accurately, as long as $(x_n^{2/3}/\gamma_n^2) \ll 1$, the second-order emission is of order $\frac{2}{3}\alpha$ and is essentially independent of electron energy, magnetic field strength, and Δn for all Δn such that $y_{nn'} \ll 1$.

The author wishes to thank T. Erber, E. Fischbach, N. Fuchs, T. K. Kuo, C. S. Shen, A. Tubis, and L. L. Van Zandt for their helpful discussions.

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*Work supported in part by Purdue Research Foundation.

[1]D. White, Phys. Rev. D 5, 1930 (1972).

[2]A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1966), Vol. II, p. 725.

[3]The expression for $V_{\vec{k}}^*(\tau)$ may be found in A. A. Sokolov and I. M. Ternov, *Synchrotron Radiation* (Pergamon, Berlin, 1968), p. 75.

[4]These calculations are carried out in detail in the Ph.D. dissertation by D. White.

[5]See Sokolov and Ternov (Ref. 3, pp. 74–77).

[6]Sokolov and Ternov (Ref. 3, pp. 84 and 85).

[7]T. Erber, Rev. Mod. Phys. 38, 654 (1966).

[8]These calculations are carried out in detail in Ref. 4. A similar integral is worked out by T. Erber in *High Magnetic Fields*, edited by H. Holm, B. Lax, and F. Bitter (Wiley, New York, 1962), pp. 717 and 718.

[9]See G. N. Watson, *Theory of Bessel Functions* (Cambridge, New York, 1962), pp. 73, 78, and 434–436.

[10]The absolute value signs are inserted because we wish to include upward transitions. The transition rates for upward transitions differ from the rates for downward transitions by a ratio of density of states with $n' < n$. For $(E_n - E_{n'})/E_n \ll 1$, this ratio is $\simeq 1$. See E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1961), p. 478.

[11]Sokolov and Ternov (Ref. 3, pp. 94 and 95).

[12]The asymptotic expression for $\int_y^\infty K_{5/3}(z)dz$ is found in Erber (Ref. 7, p. 654) and, the asymptotic expression for $K_{2/3}(y)$ is found in Sokolov and Ternov (Ref. 3, p. 30).

[13]Actually, since $y_{kn'}$, $y_{nn'}$, and y_{nk} are not integer multiples of η, the maximum value of $S_k^{(2)}$ is more like $\frac{3}{2}S_k^{(1)}$, but this too is of order $S_k^{(1)}$, and the argument is basically unchanged.

[14]It is good enough just to consider $(n-k) \gg 1$ here without choosing $(n-k)$ so large that k is near the ground state.

[15]To carry out the summation over k in Eq. (24) and Eq. (32) the author used the Purdue University CDC 6500 computer. Funds were made available by the president's fund.

PHYSICAL REVIEW D VOLUME 6, NUMBER 8 15 OCTOBER 1972

Theory of Currents*

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A modification of the Sugawara model is presented in which all components of the currents have scale dimension three, all requisite Schwinger terms are present in the stress-tensor commutators, and in which the trace of the stress tensor is a sum of terms of fixed scale dimension.

I. INTRODUCTION

In the original Sugawara model,[1] the vector and axial-vector currents are a complete set of operators (in the absence of symmetry breaking), in the sense that they alone determine the dynamics. The energy-momentum tensor of the theory is

$$\Theta^{\mu\nu}(x) = -\frac{1}{C}\{V_a^\mu(x)V_a^\nu(x) + A_a^\mu(x)A_a^\nu(x) - \tfrac{1}{2}g^{\mu\nu}[V_a^\rho(x)V_{a\rho}(x) + A_a^\rho(x)A_{a\rho}(x)]\}, \tag{1}$$

where the constant C is the Schwinger term in the V^0-V^i and A^0-A^i commutators.

$$[V_a^0(x), V_b^i(y)] = [A_a^0(x), A_b^i(y)] = if_{abc}V_c^i(x)\delta(\vec{x}-\vec{y}) - iC\partial_x^i\delta(\vec{x}-\vec{y}). \tag{2}$$

Two difficulties with the model are the implied parity degeneracy of states[2] and the fact that requisite Schwinger terms are absent in the Θ^{00}-Θ^{0i} and Θ^{0i}-Θ^{jk} commutation relations.[3] Another difficulty, from the viewpoint of asymptotic scale invariance, is that the temporal and spatial components of the currents have different scale dimensions, namely, three and one, respectively.[4]

Suppose that one poses the question as to whether the model can be modified so that all components of the currents have scale dimension three. The first thing that one must take note of is that the Schwinger term in the time-space current commutators can no longer be a c number, but must have scale dimension two.[4] We write this Schwinger term as

$$[V_a^0(x), V_b^i(y)] = [A_a^0(x), A_b^i(y)] = if_{abc}V_c^i(x)\delta(\vec{x}-\vec{y}) - 4iR(y)\partial_x^i\delta(\vec{x}-\vec{y}) . \tag{3}$$

The question that now forms is the following: "Are the vector and axial-vector currents and the operators $R(x)$ and $\partial^0 R(x)$ (plus appropriate scalar and pseudoscalar densities to account for symmetry breaking) a complete set of operators in the sense that they alone determine the dynamics?" This paper will answer this question in the affirmative. The resulting model is not parity-degenerate, and all requisite Schwinger terms are present in the stress-tensor commutators.

II. THE σ MODEL

We use as our guide the following model:

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\pi)(\partial^\mu\pi) + \tfrac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \tfrac{1}{2}\mu^2(\pi^2+\sigma^2) - \lambda(\pi^2+\sigma^2-\tfrac{1}{4}F^2)^2 - \tfrac{1}{2}\mu^2 F\sigma , \tag{4}$$

$$A^\mu = 2(\sigma\partial^\mu\pi - \pi\partial^\mu\sigma) , \tag{5}$$

$$\partial_\mu A^\mu = F\mu^2\pi , \tag{6}$$

$$\Theta^{\mu\nu} = \partial^\mu\pi\partial^\nu\pi + \partial^\mu\sigma\partial^\nu\sigma - g^{\mu\nu}\mathcal{L} - \tfrac{1}{6}(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)(\pi^2+\sigma^2) . \tag{7}$$

This is the σ model of Gell-Mann and Lévy,[5] with an isoscalar pion and with the energy-momentum tensor improved.[6] All fields in this model have scale dimension one. The operator $R(x)$ is

$$[A^0(x), A^i(y)] = 4i[\pi^2(y)+\sigma^2(y)]\partial_x^i\delta(\vec{x}-\vec{y}) = -4iR(y)\partial_x^i\delta(\vec{x}-\vec{y}) . \tag{8}$$

It is easy to see that this model can be rewritten in the following manner:

$$\Theta^{\mu\nu} = -\frac{1}{4R}\{A^\mu A^\nu + \partial^\mu R\partial^\nu R - \tfrac{1}{2}g^{\mu\nu}[A^2 + (\partial_\mu R)(\partial^\mu R)]\} + g^{\mu\nu}[-\tfrac{1}{2}\mu^2 R + \lambda(R+\tfrac{1}{4}F^2)^2 + \tfrac{1}{2}\mu^2 F\sigma] + \tfrac{1}{6}(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)R , \tag{9}$$

$$\begin{aligned} &[A^0(x), A^0(y)] = [A^i(x), A^j(y)] = 0 , \\ &[A^0(x), A^i(y)] = -4iR(y)\partial_x^i\delta(\vec{x}-\vec{y}) , \end{aligned} \tag{10}$$

$$\begin{aligned} &[A^0(x), R(y)] = [A^j(x), R(y)] = 0 , \\ &[A^0(x), \partial^0 R(y)] = 0 , \\ &[A^j(x), \partial^0 R(y)] = -4iA^j(x)\delta(\vec{x}-\vec{y}) , \end{aligned} \tag{11}$$

$$\begin{aligned} &[R(x), R(y)] = [\partial^0 R(x), \partial^0 R(y)] = 0 , \\ &[R(x), \partial^0 R(y)] = -4iR(x)\delta(\vec{x}-\vec{y}) , \end{aligned} \tag{12}$$

$$\begin{aligned} &[A^0(x), \sigma(y)] = 2i\pi(x)\delta(\vec{x}-\vec{y}) , \\ &[A^0(x), \pi(y)] = -2i\sigma(x)\delta(\vec{x}-\vec{y}) , \\ &[A^j(x), \sigma(y)] = [A^j(x), \pi(y)] = 0 , \\ &[\sigma(x), \sigma(y)] = [\sigma(x), \pi(y)] = [\pi(x), \pi(y)] = 0 , \end{aligned} \tag{13}$$

$$\begin{aligned} &[\sigma(x), R(y)] = [\pi(x), R(y)] = 0 , \\ &[\sigma(x), \partial^0 R(y)] = -2i\sigma(x)\delta(\vec{x}-\vec{y}) , \\ &[\pi(x), \partial^0 R(y)] = -2i\pi(x)\delta(\vec{x}-\vec{y}) . \end{aligned} \tag{14}$$

The equations of motion implied by (9)–(14) are

$$\partial_\mu A^\mu = F\mu^2\pi , \tag{15}$$

$$\partial^\mu A^\nu - \partial^\nu A^\mu = -\frac{1}{R}(A^\mu\partial^\nu R - A^\nu\partial^\mu R) , \tag{16}$$

$$\Box R = \frac{1}{2R}[A^2 + (\partial_\mu R)(\partial^\mu R)] - 2\mu^2 R + 8\lambda R(R+\tfrac{1}{4}F^2) + \mu^2 F\sigma , \tag{17}$$

$$\partial^\mu\sigma = \frac{1}{2R}(\pi A^\mu + \sigma\partial^\mu R) , \tag{18}$$

$$\partial^\mu\pi = -\frac{1}{2R}(\sigma A^\mu - \pi\partial^\mu R) . \tag{19}$$

III. THE MODEL

Using Sec. II as a guide, we assume the following equal-time commutation relations:

$$\begin{aligned} &[V_a^0(x), V_b^0(y)] = [A_a^0(x), A_b^0(y)] = if_{abc}V_c^0(x)\delta(\vec{x}-\vec{y}) , \\ &[V_a^0(x), V_b^i(y)] = [A_a^0(x), A_b^i(y)] = if_{abc}V_c^i(x)\delta(\vec{x}-\vec{y}) - 4i\delta_{ab}R(y)\partial_x^i\delta(\vec{x}-\vec{y}) , \\ &[V_a^0(x), A_b^\mu(y)] = [A_a^0(x), V_b^\mu(y)] = if_{abc}A_c^\mu(x)\delta(\vec{x}-\vec{y}) , \\ &[V_a^i(x), V_b^j(y)] = [V_a^i(x), A_b^j(y)] = [A_a^i(x), A_b^j(y)] = 0 , \end{aligned} \tag{20}$$

$$\begin{aligned} &[V_a^\mu(x), R(y)] = [A_a^\mu(x), R(y)] = 0 , \\ &[V_a^0(x), \partial^0 R(y)] = [A_a^0(x), \partial^0 R(y)] = 0 , \\ &[V_a^i(x), \partial^0 R(y)] = -4iV_a^i(x)\delta(\vec{x}-\vec{y}) , \\ &[A_a^i(x), \partial^0 R(y)] = -4iA_a^i(x)\delta(\vec{x}-\vec{y}) , \end{aligned} \tag{21}$$

$$\begin{aligned} &[R(x), R(y)] = [\partial^0 R(x), \partial^0 R(y)] = 0 , \\ &[R(x), \partial^0 R(y)] = -4iR(x)\delta(\vec{x}-\vec{y}) , \end{aligned} \tag{22}$$

$$[V_a^0(x), S_b(y)] = if_{abc}S_c(x)\delta(\vec{x}-\vec{y}),$$
$$[V_a^0(x), P_b(y)] = if_{abc}P_c(x)\delta(\vec{x}-\vec{y}),$$
$$[A_a^0(x), S_b(y)] = id_{abc}P_c(x)\delta(\vec{x}-\vec{y}),$$
$$[A_a^0(x), P_b(y)] = -id_{abc}S_c(x)\delta(\vec{x}-\vec{y}), \quad (23)$$
$$[V_a^i(x), S_b(y)] = [V_a^i(x), P_b(y)] = [A_a^i(x), S_b(y)] = [A_a^i(x), P_b(y)] = 0,$$

$$[S_a(x), S_b(y)] = [S_a(x), P_b(y)] = [P_a(x), P_b(y)] = 0,$$
$$[S_a(x), R(y)] = [P_a(x), R(y)] = 0,$$
$$[S_a(x), \partial^0 R(y)] = -2diS_a(x)\delta(\vec{x}-\vec{y}), \quad (24)$$
$$[P_a(x), \partial^0 R(y)] = -2diP_a(x)\delta(\vec{x}-\vec{y}).$$

The appropriate generalization of (9) is

$$\Theta^{\mu\nu}(x) = -\frac{1}{4R(x)}\{V_a^\mu(x)V_a^\nu(x) + A_a^\mu(x)A_a^\nu(x) + \partial^\mu R(x)\partial^\nu R(x) - \tfrac{1}{2}g^{\mu\nu}[V_a^\rho(x)V_{a\rho}(x) + A_a^\rho(x)A_{a\rho}(x) + \partial^\rho R(x)\partial_\rho R(x)]\}$$
$$+ g^{\mu\nu}\left(\sum_{n=0}^{N}\lambda_n R^n(x) + \epsilon_0 S_0(x) + \epsilon_8 S_8(x)\right) + \tfrac{1}{6}(\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2)R(x). \quad (25)$$

The equations of motion implied by (20)–(25) are

$$\partial_\mu V_a^\mu(x) = -\epsilon_8 f_{ab8}S_b(x),$$
$$\partial_\mu A_a^\mu(x) = [(\tfrac{2}{3})^{1/2}\epsilon_0\delta_{ab} + \epsilon_8 d_{ab8}]P_b(x), \quad (26)$$

$$\partial^\mu V_a^\nu(x) - \partial^\nu V_a^\mu(x) = -\frac{1}{4R(x)}\{f_{abc}[V_b^\mu(x)V_c^\nu(x) + A_b^\mu(x)A_c^\nu(x)] + 4[V_a^\mu(x)\partial^\nu R(x) - V_a^\nu(x)\partial^\mu R(x)]\}, \quad (27)$$

$$\partial^\mu A_a^\nu(x) - \partial^\nu A_a^\mu(x) = -\frac{1}{4R(x)}\{f_{abc}[A_b^\mu(x)V_c^\nu(x) + V_b^\mu(x)A_c^\nu(x)] + 4[A_a^\mu(x)\partial^\nu R(x) - A_a^\nu(x)\partial^\mu R(x)]\}, \quad (28)$$

$$\Box R(x) = \frac{1}{2R(x)}[V_a^\rho(x)V_{a\rho}(x) + A_a^\rho(x)A_{a\rho}(x) + \partial^\rho R(x)\partial_\rho R(x)] + 2\left(\sum_{n=0}^{N}2n\lambda_n R^n(x) + d[\epsilon_0 S_0(x) + \epsilon_8 S_8(x)]\right), \quad (29)$$

$$\partial^\mu S_a(x) = \frac{1}{4R(x)}[f_{abc}S_b(x)V_c^\mu(x) + d_{abc}P_b(x)A_c^\mu(x) + 2dS_a(x)\partial^\mu R(x)], \quad (30)$$

$$\partial^\mu P_a(x) = \frac{1}{4R(x)}[f_{abc}P_b(x)V_c^\mu(x) - d_{abc}S_b(x)A_c^\mu(x) + 2dP_a(x)\partial^\mu R(x)]. \quad (31)$$

Using (26)–(31), one obtains for the divergence and trace of $\Theta^{\mu\nu}(x)$

$$\partial_\mu\Theta^{\mu\nu}(x) = 0, \quad (32)$$

$$\Theta^\rho{}_\rho(x) = \sum_{n=0}^{N}(4-2n)\lambda_n R^n(x) + (4-d)[\epsilon_0 S_0(x) + \epsilon_8 S_8(x)]. \quad (33)$$

The various operator–stress-tensor commutation relations in this model are

$$[\Theta^{00}(x), V_a^0(y)] = i[-\partial_\mu V_a^\mu(x) + V_a^i(x)\partial_x^i]\delta(\vec{x}-\vec{y}),$$
$$[\Theta^{0i}(x), V_a^0(y)] = iV_a^0(x)\partial_x^i\delta(\vec{x}-\vec{y}),$$
$$[\Theta^{00}(x), V_a^j(y)] = i[-\partial^0 V_a^j(x) + V_a^0(y)\partial_x^j]\delta(\vec{x}-\vec{y}), \quad (34)$$
$$[\Theta^{0i}(x), V_a^j(y)] = i[-\partial^i V_a^j(x) + V_a^i(y)\partial_x^j + \tfrac{2}{3}V_a^j(y)\partial_x^i]\delta(\vec{x}-\vec{y}),$$
$$[V_a^0(x), \Theta^{ij}(y)] = i[g^{ij}(1-\tfrac{1}{3}d)\partial_\mu V_a^\mu(x) + \tfrac{1}{3}g^{ij}V_a^k(y) - g^{jk}V_a^i(y) - g^{ik}V_a^j(y)]\partial_x^k\delta(\vec{x}-\vec{y}),$$

$$[\Theta^{00}(x), R(y)] = -i\partial^0 R(x)\delta(\vec{x}-\vec{y}),$$
$$[\Theta^{0i}(x), R(y)] = i[-\partial^i R(x) + \tfrac{2}{3}R(y)\partial_x^i]\delta(\vec{x}-\vec{y}),$$
$$[\Theta^{00}(x), \partial^0 R(y)] = i[-(\partial^0)^2 R(x) + \partial^j R(y)\partial_x^j - \tfrac{2}{3}R(y)(\partial_x^j)^2]\delta(\vec{x}-\vec{y}), \quad (35)$$
$$[\Theta^{0i}(x), \partial^0 R(y)] = i[-\partial^i\partial^0 R(x) + \partial^0 R(y)\partial_x^i]\delta(\vec{x}-\vec{y}),$$

$$[\Theta^{00}(x), S_a(y)] = -i\partial^0 S_a(x)\delta(\vec{x}-\vec{y}),$$
$$[\Theta^{0i}(x), S_a(y)] = i[-\partial^i S_a(x) + \tfrac{1}{3}dS_a(y)\partial_x^i]\delta(\vec{x}-\vec{y}), \quad (36)$$

with corresponding results for $A_a^\mu(x)$ and $P_a(x)$. The last two commutation relations in (34) were derived in a previous paper by requiring that the spatial components of the currents have scale dimension three, and that the V_a^0-Θ^{00}-Θ^{0i} Jacobi identity be satisfied.[7]

Finally, for the stress-tensor commutators, we have

$$[\Theta^{00}(x),\Theta^{00}(y)]=i[\Theta^{0i}(x)+\Theta^{0i}(y)]\partial_x^i\,\delta(\vec{x}-\vec{y})\,, \tag{37}$$

$$[\Theta^{00}(x),\Theta^{0i}(y)]=i[\Theta^{ij}(x)-g^{ij}\Theta^{00}(y)]\partial_x^j\,\delta(\vec{x}-\vec{y})-\tfrac{1}{6}i(\partial_x^i\partial_x^j\partial_y^j-\tfrac{1}{3}\partial_y^i\partial_x^j\partial_x^j)[R(x)\delta(\vec{x}-\vec{y})]\,, \tag{38}$$

$$[\Theta^{0i}(x),\Theta^{0j}(y)]=i[\Theta^{0j}(x)\partial_x^i+\Theta^{0i}(y)\partial_x^j]\delta(\vec{x}-\vec{y})\,, \tag{39}$$

$$[\Theta^{00}(x),\Theta^{ij}(y)]=-i\partial^0\Theta^{ij}(x)\delta(\vec{x}-\vec{y})+i[\Theta^{0i}(y)\partial_x^j+\Theta^{0j}(y)\partial_x^i]\delta(\vec{x}-\vec{y})-\tfrac{1}{6}i(\partial_x^i\partial_x^j+\tfrac{1}{3}g^{ij}\partial_x^k\partial_x^k)[\partial^0R(x)\delta(\vec{x}-\vec{y})]\,, \tag{40}$$

$$\begin{aligned}[\Theta^{0i}(x),\Theta^{jk}(y)]=&-i\partial^i\Theta^{jk}(x)\delta(\vec{x}-\vec{y})+i\{\Theta^{ij}(y)\partial_x^k+\Theta^{ik}(y)\partial_x^j+\tfrac{2}{3}\Theta^{jk}(y)\partial_x^i+\tfrac{1}{3}g^{jk}[\Theta^{il}(y)\partial_x^l-\tfrac{1}{3}\Theta^{ll}(y)\partial_x^i]\}\delta(\vec{x}-\vec{y})\\
&+i\left(\tfrac{1}{3}\sum_{n=0}^{N}2n(4-2n)\lambda_nR^n(y)+\tfrac{1}{3}d(4-d)[\epsilon_0S_0(y)+\epsilon_8S_8(y)]-\Theta^{\rho}{}_{\rho}(y)\right)(\tfrac{1}{3}g^{jk}\partial_x^i)\delta(\vec{x}-\vec{y})\\
&-i\left(\frac{1}{24R(y)}[V_a^{\rho}(y)V_{a\rho}(y)+A_a^{\rho}(y)A_{a\rho}(y)+\partial^{\rho}R(y)\partial_{\rho}R(y)]+\sum_{n=0}^{N}(1-\tfrac{2}{3}n)\lambda_nR^n(y)\right.\\
&\qquad\left.+(1-\tfrac{1}{3}d)[\epsilon_0S_0(y)+\epsilon_8S_8(y)]\right)(g^{ij}\partial_x^k+g^{ik}\partial_x^j-\tfrac{2}{3}g^{jk}\partial_x^i)\delta(\vec{x}-\vec{y})\\
&-\tfrac{1}{6}i\{\tfrac{1}{2}\partial^i\partial^jR(y)\partial_x^k+\tfrac{1}{2}\partial^i\partial^kR(y)\partial_x^j-\tfrac{1}{3}\partial^j\partial^kR(y)\partial_x^i+\tfrac{1}{3}g^{jk}[\partial^i\partial^lR(y)\partial_x^l-\tfrac{1}{3}\partial^l\partial^lR(y)\partial_x^i]\}\delta(\vec{x}-\vec{y})\\
&+\tfrac{1}{6}i[\tfrac{1}{2}\partial^jR(y)\partial_x^i\partial_x^k+\tfrac{1}{2}\partial^kR(y)\partial_x^i\partial_x^j+\tfrac{1}{3}g^{jk}\partial^lR(y)\partial_x^i\partial_x^l]\delta(\vec{x}-\vec{y})\\
&+\tfrac{1}{6}i[\tfrac{1}{2}\partial_y^i(\partial_y^j\partial_x^k+\partial_y^k\partial_x^j)-\tfrac{1}{3}\partial_x^i\partial_y^j\partial_x^k]R(x)\delta(\vec{x}-\vec{y})\,.\end{aligned} \tag{41}$$

All requisite Schwinger terms are present in (37)–(41).[3]

IV. SUM RULES

In this section we derive two sum rules implied by the foregoing model. The following vacuum commutators define the relevant spectral functions:

$$\langle 0|[V_a^{\mu}(x),V_b^{\nu}(0)]|0\rangle=i\int_0^{\infty}d\mu^2[\rho_{ab}^{(V)}(\mu^2)(g^{\mu\nu}+\mu^{-2}\partial^{\mu}\partial^{\nu})+\partial^{\mu}\partial^{\nu}\rho_{ab}^{(S)}(\mu^2)]\Delta(x,\mu^2)\,, \tag{42}$$

$$\langle 0|[A_a^{\mu}(x),A_b^{\nu}(0)]|0\rangle=i\int_0^{\infty}d\mu^2[\rho_{ab}^{(A)}(\mu^2)(g^{\mu\nu}+\mu^{-2}\partial^{\mu}\partial^{\nu})+\partial^{\mu}\partial^{\nu}\rho_{ab}^{(P)}(\mu^2)]\Delta(x,\mu^2)\,, \tag{43}$$

$$\begin{aligned}\langle 0|[\Theta^{\mu\nu}(x),\Theta^{\rho\lambda}(0)]|0\rangle=i\int_0^{\infty}d\mu^2\{&\rho_2(\mu^2)[\tfrac{1}{2}(g^{\mu\rho}+\mu^{-2}\partial^{\mu}\partial^{\rho})(g^{\nu\lambda}+\mu^{-2}\partial^{\nu}\partial^{\lambda})+\tfrac{1}{2}(g^{\mu\lambda}+\mu^{-2}\partial^{\mu}\partial^{\lambda})(g^{\nu\rho}+\mu^{-2}\partial^{\nu}\partial^{\rho})\\
&-\tfrac{1}{3}(g^{\mu\nu}+\mu^{-2}\partial^{\mu}\partial^{\nu})(g^{\rho\lambda}+\mu^{-2}\partial^{\rho}\partial^{\lambda})]\\
&+\rho_0(\mu^2)(g^{\mu\nu}+\mu^{-2}\partial^{\mu}\partial^{\nu})(g^{\rho\lambda}+\mu^{-2}\partial^{\rho}\partial^{\lambda})\}\Delta(x,\mu^2)\,,\end{aligned} \tag{44}$$

where

$$\partial^0\Delta(x,\mu^2)|_{x^0=0}=-\delta(\vec{x})\,,\qquad(\Box+\mu^2)\Delta(x,\mu^2)=0\,. \tag{45}$$

From (20), (38), and (42)–(45), there follows

$$\langle 0|[V_a^0(x),V_a^i(0)]_{x^0=0}|0\rangle=\langle 0|[A_a^0(x),A_a^i(0)]_{x^0=0}|0\rangle=-4i\delta_{ab}\langle 0|R(0)|0\rangle\partial^i\,\delta(\vec{x})\,, \tag{46}$$

$$\langle 0|[\Theta^{00}(x),\Theta^{0i}(0)]_{x^0=0}|0\rangle=-\tfrac{1}{9}i\langle 0|R(0)|0\rangle\nabla^2\partial^i\,\delta(\vec{x})\,, \tag{47}$$

$$\int_0^{\infty}d\mu^2[\rho_{ab}^{(V)}(\mu^2)\mu^{-2}+\rho_{ab}^{(S)}(\mu^2)]=\int_0^{\infty}d\mu^2[\rho_{ab}^{(A)}(\mu^2)\mu^{-2}+\rho_{ab}^{(P)}(\mu^2)]=C\delta_{ab}\,, \tag{48}$$

$$\int_0^{\infty}d\mu^2\mu^{-4}[\tfrac{2}{3}\rho_2(\mu^2)+\rho_0(\mu^2)]=\tfrac{1}{36}C\,. \tag{49}$$

Equation (48) is the Weinberg sum rule.[8]

V. CONCLUDING REMARKS

The model presented in this paper is a complete dynamical theory with the following properties:

(1) $V_a^{\mu}(x)$, $A_a^{\mu}(x)$, $R(x)$, $\partial^0R(x)$, $S_a(x)$, and $P_a(x)$ are a complete set of operators.

(2) The terms which break SU(3)×SU(3) belong to the representation $(3,3^*)+(3^*,3)$, and have scale dimension d.

(3) All components of $V_a^{\mu}(x)$ and $A_a^{\mu}(x)$ have scale

dimension three.

(4) $\Theta^{\rho}{}_{\rho}(x)$ is a sum of terms of fixed scale dimension; the $R^n(x)$ terms breaking only scale invariance, and the $S_0(x)$ and $S_8(x)$ terms breaking both scale invariance and SU(3)×SU(3).

(5) All requisite Schwinger terms are present.

Properties (2)–(4) are, at present, all deemed desirable in a theory of strong interactions. What remains to be explored are the dynamical consequences of the model.

*Supported in part by the National Science Foundation.

[1]H. Sugawara, Phys. Rev. 170, 1659 (1968).
[2]R. Dashen and Y. Frishman, Phys. Rev. Letters 22, 572 (1969).
[3]D. Boulware and S. Deser, J. Math. Phys. 8, 1468 (1967).
[4]M. A. B. Bég, J. Bernstein, D. Gross, R. Jackiw, and A. Sirlin, Phys. Rev. Letters 25, 1231 (1970).
[5]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[6]C. G. Callan, Jr., S. Coleman, and R. Jackiw, Ann. Phys. (N. Y.) 59, 42 (1970).
[7]R. Perrin, Phys. Rev. D 6, 607 (1972).
[8]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

PHYSICAL REVIEW D VOLUME 6, NUMBER 8 15 OCTOBER 1972

Representation of a Theory of Currents*

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(Received 9 June 1972)

A canonical representation is given for a recently proposed theory of currents. The solutions differ from the analogous representation of Sugawara's theory in that one less constraint is present. This allows one to break the additional O(4) symmetry which would otherwise be present. The difficulties in obtaining perturbative solutions are the same in both theories.

I. INTRODUCTION

In the preceding paper,[1] a theory of currents is presented in which all components of the currents have scale dimension three, all requisite Schwinger terms are present in the stress-tensor commutators, and in which the trace of the stress tensor is a sum of terms of fixed scale dimension. In this paper we use the techniques of Bardakci and Halpern[2] to solve the equations of motion of the theory, and give explicit Lagrangian representations of the solutions for the cases of SU(2) and SU(2)×SU(2). One less constraint is present than in Sugawara's theory.[3] This additional degree of freedom allows one to break the additional O(4) symmetry before the inclusion of partial conservation of axial-vector current (PCAC). As in the Sugawara theory, there is no obvious way to do perturbation theory.

II. FORMAL SOLUTION

The model presented in I is defined by the energy-momentum tensor

$$\Theta^{\mu\nu}(x)=-\frac{1}{4R(x)}\{V_a^\mu(x)V_a^\nu(x)+A_a^\mu(x)A_a^\nu(x)+\partial^\mu R(x)\partial^\nu R(x)-\tfrac{1}{2}g^{\mu\nu}[V_a^\rho(x)V_{a\rho}(x)+A_a^\rho(x)A_{a\rho}(x)+\partial^\rho R(x)\partial_\rho R(x)]\}$$
$$+g^{\mu\nu}\left(\sum_{n=0}^{N}\lambda_n R^n(x)+\epsilon_0 S_0(x)+\epsilon_8 S_8(x)\right)+\tfrac{1}{6}(\partial^\mu\partial^\nu-g^{\mu\nu}\partial^2)R(x) \quad (1)$$

and the equal-time commutation relations (I20)–(I24) [i.e., Eqs. (20)–(24) of paper I]. The resulting equations of motion, (I26)–(I31), can be consolidated by writing[2]

$$V^\mu(x)=\tfrac{1}{2}\lambda_a V_a^\mu(x),\quad A^\mu(x)=\tfrac{1}{2}\lambda_a A_a^\mu(x),$$
$$S(x)=\tfrac{1}{2}\lambda_a S_a(x),\quad P(x)=\tfrac{1}{2}\lambda_a P_a(x). \quad (2)$$