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Consistent Solution to the K_{l3} Problem in the $(3,\bar{3})$ Model

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It is shown that the $(3,\bar{3})$ model can provide a consistent solution to the K_{l3} problem, provided that $\lambda_+ \approx 0.08$. Although only one experiment has given this result at the present time, it is only in this region that the following criteria can be satisfied simultaneously: (i) $\xi(0)$ is close to the experimental mean of -0.65, (ii) $f_+(0)$ is within 10% of unity, i.e., second-order SU(3) corrections are small, (iii) the vacuum is approximately SU(3)-invariant, (iv) the mass of the κ meson lies in the region where there is some experimental evidence for such a particle, and (v) the Callan-Treiman relation is reasonably well satisfied. It is also pointed out that in some earlier solutions where a much lower value of λ_+ was fitted, an important inequality involving $f_+(0)$ and m_κ was violated, thus rendering these models inconsistent.

I. INTRODUCTION

Recently, among the many papers devoted to the K_{l3} problem,[1-7] contradictory claims[2,3] have been made concerning the success of models involving current algebra and the usual $(3,\bar{3})$-symmetry-breaking Hamiltonian H'. In the present paper, we point out a basic inconsistency appearing in both solutions, but then show that the $(3,\bar{3})$ model can provide a consistent solution with the following desirable features: (i) $\xi(0)$ is relatively large and negative, in agreement with the present mean experimental value,[1] (ii) $f_+ \equiv f_+(0)$ is very close to the SU(3)-symmetric value of unity, as required by the Ademollo-Gatto theorem,[8] (iii) the vacuum is almost SU(3)-invariant, which is consistent with having approximate SU(3) multiplets of particles, (iv) the mass of the κ meson lies within the present rather vague experimental limits.[9] The unusual feature of our solution is that it needs a value of $\lambda_+ \approx 0.08$, and although this is consistent with one recent experiment,[10] it is certainly larger than the average experimental result; one consequence is that the ratio f_K/f_π is approximately 1.42, which is slightly higher than expected. Nevertheless, the Callan-Treiman relation[11] is now reasonably well satisfied, since the model predicts λ_0 to be positive, but much smaller than λ_+.

In fact, it may be thought that this last is a trivial result, but this is not the case, as may be seen from the following consideration. The ratio $f_K/f_\pi f_+ \equiv N$ is determined experimentally from the decay rates for the $K_{\mu 2}$, $\pi_{\mu 2}$, and K_{e3} processes, but the value for the last rate depends on[1,12] λ_+; in addition, the relevant equations in the $(3,\bar{3})$ model involve N, and hence, λ_+. Accordingly, for fixed values of f_+ and m_κ, a variation of λ_+ (i.e., N) produces a corresponding variation in λ_0, and there is no guarantee that for "reasonable" values of f_+ and m_κ, suitable combinations of λ_0 and λ_+ will occur which give the correct value for $\xi(0)$ or satisfy the Callan-Treiman relation, nor is it certain that the vacuum will be approximately SU(3)-invariant. Fortunately, it is possible simultaneously to satisfy all of these constraints in a reasonable fashion, although a brief consideration of Table I shows that this is not trivial. It is interesting that such consistent solutions exist, in the light of the other successes of current algebra and partial conservation of axial-vector current (PCAC).[13]

As far as the earlier, apparently contradictory, attempts[2,3] to fit the negative value of λ_0 in the con-

TABLE I. Variation of ω_K/ω_π and K_{l3} parameters with $N \equiv f_K/f_\pi f_+$ for $\kappa^2 = 1.0$ GeV^2, 1.6 GeV^2, and $f_+ \equiv f_+(0) = 1$; when $\omega_K/\omega_\pi = 1$, the vacuum is exactly SU(3)-invariant.

$f_+ = 1$		$\kappa^2 = 1.0$ GeV^2			$\kappa^2 = 1.6$ GeV^2		
N	λ_+	λ_0	ξ	ω_K/ω_π	λ_0	ξ	ω_K/ω_π
1.25	0.011	0.034	0.29	17.3	0.025	0.18	12.2
		0.015	0.06	1.20	0.018	0.09	2.60
1.30	0.032	0.036	0.03	17.4	0.029	−0.03	14.2
		0.018	−0.17	1.29	0.021	−0.14	1.56
1.35	0.052	0.039	−0.18	17.3	0.031	−0.26	12.7
		0.021	−0.38	1.40	0.025	−0.35	1.94
1.40	0.072	0.041	−0.39	17.0	0.033	−0.45	10.8
		0.024	−0.59	1.52	0.028	−0.54	2.41
1.42	0.080	0.041	−0.48	16.7	0.034	−0.57	9.4
		0.026	−0.67	1.62	0.030	−0.62	3.09
1.45	0.092	0.042	−0.62	16.4	0.035	−0.72	7.9
		0.028	−0.80	1.69	0.032	−0.75	3.48
1.50	0.112	0.044	−0.85	15.5	No real roots for λ_0.		
		0.031	−1.02	1.95			

text of the $(3, \bar{3})$ model are concerned, neither paper considers an inequality,[6] involving f_+ and m_κ, which is badly violated by their solutions, i.e., their solutions are not self-consistent. On the one hand, in a hard-meson analysis which includes a constant correction to pole dominance of the three two-point functions of the current divergences and imposes quadratic smoothness on the appropriate three-point function, Olshansky and Kang[2] show that the negative value for λ_0 leads to rather low values for f_+, as well as indicating that pole dominance is a much better approximation for the K and κ than for the π, i.e., quite the opposite of the expected situation. But in a similar type of analysis, Chan[3] claims that a satisfactory fit for $\xi(0)$ can be obtained in the $(3, \bar{3})$ model (he does not mention f_+), although this involves breaking the SU(3) symmetry of the vacuum to an appreciable extent. The last-mentioned consequence could possibly be considered sufficient grounds for rejecting the solution. But, in any case, both solutions [which are equivalent as far as $f_+(0)$ and $f'(0)$ are concerned] fail because the negative value of λ_0, together with $m_\kappa^2 = 1.1$ GeV^2, corresponds to $f_+ \simeq 0.70$, whereas the above-mentioned inequality leads to the condition $f_+ \geq 0.93$. In fact, when $N = 1.28$, consistent solutions with negative values for λ_0 cannot be obtained unless $m_\kappa \leq m_K$, a condition which is quite unacceptable at the present time. Finally, although Olshansky and Kang claim at the end of Ref. 2 that a larger value of λ_+ does lead to a consistent solution, they do not consider the effects of the corresponding increase in N, and thus make the problem appear much simpler than is actually the case.

In the remainder of the paper, we begin in Sec. II by defining the relevant quantities, and summarizing the hard-meson approach, while in Sec. III, we discuss the consistency of the model and consider the solutions for different values of N. A brief discussion is given in Sec. IV.

II. BASIC $(3, \bar{3})$ MODEL

In this section, we begin by giving the basic definitions and equations for the $(3, \bar{3})$ model, but details of the hard-meson analysis will be omitted since these appear in the paper by Olshansky and Kang.[2]

First of all, we define the various K_{l3} form factors and associated parameters.

$$\langle \pi^0(p) | V_\mu^{4+i5}(0) | K^-(q) \rangle \equiv \frac{1}{\sqrt{2}} [(q+p)_\mu f_+(t) + (q-p)_\mu f_-(t)],$$

$$\langle \pi^0(p) | \partial^\mu V_\mu^{4+i5}(0) | K^-(q) \rangle = \frac{-i}{\sqrt{2}} (K^2 - \pi^2) \left[f_+(t) + \frac{t}{K^2 - \pi^2} f_-(t) \right] \equiv \frac{-i}{\sqrt{2}} (K^2 - \pi^2) f(t), \tag{1}$$

where K^2 and π^2 denote the squared masses of the corresponding particles and $t \equiv (q-p)^2$. The usual parametrization of the form factors is

$$f_{\pm}(t)=f_{\pm}(0)\left(1+\frac{\lambda_{\pm}}{\pi^2}t+\cdots\right),$$

$$f(t)=f_+(0)\left(1+\frac{\lambda_0}{\pi^2}t+\cdots\right), \tag{2}$$

$$\xi\equiv\xi(0)\equiv\frac{f_-(0)}{f_+(0)}=\frac{K^2-\pi^2}{\pi^2}(\lambda_0-\lambda_+).$$

Next, we specify the symmetry-breaking Hamiltonian density H' by[14,15]

$$H'=\epsilon_0 u_0+\epsilon_8 u_8, \tag{3}$$

where u_0 and u_8 are the usual scalar densities transforming according to the $(3,\bar{3})\oplus(\bar{3},3)$ representation of SU(3)⊗SU(3). Their equal-time commutation relations with the SU(3)⊗SU(3) generators $V^i(t)$ and $A^i(t)$ [where $V^i(t)\equiv\int d^3x V_0^i(x)$] are

$$\begin{aligned}
&[V^i(x_0),u^j(x)]=if^{ijk}u^k(x),\\
&[V^i(x_0),v^j(x)]=if^{ijk}v^k(x),\\
&[A^i(x_0),u^j(x)]=-id^{ijk}v^k(x),\\
&[A^i(x_0),v^j(x)]=id^{ijk}u^k(x).
\end{aligned} \tag{4}$$

From these relations, the current divergences take the form

$$\begin{aligned}
&\partial^\mu A_\mu^3=-\epsilon_\pi v^3,\\
&\partial^\mu A_\mu^{4\pm i5}=-\epsilon_K v^{4\pm i5},\\
&\partial^\mu V_\mu^{4\pm i5}=\pm i\epsilon_\kappa u^{4\pm i5},
\end{aligned} \tag{5}$$

where $\epsilon_\pi\equiv(1/\sqrt{3})(\sqrt{2}\,\epsilon_0+\epsilon_8)$, $\epsilon_\kappa\equiv\frac{1}{2}\sqrt{3}\,\epsilon_8$, and $\epsilon_K\equiv\epsilon_\pi-\epsilon_\kappa$. Also, the decay constants f_i are defined by

$$\begin{aligned}
&\langle 0|\partial^\mu A_\mu^3(0)|\pi^0\rangle=f_\pi\pi^2,\\
&\langle 0|\partial^\mu A_\mu^{4+i5}(0)|K^-\rangle=\sqrt{2}\,f_K K^2,\\
&\langle 0|\partial^\mu V_\mu^{4+i5}(0)|\kappa^-\rangle=i\sqrt{2}\,f_\kappa\kappa^2.
\end{aligned} \tag{6}$$

Next if we assume pole dominance of the usual two-point functions $\Delta_i(p^2)$ $(i=\pi,K,\kappa)$, where, for example,

$$\begin{aligned}
\Delta_K(p^2)&\equiv-\tfrac{1}{2}i\int d^4x\,e^{ipx}\\
&\quad\times\langle 0|T\{\partial^\mu A_\mu^{4+i5}(x)\partial^\nu A_\nu^{4-i5}(0)\}|0\rangle\\
&=\frac{f_K{}^2K^4}{p^2-K^2},
\end{aligned} \tag{7}$$

then partial integration leads to the relations

$$\begin{aligned}
&\Delta_\pi(0)=-f_\pi{}^2\pi^2=\epsilon_\pi\omega_\pi,\\
&\Delta_K(0)=-f_K{}^2K^2=\epsilon_K\omega_K,\\
&\Delta_\kappa(0)=-f_\kappa{}^2\kappa^2=\epsilon_\kappa\omega_\kappa,
\end{aligned} \tag{8}$$

where $\omega_i\equiv\langle 0|u_i|0\rangle$, and ω_π, ω_K, and ω_κ are defined in the same way as the ϵ_π etc. above. From these last equations, we find

$$[(f_K{}^2K^2+f_\pi{}^2\pi^2)-f_\kappa{}^2\kappa^2]^2-4f_K{}^2K^2f_\pi{}^2\pi^2$$
$$=(\epsilon_\kappa\omega_\pi-\epsilon_\pi\omega_\kappa)^2\geq 0, \tag{9}$$

and this leads to the alternative inequalities[6]

$$f_\kappa\kappa\leq f_K K-f_\pi\pi \tag{10a}$$

or

$$f_\kappa\kappa\geq f_K K+f_\pi\pi, \tag{10b}$$

where we assume that all of the f_i are positive, and that $f_K K\geq f_\pi\pi$. However, the second inequality is not satisfied in the SU(3) limit, so we shall follow the usual practice[6] of rejecting it in favor of the first condition.

For the hard-meson analysis, the three-point function $G(p^2,q^2,t)$ is used, where

$$G(p^2,q^2,t)=i^3\frac{\pi^2-p^2}{f_\pi\pi^2}\frac{K^2-q^2}{\sqrt{2}\,f_K K^2}\frac{\kappa^2-t}{\sqrt{2}\,f_\kappa\kappa^2}\iint d^4x\,d^4y\,e^{ipx-iqy}\langle 0|T\{\partial^\mu A_\mu^3(x)\partial^\nu V_\nu^{4+i5}(0)\partial^\lambda A_\lambda^{4-i5}(y)\}|0\rangle; \tag{11}$$

this is related to $f(t)$ by

$$(K^2-\pi^2)f(t)=\frac{2f_\kappa\kappa^2}{\kappa^2-t}G(\pi^2,K^2,t). \tag{12}$$

The standard procedure of partial integration, etc.,[16] eventually leads to the results[17]

$$f_+(0)=\frac{1}{2f_K f_\pi}(f_K{}^2+f_\pi{}^2-f_\kappa{}^2), \tag{13}$$

$$\begin{aligned}
f'(0)&=\frac{f_+(0)}{\kappa^2}+\frac{1}{K^2-\pi^2}\frac{1}{2f_K f_\pi}\left(f_K{}^2-f_\pi{}^2+f_\kappa{}^2+2f_K{}^2\frac{K^2}{\kappa^2}\frac{\epsilon_\kappa}{\epsilon_K}\right)\\
&=\frac{f_+(0)}{\kappa^2}+\frac{1}{K^2-\pi^2}\frac{1}{2f_K f_\pi}\left[f_K{}^2-f_\pi{}^2+f_\kappa{}^2-2f_K{}^2\frac{K^2}{\kappa^2}\left(1-\frac{f_\pi{}^2\pi^2}{f_K{}^2K^2}\frac{\omega_K}{\omega_\pi}\right)\right],
\end{aligned} \tag{14}$$

where the second version for $f'(0)$ is valid even when a (1, 8) term is included in H' (as pointed out by Chan[3]).

Finally, the quantity $N \equiv f_K/f_\pi f_+$ can be determined from the decay rates for the K_{e3}, $K_{\mu 2}$, and $\pi_{\mu 2}$ processes in the context of the single-angle Cabibbo theory.[12] Firstly, the Dalitz-plot expression for $\Gamma(K_{e3})$ is[1]

$$\Gamma(K_{e3}) = A f_+^2 \sin^2\theta\,(1 + \alpha\lambda_+ + \beta\lambda_+^2)\,, \tag{15}$$

where A, α, and β are factors depending on masses, etc., with numerical values $A/\Gamma(K_{e3}) = 20.75$, $\alpha = 3.6995$, and $\beta = 5.4777$. Then

$$N^2 = \tan^2\theta_A^M\left(\frac{1}{f_+^2\sin^2\theta} - \frac{1}{f_+^2}\right)$$

$$= \tan^2\theta_A^M\left[\frac{A}{\Gamma(K_{e3})}(1 + \alpha\lambda_+ + \beta\lambda_+^2) - \frac{1}{f_+^2}\right], \tag{16}$$

where $\tan\theta_A^M$ is a number determined from the ratio $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$; when λ_+ takes the value 0.023 corresponding to K^* dominance of $f_+(t)$ and $f_+ = 1$, then $N = 1.28$.

III. NUMERICAL RESULTS

In this section, we begin by discussing the effects of the inequality (10a) and then consider the variation of the results with N. First of all, the quantity f_κ^2 can be eliminated by using Eq. (13):

$$\left(\frac{f_\kappa}{f_\pi}\right)^2 = 1 + N(N-2)f_+^2\,. \tag{17}$$

Thus, from Eqs. (10) and (17), we obtain the important relation

$$\kappa \leq \frac{KNf_+ - \pi}{[1 - N(2-N)f_+^2]^{1/2}}\,. \tag{18}$$

From (17), we see that f_+ has an upper bound given by

$$f_+^2 \leq 1/N(2-N) \quad (\text{for } 0 < N < 2) \tag{19}$$

and a rearrangement of (18) also yields a lower bound for f_+ which now depends on κ.

Next, from Eq. (14), we can obtain the following expression for ω_K/ω_π, viz.,

$$\frac{\omega_K}{\omega_\pi} = \frac{Nf_+^2}{\pi^2}[Y + \kappa^2 - N(\kappa^2 - K^2)]\,, \tag{20}$$

where

$$Y \equiv (K^2 - \pi^2)(\lambda_0\kappa^2/\pi^2 - 1)\,. \tag{21}$$

This ratio is a useful quantity for measuring the SU(3) symmetry-breaking of the vacuum since $\omega_K/\omega_\pi = 1$ in the SU(3) limit. In addition, we can obtain an alternative expression involving ω_K/ω_π from Eq. (8):

$$\frac{f_K^2K^2}{\omega_K} = \frac{f_\pi^2\pi^2}{\omega_\pi} - \frac{f_\kappa^2\kappa^2}{\omega_\kappa}\,. \tag{22}$$

By substituting for f_κ^2 and ω_K/ω_π, we derive the following quadratic equation for Y in terms of N, f_+, and κ^2, viz.,

$$Y^2 - Y[N(\kappa^2 - K^2) - (\kappa^2 - \pi^2)/Nf_+^2] + \kappa^2[N(\kappa^2 - K^2) + (\kappa^2 - \pi^2)/Nf_+^2 - \kappa^2 - (\kappa^2 - K^2 - \pi^2)/f_+^2] = 0\,. \tag{23}$$

To illustrate how we can use (18), (19), and (23), we begin by putting $N = 1.28$, the conventional value used in Refs. 2 and 3. For a fixed value of κ^2, f_+ is constrained to lie in a narrow range, and we can use this information to determine the corresponding limits on λ_0 from Eq. (23). The results for four values of κ^2 are given below:

$\kappa^2 = 1.60$,	$0.97 < f_+ < 1.05$,	$0.019 < \lambda_0 < 0.034$;
$\kappa^2 = 1.00$,	$0.93 < f_+ < 1.05$,	$0.017 < \lambda_0 < 0.041$;
$\kappa^2 = 0.50$,	$0.86 < f_+ < 1.05$,	$0.011 < \lambda_0 < 0.020$,
		$0.023 < \lambda_0 < 0.059$;
$\kappa^2 = 0.25$,	$0.76 < f_+ < 1.05$,	$-0.001 < \lambda_0 < 0.020$,
		$0.026 < \lambda_0 < 0.095$.

The first two values of κ^2 correspond to the limits of the region in which there is some slight experimental evidence for the existence of a κ meson, while the other values are used simply to illustrate how low κ^2 has to become in order that λ_0 attain negative values. Clearly, the mean experimental value for λ_0 of -0.024 cannot be obtained for realistic values of κ^2 when $N = 1.28$, so some modifications must be made.

One change which is partially successful is the inclusion of a (1, 8) term in H', since the two additional parameters introduced allow sufficient freedom to fit a negative λ_0 with κ^2 around 1.0 GeV^2: however, as is obvious from Chan's paper, this corresponds to a large breakdown of the vacuum SU(3) symmetry, since ω_K/ω_π is not close to unity.

Another, somewhat different, alteration is to allow N to vary over a range of values,[18] and to check whether it is possible to obtain a "good" solution, i.e., a self-consistent solution which involves values of parameters which are close, ei-

TABLE II. Variation of ω_K/ω_π and K_{l3} parameters with f_+ for $\kappa^2=1.0$ GeV^2, 1.6 GeV^2, and $N=1.42$.

$N=1.42$		$\kappa^2=1.0$ GeV^2			$\kappa^2=1.6$ GeV^2		
f_+	λ_+	λ_0	ξ	ω_K/ω_π	λ_0	ξ	ω_K/ω_π
0.95	0.081	0.027	−0.67	2.90	No real roots for λ_0.		
1.00	0.080	0.026	−0.67	1.62	0.030	−0.62	3.09
1.05	0.079	0.025	−0.67	1.21	0.029	−0.63	1.37
1.10	0.078	0.025	−0.66	0.98	0.028	−0.62	0.97

ther to present experimental numbers, or else – as in the case of f_+ and ω_K/ω_π – to some desirable theoretical value.

In more detail, we take κ^2, f_+, and N as our independent variables and require that κ^2 lie in the "experimental" range between 1.0 and 1.6 GeV^2, while f_+ is restricted to the region close to unity, since the Ademollo-Gatto theorem[8] tells us that $|f_+-1|$ is of order $O(\epsilon_\kappa^2)$. As for N, we should expect that the combination $Nf_+=f_\pi$ is not too far removed from its SU(3)-symmetric value of unity; in fact, to obtain negative values for ξ in the model, we focus our attention on the range $1.25 \leq N \leq 1.50$. Our dependent variables are ξ, ω_K/ω_π, and λ_+: experimental values for ξ range widely, although more recent values appear to be clustering around −0.65; as mentioned before, $|\omega_K/\omega_\pi -1|$ should be small in order to keep the vacuum almost SU(3)-invariant; and the experimental range for λ_+ is from 0.015 to 0.080, corresponding to N lying between 1.26 and 1.42. For each combination of κ^2, N, and f_+, we find λ_+ from Eq. (16) – actually, only the positive root is relevant – and λ_0 from Eqs. (23) and (21); then ξ is evaluated from Eq. (3).

To illustrate general trends, we keep $f_+=1$ and consider the variation of λ_+, λ_0, ξ, and ω_K/ω_π for $\kappa^2=1.0$ and 1.6 GeV^2 as N goes from 1.25 to 1.50. The results are given in Table I and we see immediately that for $-0.7 \leq \xi \leq -0.6$, we require N between 1.40 and 1.45, while the value of κ^2 is not critical. However, for ω_K/ω_π near unity (i.e., less than about 1.5), only the lower roots for λ_0 are relevant, and even then, we need κ^2 around 1.0 GeV^2. From the pattern of results, it appears that values of κ^2 below 1.0 GeV^2 would yield $\xi \simeq -0.65$ with $|\omega_K/\omega_\pi-1|$ much smaller, and also for slightly smaller values of N, but there is no evidence for a κ meson in this region.

Next, to demonstrate the variation of the parameters with f_+, we fix $N=1.42$, take $\kappa^2=1.0$ and 1.6 GeV^2, and consider the results (for the lower roots of λ_0 only) for $f_+=0.95$, 1.00, 1.05, and 1.10; in fact, for $N=1.42$, we find $0.66\ (0.64) \leq f_+ \leq 1.10$ where the lower limit corresponds to $\kappa^2 = 1.0\ (1.6)$ GeV^2. The results, which appear in Table II, indicate that the effect on ω_K/ω_π is considerable, although ξ is hardly affected, and we are thus able to obtain "good" solutions over the whole range of κ^2. Finally, we can check how well the Callan-Treiman relation is satisfied. The usual form of the equation,

$$f(K^2) \approx f_K/f_\pi\,, \tag{24}$$

may be rearranged to give

$$\lambda_0 K^2/\pi^2 = N-1\,. \tag{25}$$

Thus, for $N=1.42$ and $f_+=1.05$, the left-hand side of Eq. (25) takes the values 0.34 (0.39) for $\kappa^2=1.0$ (1.6) GeV^2, so that agreement is reasonably good, especially for the larger value of κ^2.

IV. DISCUSSION

Hence, we see that the value of $\xi \approx -0.65$ can be obtained in the simple pole dominance $(3,\bar{3})$ model without involving a large breakdown of the SU(3) symmetry of the vacuum or a large deviation of f_+ from unity, as long as we take $\lambda_+ \approx 0.08$. The extent of this region of suitable solutions is quite limited, however, as can be seen from Tables I and II. The main restriction is in the allowed values of N, since λ_+ varies much more rapidly with N than does λ_0, and so ξ is around −0.65 for only a very narrow range of N. Next, although ξ appears particularly insensitive to changes in f_+ around $f_+=1$, our main concern is to obtain ω_K/ω_π close to unity; for $\kappa^2 \approx 1.0$ GeV^2, this can be arranged for $1.0<f_+<1.1$, but for larger values of κ^2, the appropriate range of f_+ decreases in size as the lower bound rises toward 1.05. At the same time, f_+ is bounded above according to (19) so that there is not too much room for manoeuvre here either.

Thus, for the simple $(3,\bar{3})$ model, a good all-round solution to the K_{l3} problem does exist, provided only that the value of λ_+ is near 0.08, i.e., is somewhat larger than the present mean experimental value.

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[17]These results were first obtained, respectively, by Glashow and Weinberg,[14] and Gerstein and Schnitzer.[16]

[18]The author is grateful to Professor S. L. Glashow for making this suggestion, and for pointing out the connection between N and λ_+.

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Errata

Single π^+ Electroproduction at $W\approx 2$ GeV and the Pion Form Factor, R. C. E. Devenish and D. H. Lyth [Phys. Rev. D 5, 47 (1972)]. The B_i amplitudes in Eqs. (A7) of Appendix A are incorrectly numbered. Only the six B_i amplitudes chosen to be independent should appear, namely B_1, B_2, B_3, B_5, B_6, and B_8. The correct equations are obtained by making the following replacement: B_5 for B_4, B_6 for B_5, and B_8 for B_6.

Lepton-Number and Chirality Nonconservation in Weak Processes, H. Primakoff and S. P. Rosen [Phys. Rev. D 5, 1784 (1972)]. Page 1792, item (1): $\psi_{\mu^-}(x)$ should be replaced by $\psi_{\mu^+}(x)$. Page 1792, item (5): The first sentence should read: The pair of successive processes,

$$\nu_\mu \to e^- + (\Delta^{++} + \bar{p}),\ \pi^+, \ldots \to \nu_e \qquad \text{[Eqs. (28) and (3)]},$$

where the intermediate lepton-hadron states are virtual, provides a mechanism for Pontecorvo's "neutrino oscillations."[16] [Transitions of the type $\nu_\mu \to \bar{\nu}_\mu, \bar{\nu}_e$ are forbidden in vacuo by angular momentum conservation because ν_μ has average helicity $(-h)$ and $\bar{\nu}_\mu, \bar{\nu}_e$ have average helicity $(+h)$ where $h = (1-|\eta|^2)/(1+|\eta|^2)$.]