dimension three.

(4) $\Theta^{\rho}{}_{\rho}(x)$ is a sum of terms of fixed scale dimension; the $R^n(x)$ terms breaking only scale invariance, and the $S_0(x)$ and $S_8(x)$ terms breaking both scale invariance and SU(3)×SU(3).

(5) All requisite Schwinger terms are present.

Properties (2)–(4) are, at present, all deemed desirable in a theory of strong interactions. What remains to be explored are the dynamical consequences of the model.

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Representation of a Theory of Currents*

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A canonical representation is given for a recently proposed theory of currents. The solutions differ from the analogous representation of Sugawara's theory in that one less constraint is present. This allows one to break the additional O(4) symmetry which would otherwise be present. The difficulties in obtaining perturbative solutions are the same in both theories.

I. INTRODUCTION

In the preceding paper,[1] a theory of currents is presented in which all components of the currents have scale dimension three, all requisite Schwinger terms are present in the stress-tensor commutators, and in which the trace of the stress tensor is a sum of terms of fixed scale dimension. In this paper we use the techniques of Bardakci and Halpern[2] to solve the equations of motion of the theory, and give explicit Lagrangian representations of the solutions for the cases of SU(2) and SU(2)×SU(2). One less constraint is present than in Sugawara's theory.[3] This additional degree of freedom allows one to break the additional O(4) symmetry before the inclusion of partial conservation of axial-vector current (PCAC). As in the Sugawara theory, there is no obvious way to do perturbation theory.

II. FORMAL SOLUTION

The model presented in I is defined by the energy-momentum tensor

$$\Theta^{\mu\nu}(x)=-\frac{1}{4R(x)}\{V_a^\mu(x)V_a^\nu(x)+A_a^\mu(x)A_a^\nu(x)+\partial^\mu R(x)\partial^\nu R(x)-\tfrac{1}{2}g^{\mu\nu}[V_a^\rho(x)V_{a\rho}(x)+A_a^\rho(x)A_{a\rho}(x)+\partial^\rho R(x)\partial_\rho R(x)]\}$$
$$+g^{\mu\nu}\left(\sum_{n=0}^{N}\lambda_n R^n(x)+\epsilon_0 S_0(x)+\epsilon_8 S_8(x)\right)+\tfrac{1}{6}(\partial^\mu\partial^\nu-g^{\mu\nu}\partial^2)R(x) \qquad (1)$$

and the equal-time commutation relations (I20)–(I24) [i.e., Eqs. (20)–(24) of paper I]. The resulting equations of motion, (I26)–(I31), can be consolidated by writing[2]

$$V^\mu(x)=\tfrac{1}{2}\lambda_a V_a^\mu(x),\quad A^\mu(x)=\tfrac{1}{2}\lambda_a A_a^\mu(x),$$
$$S(x)=\tfrac{1}{2}\lambda_a S_a(x),\quad P(x)=\tfrac{1}{2}\lambda_a P_a(x). \qquad (2)$$

$$J^\mu(x)=V^\mu(x)+\gamma_5 A^\mu(x),$$
$$M(x)=\gamma^0 S(x)+i\gamma^0\gamma_5 P(x),\qquad (3)$$
$$\gamma^0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad \gamma_5=\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

$$V_a^\mu(x)=\mathrm{Tr}\,(\tfrac{1}{2}\lambda_a J^\mu(x)),\quad A_a^\mu(x)=\mathrm{Tr}\,(\tfrac{1}{2}\lambda_a\gamma_5 J^\mu(x)),$$
$$S_a(x)=\mathrm{Tr}\,(\tfrac{1}{2}\lambda_a\gamma^0 M(x)),\quad P_a(x)=i\,\mathrm{Tr}\,(\tfrac{1}{2}\lambda_a\gamma^0\gamma_5 M(x)).\qquad (4)$$

Equations (I27), (I28) and (I30), (I31), now become

$$\partial^\mu J^\nu(x)-\partial^\nu J^\mu(x)=\frac{i}{4R(x)}\{[J^\mu(x),J^\nu(x)]+4i[J^\mu(x)\partial^\nu R(x)-J^\nu(x)\partial^\mu R(x)]\},\qquad (5)$$
$$\partial^\mu M(x)=-\frac{i}{4R(x)}\{[M(x),J^\mu(x)]+2id\partial^\mu R(x)\}.\qquad (6)$$

The solutions to (5) and (6) follow immediately from the work of Bardakci and Halpern.[2] They are

$$J^\mu(x)=4iR(x)[U^{-1}(x)\partial^\mu U(x)-\tfrac{1}{6}\mathrm{Tr}\,(U^{-1}(x)\partial^\mu U(x))-\tfrac{1}{6}\gamma_5\,\mathrm{Tr}\,(\gamma_5 U^{-1}(x)\partial^\mu U(x))],\qquad (7)$$
$$M(x)=R^{d/2}(x)U^{-1}(x)C\,U(x),\qquad (8)$$

where U is any nonsingular 6×6 matrix, and C is any constant 6×6 matrix. The traces are subtracted in (7) to remove the singlet currents.

The next step is to represent the theory in Lagrangian form. We shall do this in the next two sections for the cases of SU(2) and SU(2)×SU(2). The form we adopt for $\mathcal{L}(x)$ is that suggested by (1):

$$\mathcal{L}(x)=-\frac{1}{8R(x)}[V_a^\mu(x)V_{a\mu}(x)+A_a^\mu(x)A_{a\mu}(x)+\partial^\mu R(x)\partial_\mu R(x)]-\sum_{n=0}^{N}\lambda_n R^n(x)-\epsilon_0 S_0(x)-\epsilon_8 S_8(x)$$
$$=-\frac{1}{8R(x)}[\mathrm{Tr}\,(J^\mu(x)J_\mu(x))+\partial^\mu R(x)\partial_\mu R(x)]-\sum_{n=0}^{N}\lambda_n R^n(x)-\epsilon_0 S_0(x)-\epsilon_8 S_8(x).\qquad (9)$$

III. SU(2)

We consider here an SU(2) version of the foregoing theory, described by

$$\mathcal{L}=-\frac{1}{8R}[V_a^\mu V_{a\mu}+\partial^\mu R\partial_\mu R]-\sum_{n=0}^{N}\lambda_n R^n$$
$$=-\frac{1}{8R}[2\,\mathrm{Tr}(V^\mu V_\mu)+\partial^\mu R\partial_\mu R]-\sum_{n=0}^{N}\lambda_n R^n,\qquad (10)$$

where

$$V^\mu=\tfrac{1}{2}\tau_a V_a^\mu$$
$$=4iR[U^{-1}\partial^\mu U-\tfrac{1}{2}\mathrm{Tr}(U^{-1}\partial^\mu U)].\qquad (11)$$

Introducing the variables[2]

$$U=\tfrac{1}{2}\tau_0 u_0+i\tfrac{1}{2}\tau_a u_a,$$
$$\tau_0=\begin{pmatrix}1&0\\0&1\end{pmatrix},$$
$$U^{-1}=\frac{4}{u^2}(\tfrac{1}{2}\tau_0 u_0-i\tfrac{1}{2}\tau_a u_a),\qquad (12)$$
$$u^2=u_0^{\,2}+u_a^{\,2}=u_0^{\,2}+\vec{u}^2=u_\alpha u_\alpha,$$

the Lagrangian and currents become

$$\mathcal{L}=-8R\left[\frac{1}{u^2}(\partial^\mu u_\alpha\partial_\mu u_\alpha)-\frac{1}{4u^4}\partial^\mu(u^2)\partial_\mu(u^2)\right]$$
$$-\frac{1}{8R}\partial^\mu R\partial_\mu R-\sum_{n=0}^{N}\lambda_n R^n,\qquad (13)$$

$$V_a^\mu=-\frac{8R}{u^2}[\epsilon_{abc}u_b\partial^\mu u_c+(u_0\partial^\mu u_a-u_a\partial^\mu u_0)].\qquad (14)$$

In the Sugawara theory,[3] which can be obtained by setting $4R=C$, the Lagrangian can be written[2]

$$\mathcal{L}=-2C\partial^\mu\hat{u}_\alpha\partial_\mu\hat{u}_\alpha,\quad \hat{u}_\alpha=u_\alpha/(u^2)^{1/2}.\qquad (15)$$

This implies the constraint $u^2=$ constant. In the present theory, because of the presence of $R(x)$, there is no constraint.

Writing

$$R=F(u_0^{\,2},\vec{u}^2),\qquad \partial^\mu R=F_1\partial^\mu u_0^{\,2}+F_2\partial^\mu\vec{u}^2,\qquad (16)$$

we have

$$\pi_0=\frac{\partial\mathcal{L}}{\partial(\partial^0 u_0)}$$
$$=-\frac{16F}{u^2}\partial^0 u_0+\left(\frac{16F}{u^4}-\frac{F_1^{\,2}}{F}\right)u_0^{\,2}\partial^0 u_0$$
$$+\left(\frac{16F}{u^4}-\frac{F_1F_2}{F}\right)u_0 u_a\partial^0 u_a,$$

$$\pi_a = \frac{\partial \mathcal{L}}{\partial(\partial^0 u_a)}$$
$$= -\frac{16F}{u^2}\partial^0 u_a + \left(\frac{16F}{u^4} - \frac{F_2{}^2}{F}\right)u_a u_b \partial^0 u_b + \left(\frac{16F}{u^4} - \frac{F_1 F_2}{F}\right)u_0 u_a \partial^0 u_0\,, \qquad (17)$$

$$[u_\alpha(x), \pi_\beta(y)] = i\delta_{\alpha\beta}\delta(\vec{x}-\vec{y})\,. \qquad (18)$$

From (14), (16), and (17), it follows that

$$\partial^0 R = -\frac{2Fu_\alpha \pi_\alpha}{F_1 u_0{}^2 + F_2 \vec{u}^2}\,, \qquad (19)$$

$$V_a^0 = \tfrac{1}{2}\epsilon_{abc}u_b\pi_c + \tfrac{1}{2}(u_0\pi_a - u_a\pi_0) + \frac{F_2 - F_1}{4F}u_0 u_a \partial^0 R\,. \qquad (20)$$

Using

$$[g(u(x)), \pi_\beta(y)] = \frac{\partial g}{\partial u_\alpha}[u_\alpha(x), \pi_\beta(y)] = i\frac{\partial g}{\partial u_\beta}\delta(\vec{x}-\vec{y}) \qquad (21)$$

and (18)–(20), it is straightforward to show that the commutation relations (I20)–(I22) are satisfied.

The commutators of u_0 and u_a with V_a^0 are

$$[V_a^0(x), u_0(y)] = \tfrac{1}{2}i\left(\frac{F_2 u^2}{F_1 u_0{}^2 + F_2\vec{u}^2}\right)u_a\delta(\vec{x}-\vec{y})\,,$$
$$[V_a^0(x), u_b(y)] = \left[\tfrac{1}{2}i\epsilon_{abc}u_c - \tfrac{1}{2}i\left(\delta_{ab} - \frac{(F_2 - F_1)u_a u_b}{F_1 u_0{}^2 + F_2\vec{u}^2}\right)u_0\right]\delta(\vec{x}-\vec{y})\,, \qquad (22)$$

showing that these fields transform nonlinearly under the isospin group. The only exception to this occurs when $F_1 = F_2$, which corresponds to $R = R(u^2)$. However, this choice for R is unacceptable since it implies an additional O(4) symmetry in the theory.

Finally, we note that Lagrange's equations of motion for the fields u_0 and u_a imply that R satisfies the equation of motion (I29).

IV. SU(2)×SU(2)

The extension of the preceding results to SU(2)×SU(2) proceeds straightforwardly, the only difference being that a constraint exists among the variables. The Lagrangian is

$$\mathcal{L} = -\frac{1}{8R}[V_a^\mu V_{a\mu} + A_a^\mu A_{a\mu} + \partial^\mu R\partial_\mu R] - \sum_{n=0}^{N}\lambda_n R^n$$
$$= -\frac{1}{8R}[\mathrm{Tr}(J^\mu J_\mu) + \partial^\mu R\partial_\mu R] - \sum_{n=0}^{N}\lambda_n R^n\,, \qquad (23)$$

where

$$J^\mu = \tfrac{1}{2}\tau_a V_a^\mu + \gamma_5\tfrac{1}{2}\tau_a A_a^\mu$$
$$= 4iR[U^{-1}\partial^\mu U - \tfrac{1}{4}\mathrm{Tr}(U^{-1}\partial^\mu U) - \tfrac{1}{4}\gamma_5\mathrm{Tr}(\gamma_5 U^{-1}\partial^\mu U)]\,. \qquad (24)$$

Introducing the variables[2]

$$U = (\tfrac{1}{2}\tau_0\phi_{+0} + i\tfrac{1}{2}\tau_a\phi_{+a})\tfrac{1}{2}(1+\gamma_5) + (\tfrac{1}{2}\tau_0\phi_{-0} + i\tfrac{1}{2}\tau_a\phi_{-a})\tfrac{1}{2}(1-\gamma_5)\,,$$
$$U^{-1} = \frac{4}{\phi_+{}^2}(\tfrac{1}{2}\tau_0\phi_{+0} - i\tfrac{1}{2}\tau_a\phi_{+a})\tfrac{1}{2}(1+\gamma_5) + \frac{4}{\phi_-{}^2}(\tfrac{1}{2}\tau_0\phi_{-0} - i\tfrac{1}{2}\tau_a\phi_{-a})\tfrac{1}{2}(1-\gamma_5)\,, \qquad (25)$$

the Lagrangian and currents become

$$\mathcal{L} = -4R\left[\frac{1}{\phi_+{}^2}\partial^\mu\phi_{+\alpha}\partial_\mu\phi_{+\alpha} - \frac{1}{4\phi_+{}^4}\partial^\mu(\phi_+{}^2)\partial_\mu(\phi_+{}^2) + (+\to -)\right] - \frac{1}{8R}\partial^\mu R\partial_\mu R - \sum_{n=0}^{N}\lambda_n R^n\,. \qquad (26)$$

$$V_a^\mu + A_a^\mu = -\frac{8R}{\phi_+{}^2}[\epsilon_{abc}\phi_{+b}\partial^\mu\phi_{+c} + (\phi_{+0}\partial^\mu\phi_{+a} - \phi_{+a}\partial^\mu\phi_{+0})]\,,$$
$$V_a^\mu - A_a^\mu = -\frac{8R}{\phi_-{}^2}[\epsilon_{abc}\phi_{-b}\partial^\mu\phi_{-c} + (\phi_{-0}\partial^\mu\phi_{-a} - \phi_{-a}\partial^\mu\phi_{-0})]\,. \qquad (27)$$

Writing

$$R = F(\tfrac{1}{2}(\phi_{+0}{}^2 + \phi_{-0}{}^2),\ \phi_{+0}\phi_{-0},\ \tfrac{1}{2}(\vec{\phi}_+{}^2 + \vec{\phi}_-{}^2),\ \vec{\phi}_+\cdot\vec{\phi}_-)\,,$$
$$\partial^\mu R = \tfrac{1}{2}F_1\partial^\mu(\phi_{+0}{}^2 + \phi_{-0}{}^2) + F_2\partial^\mu(\phi_{+0}\phi_{-0}) + \tfrac{1}{2}F_3\partial^\mu(\vec{\phi}_+{}^2 + \vec{\phi}_-{}^2) + F_4\partial^\mu(\vec{\phi}_+\cdot\vec{\phi}_-)\,, \qquad (28)$$

we can calculate the canonical momenta $\pi_{+\alpha}$ and $\pi_{-\alpha}$. However, not all of these momenta are independent. One finds

$$\begin{aligned}\phi_{+\alpha}\pi_{+\alpha} &= -\frac{1}{4F}(F_1\phi_{+0}^2+F_2\phi_{+0}\phi_{-0}+F_3\vec{\phi}_+^2+F_4\vec{\phi}_+\cdot\vec{\phi}_-)\partial^0 R\,,\\ \phi_{-\alpha}\pi_{-\alpha} &= -\frac{1}{4F}(F_1\phi_{-0}^2+F_2\phi_{+0}\phi_{-0}+F_3\vec{\phi}_-^2+F_4\vec{\phi}_+\cdot\vec{\phi}_-)\partial^0 R\,,\end{aligned} \tag{29}$$

which implies one constraint among the variables. We have not been able to find the constraint for arbitrary R. We consider, instead, as a particular example, $R=F[\frac{1}{2}(\phi_+^2+\phi_-^2)]$. A consideration of the constraints present in the Sugawara theory ($\phi_+^2=\phi_-^2=1$) (Ref. 2) leads one to conjecture that the correct constraint in this case is[4]

$$\phi_+^2=\phi_-^2\equiv\phi^2\,. \tag{30}$$

The validity of (30) is borne out by the consistency of all of the consequent results. Redefining

$$\begin{aligned}&R=F(\phi^2)\,,\\ &\partial^\mu R=F'\partial^\mu\phi^2\,,\end{aligned} \tag{31}$$

and rewriting $\mathfrak{L}$ in terms of ϕ, ϕ_{+a}, and ϕ_{-a}, we have

$$\begin{aligned}\pi &= \frac{\partial\mathfrak{L}}{\partial(\partial^0\phi)}\\ &= -\frac{4F}{\phi(\phi^2-\vec{\phi}_+^2)}\partial^0(\phi^2-\vec{\phi}_+^2)\\ &\quad -\frac{4F}{\phi(\phi^2-\vec{\phi}_-^2)}\partial^0(\phi^2-\vec{\phi}_-^2)+\frac{8F}{\phi^3}\partial^0\phi^2-\frac{F'\phi}{2F}\partial^0 R\,,\\ \pi_{+a} &= \frac{\partial\mathfrak{L}}{\partial(\partial^0\phi_{+a})}\\ &= -\frac{8F}{\phi^2}\partial^0\phi_{+a}+\frac{4F\phi_{+a}}{\phi^2(\phi^2-\vec{\phi}_+^2)}\partial^0(\phi^2-\vec{\phi}_+^2)\,,\\ \pi_{-a} &= \frac{\partial\mathfrak{L}}{\partial(\partial^0\phi_{-a})}\\ &= -\frac{8F}{\phi^2}\partial^0\phi_{-a}+\frac{4F\phi_{-a}}{\phi^2(\phi^2-\vec{\phi}_-^2)}\partial^0(\phi^2-\vec{\phi}_-^2)\,,\end{aligned} \tag{32}$$

$$\begin{aligned}&[\phi(x),\pi(y)]=i\delta(\vec{x}-\vec{y})\,,\\ &[\phi_{+a}(x),\pi_{+b}(y)]=i\delta_{ab}\delta(\vec{x}-\vec{y})\,,\\ &[\phi_{-a}(x),\pi_{-b}(y)]=i\delta_{ab}\delta(\vec{x}-\vec{y})\,.\end{aligned} \tag{33}$$

In terms of the momenta, the operator $\partial^0 R$ and the time components of the currents are, respectively,

$$\partial^0 R=-\frac{2F}{F'\phi^2}(\phi\pi+\phi_{+a}\pi_{+a}+\phi_{-a}\pi_{-a})\,, \tag{34}$$

$$\begin{aligned}&V_a^0+A_a^0=\epsilon_{abc}\phi_{+b}\pi_{+c}+(\phi^2-\vec{\phi}_+^2)^{1/2}\pi_{+a}\,,\\ &V_a^0-A_a^0=\epsilon_{abc}\phi_{-b}\pi_{-c}+(\phi^2-\vec{\phi}_-^2)^{1/2}\pi_{-a}\,.\end{aligned} \tag{35}$$

From (26), and (32)–(35), it is straightforward to show that the commutation relations (I20)–(I22) and the equation of motion (I29) are satisfied. We note, however, that with the above choice of R, the Lagrangian possesses an additional O(4) symmetry. It would, therefore, be desirable to obtain a representation with a more general R.

To introduce chiral symmetry breaking, we add a term $-\epsilon S$ to the Lagrangian,[5] where the scalar density S and the associated pseudoscalar densities P_a are assumed to satisfy

$$\begin{aligned}&[V_a^0(x),S(y)]=0\,,\\ &[V_a^0(x),P_b(y)]=i\epsilon_{abc}P_c(x)\delta(\vec{x}-\vec{y})\,,\\ &[A_a^0(x),S(y)]=iP_a(x)\delta(\vec{x}-\vec{y})\,,\\ &[A_a^0(x),P_b(y)]=-i\delta_{ab}S(x)\delta(\vec{x}-\vec{y})\,,\end{aligned} \tag{36}$$

and the commutation relation (I24). The divergence of the axial-vector current is now

$$\partial_\mu A_a^\mu=\epsilon P_a\,. \tag{37}$$

Using (4) and (8), with $C=\frac{1}{2}\gamma^0(1+\gamma_5)$, defines scalar and pseudoscalar densities

$$\begin{aligned}S&=R^{d/2}[(1/\phi^2)\phi_+\cdot\phi_-]\,,\\ P_a&=R^{d/2}\{(1/\phi^2)[\epsilon_{abc}\phi_{-b}\phi_{+c}+(\phi^2-\vec{\phi}_-^2)^{1/2}\phi_{+a}\\ &\qquad -(\phi^2-\vec{\phi}_+^2)^{1/2}\phi_{-a}]\}\,.\end{aligned} \tag{38}$$

From (34) and (35), it follows that these densities satisfy (I24) and (36), and are, therefore, a representation of the above form of symmetry breaking.

The alternate form of symmetry breaking, discussed by Bardakci and Halpern,[2] can also be represented in the form (8). Using $C=\frac{1}{2}\tau_1$ defines isovector, scalar, and pseudoscalar densities

$$S_a = \mathrm{Tr}(\tfrac{1}{2}\tau_a M)$$
$$= R^{d/2}(1/2\phi^2)\{(\phi_{+1}\phi_{+a}+\phi_{-1}\phi_{-a})+\epsilon_{1ab}[(\phi^2-\vec{\phi}_+^{\,2})^{1/2}\phi_{+b}+(\phi^2-\vec{\phi}_-^{\,2})^{1/2}\phi_{-b}]+\delta_{a1}(\phi^2-\vec{\phi}_+^{\,2}-\vec{\phi}_-^{\,2})\},$$
$$P_a = \mathrm{Tr}(\tfrac{1}{2}\tau_a\gamma_5 M) \tag{39}$$
$$= R^{d/2}(1/2\phi^2)\{(\phi_{+1}\phi_{+a}-\phi_{-1}\phi_{-a})+\epsilon_{1ab}[(\phi^2-\vec{\phi}_+^{\,2})^{1/2}\phi_{+b}-(\phi^2-\vec{\phi}_-^{\,2})^{1/2}\phi_{-b}]-\delta_{a1}(\vec{\phi}_+^{\,2}-\vec{\phi}_-^{\,2})\}.$$

From (34) and (35), one obtains for the commutators of S_a and P_a with A_a^0 and $\partial^0 R$

$$[A_a^0(x), S_b(y)] = i\epsilon_{abc}P_c(x)\delta(\vec{x}-\vec{y}),$$
$$[A_a^0(x), P_b(y)] = i\epsilon_{abc}S_c(x)\delta(\vec{x}-\vec{y}),$$
$$[S_a(x), \partial^0 R(y)] = -2diS_a(x)\delta(\vec{x}-\vec{y}), \tag{40}$$
$$[P_a(x), \partial^0 R(y)] = -2diP_a(x)\delta(\vec{x}-\vec{y}).$$

By adding a term $-\epsilon S_a S_a$ to the Lagrangian, one then has

$$\partial_\mu A_a^\mu = (2i\epsilon)\epsilon_{abc}S_b P_c. \tag{41}$$

This representation of PCAC corresponds to the dotted spinor representation of Ref. 2.

V. CONCLUDING REMARKS

The preceding representations of the theory of currents defined by (I20)–(I25) possess one more degree of freedom than the corresponding representations of the Sugawara theory.[2] This is due to the presence of the operator Schwinger term $R(x)$. This extra degree of freedom allows one to break the additional O(4) symmetry which would otherwise be present.

The two theories share a common problem in the fact that neither allows a straightforward way of doing perturbation theory. As an example of this in the present theory, we consider the SU(2) version described in Sec. III, with $R=F(u_0^2)$. Since $\langle R\rangle_0 \neq 0$,[6] one has $\langle u_0\rangle \neq 0$. In order to attempt a perturbation expansion, one first has to define a new field

$$\bar{u}_0 = u_0 - \langle u_0\rangle \tag{42}$$

with vanishing vacuum expectation value. One then attempts to deal with the inverse operators occurring in (13) by expanding them in the following manner:

$$\frac{1}{u^2} = \frac{1}{u_0^2+\vec{u}^2}$$
$$= [\langle u_0\rangle^2 + 2\langle u_0\rangle\bar{u}_0 + \bar{u}_0^2 + \vec{u}^2]^{-1}$$
$$= \frac{1}{\langle u_0\rangle^2}\{1 - 2\bar{u}_0/\langle u_0\rangle + 3\bar{u}_0^{\,2}/\langle u_0\rangle^2 - \vec{u}^2/\langle u_0\rangle^2 + \cdots\}. \tag{43}$$

However, since the fields u_a do not have definite isospin, the above expansion violates isospin conservation at each order. This follows from the fact that, whereas the isospin generators commute with the full Lagrangian (13), they do not commute with the above term-by-term expansion. Similar considerations apply in the SU(2)×SU(2) theory, with regard to the operators $(\phi^2-\vec{\phi}_+^{\,2})^{1/2}$ and $(\phi^2-\vec{\phi}_-^{\,2})^{1/2}$. What remains to be done is to develop a suitable approximation procedure to deal with Lagrangians such as (13) and (26).

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