the above equation does not contribute to the integral. Also since by assumption the fields vanish at $x_i = \pm\infty$ and $x_j = \pm\infty$, the last two terms give a vanishing contribution to the integral. If $\lambda = 4$ and $\mu = j$ the contribution of the last two terms of Eq. (2.14) to the angular momentum M_{4j} of Eq. (2.2) is

$$-i\int d_{4j,4} d^3x = -i\int [\tfrac{1}{2}\partial_l (T^\dagger_{lk} T_{jk} + T^\dagger_{jk} T_{lk}) - \partial_l (T^\dagger_{j4} T_{l4} + T^\dagger_{l4} T_{j4}) + \partial_j (T^\dagger_{4k} T_{4k})]\, d^3x\,, \tag{2.16}$$

which vanishes. So we get

$$M_{\lambda\mu} = -i\int (x_\lambda \Theta^P_{\mu 4} - x_\mu \Theta^P_{\lambda 4}) d^3x\,, \tag{2.17}$$

as in the usual case of a spin-one particle.

The current J_μ is given by

$$J_\mu = i(H^\dagger_{\mu\sigma} V_\sigma - V^\dagger_\sigma H_{\mu\sigma}) + i\partial_\lambda (T^\dagger_{\lambda\sigma} T_{\mu\sigma} - T^\dagger_{\mu\sigma} T_{\lambda\sigma})\,. \tag{2.18}$$

The second term of the above expression is of the form $\partial_\lambda f_{\lambda\mu}$, $f_{\lambda\mu} = -f_{\mu\lambda}$, which means that it does not contribute to the charge q of the field. We get

$$q = -ie\int J_4(x) d^3x = e\int (H^\dagger_{4\sigma} V_\sigma - V^\dagger_\sigma H_{4\sigma}) d^3x\,. \tag{2.19}$$

The first term of Eq. (2.18) is the usual current vector of a spin-one particle.[12] Therefore, we have proven that the linear-momentum four-vector P_μ, the angular momentum tensor $M_{\lambda\mu}$, and the charge q are given in terms of the fields V_μ by the same expressions as in the well-known theory of a spin-one particle. This means that the fundamental postulate of quantization[8] will quantize the field V_μ as in the usual case. Details can be found in Ref. 8. If the field $T_{\mu\nu}$ is real, in which case $J_\mu = q = 0$, the quantization is done in an analogous fashion.

Let $V'_\mu(x)$ be a vector or axial-vector field which satisfies the Klein-Gordon equation $(\Box - m^2)V'_\mu = 0$. Then its spin-one part is given by

$$V'_\mu - (1/m^2)\partial_\mu \partial_\nu V'_\nu = (1/m^2)\partial_\nu (\partial_\nu V'_\mu - \partial_\mu V'_\nu)\,. \tag{2.20}$$

Therefore, we make the identification

$$T_{\nu\mu} = (1/m)(\partial_\nu V'_\mu - \partial_\mu V'_\nu) = (1/m)(\partial_\nu V_\mu - \partial_\mu V_\nu)\,, \tag{2.21}$$

where V_μ is the spin-one part of the field V'_μ. From Eq. (2.21) we get

$$V_\mu = (1/m)\partial_\nu T_{\nu\mu}\,, \tag{2.22}$$

which is the same with Eq. (1.5). The field equation is consistent with Eqs. (2.21) and (2.22).

The field $(1/m)\partial_\nu T_{\nu\mu}$ can be decomposed into positive- and negative-frequency parts,

$$\frac{1}{m}\partial_\nu T^{(\pm)}_{\nu\mu}(x) = \frac{1}{(2\pi)^{3/2}} \times \int d^3k\, e^{\pm ikx} \left[V'^{(\pm)}_\mu(\vec{k}) + \frac{k_\mu k_\nu}{m^2} V'^{(\pm)}_\nu(\vec{k}) \right]. \tag{2.23}$$

Imposing the subsidiary condition

$$k_\nu V'^{(\pm)}_\nu(\vec{k}) = 0\,, \tag{2.24}$$

Eq. (2.23) becomes

$$\frac{1}{m}\partial_\nu T^{(\pm)}_{\nu\mu}(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k\, e^{\pm ikx} V'^{(\pm)}_\mu(\vec{k})\,. \tag{2.25}$$

The field $(1/m)\partial_\nu T^\dagger_{\nu\mu}$ is decomposed in a similar fashion. The relations (2.24) and (2.25) are identical with Eqs. (4.20) and (4.14) of Ref. 8, and so one may proceed in introducing creation and annihilation operators, etc., as in the well-known description of a spin-one particle.

III. QUANTIZATION OF INTERACTING FIELDS

In Sec. II we demanded that the field operator $V_\mu(x) = (1/m)\partial_\nu T_{\nu\mu}(x)$ satisfy the free-field equation of motion

$$(\Box - m^2)\partial_\nu T_{\nu\mu}(x) = 0 \tag{3.1}$$

and the Heisenberg equation

$$-i\partial_\mu \partial_\nu T_{\nu\lambda}(x) = [\partial_\nu T_{\nu\lambda}(x), P_\mu]\,. \tag{3.2}$$

The energy-momentum four-vector P_μ was obtained by Eq. (2.1), and then Eq. (3.2) was used to derive the commutation relations of the free-field operators. It was found that the fields $(1/m)\partial_\nu T_{\nu\mu}$ satisfy the usual commutation relations of the spin-one fields. The vector P_μ satisfies the conservation equation

$$\frac{\delta}{\delta\sigma(x)} P_\mu = 0\,, \tag{3.3}$$

where $\sigma(x)$ is a spacelike surface in space-time.[5, 14]

In the interacting-field case instead of Eq. (3.1) we have[15]

$$(\Box - m^2)\partial_\nu \underline{T}^i_{\nu\mu} = \partial_\nu \underline{J}^i_{\nu\mu} + \partial_\nu(\partial_\mu \underline{J}^i_\nu - \partial_\nu \underline{J}^i_\mu)\,, \tag{3.4}$$

$$\underline{J}^i_{\nu\mu} = \frac{\partial \underline{\mathfrak{L}}_{\text{int}}}{\partial \underline{T}^i_{\nu\mu}}\,, \tag{3.5}$$

$$\underline{J}^i_\nu = \frac{\partial \underline{\mathfrak{L}}_{\text{int}}}{\partial_\mu \underline{T}^i_{\mu\nu}}\,, \tag{3.6}$$

and also we have the equations of motion of the other fields which appear in our Lagrangian. Again we want to construct a conserved energy-momentum four-vector $\underline{P}_\mu$ such that the Heisenberg equation is satisfied for all fields. We shall follow the method of Takahashi and Umezawa.[5, 14] For each field $Q_a(x)$ we introduce an auxiliary field $Q_a[x, \sigma]$ which is related to $Q_a(x)$ by the transformation

$$Q_a[x, \sigma] = S^{-1}(\sigma) Q_a(x) S(\sigma)\,. \tag{3.7}$$

The above equation implies that the commutation relations of the fields $Q_a(x)$ and $Q_a[x, \sigma]$ are the same. We express each Heisenberg field in terms of the auxiliary fields. Therefore, from the knowledge of the commutation relations of the auxiliary fields the commutation relations of the Heisenberg fields can be calculated.

The energy-momentum four-vector $\underline{P}_\mu$ in the Heisenberg representation is given by[5, 14, 16]

$$\underline{P}_\mu = S^{-1}(\sigma)\left(P_\mu + i\delta_{\mu 4}\int d^3x\,\mathcal{H}_{\text{int}}\right)S(\sigma)\,, \tag{3.8}$$

where P_μ is the energy-momentum four-vector of the free fields, $\mathcal{H}_{\text{int}}$ is the interaction Hamiltonian in the interaction representation, and for simplicity we have chosen a flat surface $\sigma(x) = \sigma(t)$ at a specific time t. The $\underline{P}_\mu$ of Eq. (3.8) is conserved, and satisfies the Heisenberg equation.

We have mentioned above a method of quantizing the interacting fields without details, which can be found particularly in Ref. 5. We shall apply this method in the case in which, besides an isotriplet of spin-one fields, we have an isodoublet of baryon fields ψ, with an interaction Lagrangian

$$\underline{\mathfrak{L}}_{\text{int}} = (ig/m)\partial_\nu \underline{T}^i_{\nu\mu}\underline{\bar{\psi}}\gamma_\mu \tau^i \underline{\psi}\,, \quad i = 1, 2, 3\,. \tag{3.9}$$

From Eqs. (3.5), (3.6), and (3.9), we get

$$\underline{J}^i_\mu = (ig/m)\underline{\bar{\psi}}\gamma_\mu \tau^i \underline{\psi}\,, \tag{3.10}$$

$$\underline{J}^i_{\mu\nu} = 0\,, \tag{3.11}$$

$$\underline{J}^l = -\frac{\partial \underline{\mathfrak{L}}_{\text{int}}}{\partial \underline{\bar{\psi}}^l} = -(ig/m)(\partial_\nu \underline{T}^i_{\nu\mu}\gamma_\mu \tau^i \underline{\psi})_l\,, \quad l = 1, 2\,. \tag{3.12}$$

Therefore, Eqs. (26) and (35) of Ref. 1 give

$$\partial_\rho \underline{T}^i_{\rho\nu}(x) = \partial_\rho T^i_{\rho\nu}(x|\sigma) - \underline{J}^i_\nu(x) - \underline{J}^i_\rho(x) n_\rho n_\nu\,, \tag{3.13}$$

$$\underline{\psi}^l(x) = \psi^l(x|\sigma)\,, \tag{3.14}$$

where to simplify the notation we write $(Q_a[x,\sigma])_{x|\sigma} \equiv Q_a(x|\sigma)$. The fields $(1/m)\partial_\nu T^i_{\nu\mu}(x|\sigma)$ and $\psi^l(x|\sigma)$ satisfy the usual commutation or anticommutation relations of free fields. At equal times we get[8, 11]

$$[\partial_\rho T^i_{\rho\mu}(x|\sigma), \partial'_\lambda T^j_{\lambda\nu}(x'|\sigma)]\,\delta(x_0 - x_0') = \delta_{ij}\left(\delta'_{\mu 4}\delta_{\nu 4}\frac{\partial}{\partial x_\mu} + \delta'_{\nu 4}\delta_{\mu 4}\frac{\partial}{\partial x_\nu}\right)\delta^4(x - x')\,, \tag{3.15}$$

$$\{\psi^l(x|\sigma), \bar{\psi}^m(x'|\sigma)\}\,\delta(x_0 - x_0') = \delta_{lm}\gamma_4\delta^4(x - x')\,, \tag{3.16}$$

where $\delta'_{\mu 4} = 1 - \delta_{\mu 4}$, $\delta'_{\nu 4} = 1 - \delta_{\nu 4}$ and we have no summation with respect to μ and ν. From Eqs. (3.10), (3.13)–(3.16), we get

$$\begin{aligned}[\partial_\rho \underline{T}_{\rho\mu}(x), \partial'_\sigma \underline{T}^j_{\sigma\nu}(x')]\,\delta(x_0 - x_0') = {}& \delta_{ij}\left(\delta'_{\mu 4}\delta_{\nu 4}\frac{\partial}{\partial x_\mu} + \delta'_{\nu 4}\delta_{\mu 4}\frac{\partial}{\partial x_\nu}\right)\delta^4(x - x') \\ & + (2ig^2/m^2)\underline{\psi}^\dagger(x)(\epsilon_{ijk}\tau^k\delta_{\mu\nu} + \delta_{ij}\sigma_{\mu\nu})\underline{\psi}(x)\delta^4(x - x')\delta'_{\mu 4}\delta'_{\nu 4}\,,\end{aligned} \tag{3.17}$$

$$\{\underline{\psi}^l(x), \underline{\bar{\psi}}^m(x')\}\,\delta(x_0 - x_0') = \delta_{lm}\gamma_4\delta^4(x - x')\,. \tag{3.18}$$

The energy-momentum four-vector is given by Eq. (3.8). We have

$$P_\mu=(P_\mu)_V+(P_\mu)_\psi\,, \tag{3.19}$$

where the free energy-momentum four-vector of the vector field $(P_\mu)_V$, which is obtained from Eqs. (2.1) and (2.7) if we add the contribution of the neutral vector-meson field, is given by

$$(P_\mu)_V=-(i/2m^2)\int d^3x[2T^i_{\mu\lambda}T^i_{4\lambda}-\tfrac{1}{2}\delta_{\mu4}T^i_{\nu\lambda}T^i_{\nu\lambda}+m^2(2\partial_\nu T^i_{\nu\mu}\partial_\lambda T^i_{\lambda4}-\delta_{\mu4}\partial_\nu T^i_{\nu\zeta}\partial_\lambda T^i_{\lambda\zeta})] \tag{3.20}$$

and

$$(P_\mu)_\psi=-\tfrac{1}{2}i\int d^3x(\psi^\dagger\overleftrightarrow{\partial}_\mu\psi)\,. \tag{3.21}$$

To express $S^{-1}(\sigma)P_\mu S(\sigma)$ in terms of Heisenberg fields, we use Eqs. (3.10), (3.13), (3.14), and the relations[17]

$$\underline{T}^i_{\mu\nu}(x)=T^i_{\mu\nu}(x|\sigma)\,, \tag{3.22}$$

$$\partial_\mu\underline{\psi}^l(x)=\partial_\mu\psi^l(x|\sigma)+n_\mu n_\lambda\gamma_\lambda\underline{J}^l(x)\,, \tag{3.23}$$

where the current $\underline{J}^l$ is given by Eq. (3.12). We find

$$S^{-1}(\sigma)\partial_\rho T^i_{\rho\nu}(x)S(\sigma)=\partial_\rho T^i_{\rho\nu}(x|\sigma)=\partial_\rho\underline{T}^i_{\rho\nu}(x)+\underline{J}^i_\nu(x)+\underline{J}^i_\rho n_\rho n_\nu\,, \tag{3.24}$$

$$S^{-1}(\sigma)T^i_{\mu\nu}(x)S(\sigma)=T^i_{\mu\nu}(x|\sigma)=\underline{T}^i_{\mu\nu}(x)\,, \tag{3.25}$$

$$S^{-1}(\sigma)\psi^l(x)S(\sigma)=\psi^l(x|\sigma)=\underline{\psi}^l(x)\,, \tag{3.26}$$

$$S^{-1}(\sigma)\partial_\mu\psi^l(x)S(\sigma)=\partial_\mu\psi^l(x|\sigma)=\partial_\mu\underline{\psi}^l(x)-n_\mu n_\lambda\gamma_\lambda\underline{J}^l(x)\,, \tag{3.27}$$

from which we get

$$S^{-1}(\sigma)P_\mu S(\sigma)=(\underline{P}_\mu)_V+(\underline{P}_\mu)_\psi+2i\delta_{\mu4}\int d^3x\,\underline{\mathcal{L}}_{\rm int}-i\int d^3x[\partial_\lambda\underline{T}^i_{\lambda4}\underline{J}^i_\mu-\tfrac{1}{2}\delta_{\mu4}\underline{J}^i_j\underline{J}^i_j]\,,\quad j=1,2,3\,. \tag{3.28}$$

In the above equation we denote by $(\underline{P}_\mu)_V$ and $(\underline{P}_\mu)_\psi$ the expressions (3.20) and (3.21) in which all fields have been replaced by Heisenberg fields. The current $\underline{J}^i_\mu$ is given by Eq. (3.10).

The interaction Hamiltonian in the interaction representation $\mathcal{H}_{\rm int}$ is calculated in the way indicated in Ref. 1. We find for a flat surface for which $n_\mu=(0,0,0,i)$

$$\mathcal{H}_{\rm int}=-\mathcal{L}_{\rm int}+\tfrac{1}{2}J^i_\mu J^i_\mu-\tfrac{1}{2}J^i_4J^i_4\,, \tag{3.29}$$

since in our case $J^i_{\mu\nu}=0$. Using Eqs. (3.10), (3.24), (3.26), and (3.29), we get

$$i\delta_{\mu4}S^{-1}(\sigma)\int d^3x\mathcal{H}_{\rm int}S(\sigma)=-i\delta_{\mu4}\int d^3x(\underline{\mathcal{L}}_{\rm int}+\tfrac{1}{2}\underline{J}^i_j\underline{J}^i_j)\,. \tag{3.30}$$

From Eqs. (3.8), (3.28), and (3.30) we get

$$\underline{P}_\mu=(\underline{P}_\mu)_V+(\underline{P}_\mu)_\psi+i\delta_{\mu4}\int d^3x\,\underline{\mathcal{L}}_{\rm int}-i\int d^3x\,\partial_\lambda\underline{T}^i_{\lambda4}\underline{J}^i_\mu\,, \tag{3.31}$$

which is the expression for the energy-momentum four-vector in the Heisenberg representation.

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[1]E. Kyriakopoulos, Phys. Rev. 183, 1318 (1969).

[2]Y. Takahashi and R. Palmer, Phys. Rev. D 1, 2974 (1970).

[3]Equation (1.4) is also obtained if $V_\mu(x)$ is defined by

$$V_\mu(x)=(1/m')\partial_\lambda T_{\lambda\mu}(x)\,,$$

where m' is a constant with the dimensions of a mass. The existence of the interaction Hamiltonian requires taking $m'=m$.

[4]P. T. Matthews, Phys. Rev. 76, 684 (1949).

[5]Y. Takahashi, *An Introduction to Field Quantization* (Pergamon, New York, 1969).

[6]The quantity $\Gamma_{\lambda,\mu\nu\rho\sigma}(\overrightarrow{\partial},-\overleftarrow{\partial})$ which satisfies the relation (called first identity in Ref. 5)

$$\Lambda_{\mu\nu\rho\sigma}(\overrightarrow{\partial})-\Lambda_{\mu\nu\rho\sigma}(-\overleftarrow{\partial})=(\overleftarrow{\partial}_\lambda+\overrightarrow{\partial}_\lambda)\Gamma_{\lambda,\mu\nu\rho\sigma}(\overrightarrow{\partial},-\overleftarrow{\partial})$$

is given by

$$\Gamma_{\lambda,\mu\nu\rho\sigma}(\overrightarrow{\partial},-\overleftarrow{\partial})=\tfrac{1}{4}[(\delta_{\lambda\rho}\delta_{\sigma\nu}-\delta_{\lambda\sigma}\delta_{\rho\nu})(\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu)-(\delta_{\lambda\rho}\delta_{\mu\sigma}-\delta_{\lambda\sigma}\delta_{\mu\rho})(\overrightarrow{\partial}_\nu-\overleftarrow{\partial}_\nu)+(\delta_{\lambda\mu}\delta_{\nu\sigma}-\delta_{\lambda\nu}\delta_{\mu\sigma})(\overrightarrow{\partial}_\rho-\overleftarrow{\partial}_\rho)-(\delta_{\lambda\mu}\delta_{\nu\rho}-\delta_{\lambda\nu}\delta_{\mu\rho})(\overrightarrow{\partial}_\sigma-\overleftarrow{\partial}_\sigma)]\,.$$

[7]Contrary to what was argued in Ref. 1, the operator $d_{\rho\sigma\lambda\zeta}(\partial)$ satisfying the relation (second identity of Ref. 5)

$$\Lambda_{\mu\nu\rho\sigma}(\partial)d_{\rho\sigma\lambda\zeta}(\partial)=d_{\rho\sigma\lambda\zeta}(\partial)\Lambda_{\mu\nu\rho\sigma}(\partial)=(\Box-m^2)(\delta_{\mu\lambda}\delta_{\nu\zeta}-\delta_{\mu\zeta}\delta_{\nu\lambda})$$

does exist, and it is given by

$$d_{\rho\sigma\lambda\zeta}(\partial)=\frac{1}{m^2}(\partial_\rho\partial_\lambda\delta_{\sigma\zeta}+\partial_\sigma\partial_\zeta\delta_{\rho\lambda}-\partial_\rho\partial_\zeta\delta_{\sigma\lambda}-\partial_\sigma\partial_\lambda\delta_{\rho\zeta})+\frac{\Box-m^2}{m^2}(\delta_{\rho\zeta}\delta_{\sigma\lambda}-\delta_{\rho\lambda}\delta_{\sigma\zeta})\,.$$

If we quantize $T_{\mu\nu}$ directly as in Ref. 2, Matthews's rule follows. This point was reexamined after a discussion with Professor F. Hadjioannou, whom I thank.

[8]N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York,

1959).
[9]F. J. Belinfante, Physica 6, 887 (1939); 7, 305 (1940).
[10]L. Rosenfeld, Mem. Acad. Roy. Belg. 6, 30 (1940).
[11]G. Wentzel, *Quantum Theory of Fields* (Interscience, New York, 1949).
[12]W. Pauli, Rev. Mod. Phys. 13, 203 (1941).
[13]We have $G_{\nu\lambda\mu} = f_{\mu,\nu\lambda}$, where $G_{\nu\lambda\mu}$ is defined by Eq. (I.17) of Ref. 11 and $f_{\mu,\nu\lambda}$ by Eqs. (I.13) of Ref. 12.
[14]H. Umezawa, *Quantum Field Theory* (North-Holland, Amsterdam, 1956).
[15]We use underlined letters for quantities in the Heisenberg representation and nonunderlined letters for quantities in the interaction representation.
[16]Y. Takahashi and H. Umezawa, Progr. Theoret. Phys. (Kyoto) 9, 14 (1953).
[17]Equation (3.22) is obtained from Eq. (3.11) above and Eq. (28) of Ref. 1, while Eq. (3.23) is easily derived from Eq. (10.30) of Ref. 14.

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Tensor Approach to Spin-One Mesons. III. Magnetic Dipole Moment and Electric Quadrupole Moment

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In previous papers the massive spin-one mesons were described by means of an antisymmetric second-rank tensor field. In the present paper their free Lagrangian is modified in such a way that when the electromagnetic interactions are introduced by the minimal substitution the mesons get an arbitrary magnetic dipole moment. The addition of other terms in the Lagrangian allows the spin-one mesons to also have an arbitrary electric quadrupole moment. The covariance of the S matrix to order e^2 is achieved by the addition of counterterms.

I. INTRODUCTION

In two previous papers[1,2] the massive spin-one mesons were described by means of an antisymmetric second-rank tensor field $T_{\mu\nu}$. In Ref. 2 we considered the Lagrangian

$$\mathcal{L}^0_{\text{free}} = \partial_\mu T^\dagger_{\nu\lambda}\partial_\nu T_{\mu\lambda} + \tfrac{1}{2}m^2 T^\dagger_{\mu\nu}T_{\mu\nu}\,, \qquad (1.1)$$

and we imposed the antisymmetry condition $T_{\mu\nu}(x) = -T_{\mu\nu}(x)$. The above Lagrangian gives the equation of motion

$$\partial_\mu\partial_\lambda T_{\lambda\nu} - \partial_\nu\partial_\lambda T_{\lambda\mu} - m^2 T_{\mu\nu} = 0\,. \qquad (1.2)$$

Differentiating Eq. (1.2) we get

$$[(\Box - m^2)\delta_{\nu\mu} - \partial_\nu\partial_\lambda\delta_{\mu\lambda}]V_\mu(x) = 0\,, \qquad (1.3)$$

where the field $V_\mu(x)$ is defined by

$$V_\mu(x) = (1/m)\partial_\lambda T_{\lambda\mu}(x). \qquad (1.4)$$

The field $V_\mu(x)$ describes the spin-one component of a vector or axial-vector field.

In Ref. 1 we described a way of obtaining the interaction Hamiltonian in the interaction representation $\mathcal{H}_{\text{int}}$, when the field $T_{\mu\nu}$ is involved. The $\mathcal{H}_{\text{int}}$ corresponding to a specific interaction Lagrangian was calculated, and it was shown that the S matrix coming from this $\mathcal{H}_{\text{int}}$ is covariant to any order in perturbation theory. In Ref. 2 the quantization was performed in the free-field case and also in the interacting-field case.

The magnetic dipole moment μ and the electric quadrupole moment Q of the $J^{PC} = 1^{--}$ nonet of vector mesons, except the ρ^0, ω, and φ, are not known. One wants a theory which allows arbitrary values of μ and Q. The Proca theory in which the electromagnetic interactions have been introduced by the minimal substitution describes particles with the "normal" magnetic dipole moment, i.e., with $\mu = e/2m$.[3] An extension was later made by Pauli[4] and by Corben and Schwinger[5] to include particles with arbitrary μ. Further terms can be added, which allow the mesons to also have an arbitrary electric quadrupole moment.[6,7] The values of μ and Q we obtain if we introduce in the Lagrangian of Eq. (1.1) the electromagnetic interactions by the minimal substitution are fixed. It is interesting to see if by proper generalization the tensor formalism can describe particles with arbitrary μ and Q. This is indeed the case, as shown in Sec. II.

In Sec. III the covariance of the S matrix to order e^2 is examined. It is shown that the S matrix can be made covariant to this order, if we add to our Lagrangian some additional terms (counterterms). The same method has been applied in the usual description of spin-one mesons.[7-9] Finally in Sec. IV the Feynman rules are given.

II. EXTENSION TO AN ARBITRARY MAGNETIC DIPOLE MOMENT AND ELECTRIC QUADRUPOLE MOMENT

The electromagnetic interactions are usually introduced by the minimal substitution