Inclusive Single-Particle Spectra According to the Thermodynamic Model and Double- and Single-Peaked $\ln\tan\frac{1}{2}\theta$ Distributions

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Inclusive single-particle spectra are calculated according to the thermodynamic model in the form $dN/d(\ln\tan\frac{1}{2}\theta)$. At asymptotic energies the model using the weak thermodynamic bootstrap solution leads to double-peaked distributions whereas the model using the strong thermodynamic bootstrap solution leads to a central plateau.

Recently a method was described[1] for calculating approximately the distribution $dN/d(\ln\tan\frac{1}{2}\theta^*)$ of secondary particles which result from a continuous superposition of decaying fireballs. These fireballs move with different velocities in the c.m. system. According to the thermodynamic model,[2,3] the inclusive single-particle spectra are obtained in such a way. The distributions of the thermodynamic spectra in the angular variable $\xi=\ln\tan\frac{1}{2}\theta^*$ previously were calculated numerically,[4-6] and it was found that the one-peak curves obtained at small collision energy develop into double-peaked distributions at higher energies.

This behavior of the thermodynamic momentum spectra can be described and understood in model-independent terms quite easily by noticing that for inclusive single-particle spectra with exponentially damped transverse-momentum dependence, the variable $\xi=\ln\tan\frac{1}{2}\theta^*$ becomes proportional asymptotically at $s\to\infty$ to the rapidity

$$Y^*=\ln\left(\frac{p_\parallel^*+(p_\parallel^{*2}+\mu^2)^{1/2}}{\mu}\right).$$

Here $\mu=(p_\perp{}^2+m^2)^{1/2}$ is the longitudinal mass.

The inclusive single-particle spectra $d^2N/dY^*dp_\perp{}^2$ are said to have scaling behavior[7] if asymptotically

$$\frac{d^2N}{dY^*dp_\perp{}^2}\xrightarrow[s\to\infty]{}\begin{cases} g(p_\perp{}^2) & \text{for } |Y^*|<Y^*_{\max}-\Lambda \\ f(p_\perp{}^2, Y^*_{\max}-Y^*) & \\ \quad\text{for } Y^*_{\max}-\Lambda\leqslant Y^*\leqslant Y^*_{\max}, & \end{cases} \tag{1}$$

where $Y^*_{\max}=\ln(\sqrt{s}/m)$ and Λ is a finite correlation length. The region $|Y^*|<Y^*_{\max}-\Lambda$ is referred to as the central plateau or the pionization region. Distributions with $g(p_\perp{}^2)\neq 0$ are said to exhibit a nonvanishing central plateau or pionization; if $g(p_\perp{}^2)=0$, no pionization is present. The regions $Y^*_{\max}-\Lambda\leqslant Y^*\leqslant Y^*_{\max}$ are referred to as the target and projectile fragmentation regions.

The inclusive single-particle spectra of particle i according to the thermodynamic model[2,3] are given in the c.m. system by expressions like the following:

$$\left.\frac{d^3N^*}{d^3p^*}\right|_i=\int_{-1}^{1}d\lambda\, F(\lambda)\, L(\lambda,\gamma_0) f_i(p_i', T(\lambda,\gamma_0)), \tag{2}$$

where $\lambda=\mathrm{sgn}(\beta)(\gamma-1)/(\gamma_0-1)$ is a velocity parameter and $F(\lambda)$ is a velocity weight function. β and γ are Lorentz parameters of fireballs moving forward ($\lambda>0$) and backward ($\lambda<0$) in the c.m. system; γ_0 is the Lorentz parameter of the c.m. system. $L(\lambda,\gamma_0)$ is a Lorentz boost operator transforming the isotropic spectrum $f_i(p_i', T(\lambda,\gamma_0))$ from the rest system of the fireball (momenta p') to the c.m. system (momenta p^*). $T(\lambda,\gamma_0)$ is a temperature parameter in the Planck-type spectrum $f_i(p',T)$. The fact that asymptotically with $s\to\infty$ T approaches a finite limiting temperature T_0 explains the exponential damping of transverse momenta in the thermodynamic model.

Expression (2) refers to inclusive single-particle spectra calculated from the weak thermodynamic bootstrap solution.[8] It can be shown that the spectra (2) have the scaling property in the fragmentation regions, and that they lead to a vanishing central plateau; this means that they do not exhibit pionization.[9,10,11] At high energy the rapidity distributions approach zero in the central region. Due to the relation between Y^* and the variable $\xi=\ln\tan\frac{1}{2}\theta$, this is equivalent to double-peaked distributions in ξ as directly calculated.[4-6]

Zgrablich[1] calculates the ξ distributions from the continuous superpositions of isotropically decaying fireballs in the following form:

$$F(\xi)=\int_0^{\phi_0}d\phi\, U(\phi)/\cosh^2(\phi+\xi). \tag{3}$$

The term $\cosh^{-2}(\phi+\xi)$ gives the ξ distribution of one isotropically decaying fireball moving with $\gamma=e^\phi$ backwards. The integration extends only over backward-moving fireballs. The expression (3) with any function $U(\phi)$ will not lead to a curve $F(\xi)$ peaked at $\xi>0$. However, in the thermodynamic model, fireballs moving forward as well as backward in the c.m. system have to be considered.

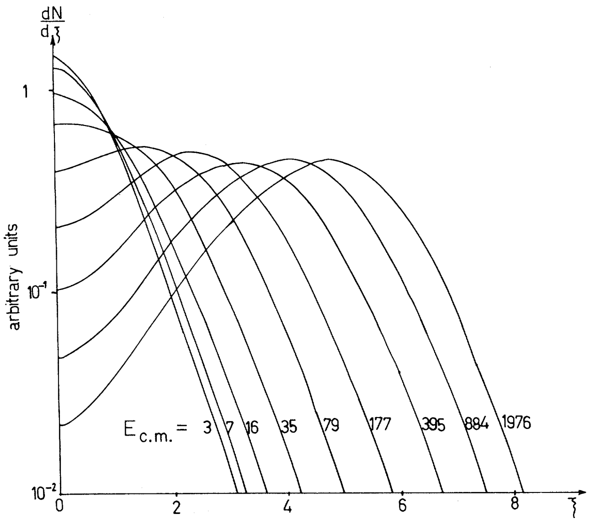

FIG. 1. The angular distribution function (4) according to the thermodynamic model (2), using the weak thermodynamic bootstrap solution.

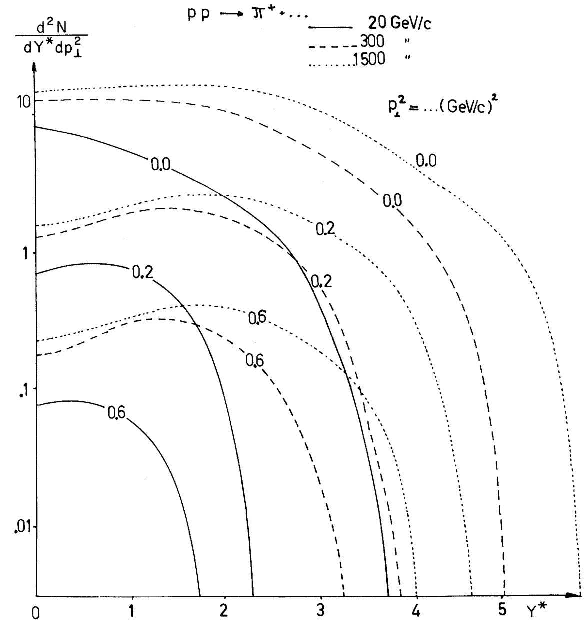

FIG. 2. Distributions $d^2N^*/dY^*dp_\perp^2$ according to the thermodynamic model, using the strong bootstrap solution (6). The curves plotted are for secondary π^+ created in proton-proton collisions.

The ξ distribution corresponding to the thermodynamic model (2) can be calculated with the following integral:

$$F(\xi)=\frac{\phi_0}{\xi}\int_{-1}^{1} dU\, F\left(\frac{\exp(\phi_0|U|)-1}{\xi}\right)\frac{\exp(\phi_0|U|)}{\cosh^2(\xi+\phi_0 U)}\,, \tag{4}$$

where $\phi=\phi_0 U=\ln[(\gamma_0-1)\lambda\,\mathrm{sgn}(\beta)+1]$ and $\xi=\gamma_0-1$. In Fig. 1 we plot the curves $F(\xi)$ for $\xi>0$ and different c.m. system energies $E_{\mathrm{c.m.}}=\sqrt{s}$, which were obtained using the velocity weight function

$$F(\lambda)=\frac{1}{N}(1-\lambda)e^{-A\lambda},\quad A=5.6. \tag{5}$$

The curves in Fig. 1 agree with previously found curves,[4-6] but disagree with Zgrablich.[1] The one-peaked distribution at low energy becomes a two-peaked curve with increasing energy.

All results given so far refer to the inclusive spectra using the weak thermodynamic bootstrap solution. A more recent version[10] of the thermodynamic, inclusive, single-particle spectra is being calculated using the strong statistical-thermodynamic bootstrap solution.[12-14] In this model the thermodynamic single-particle spectra are described by

$$\left.\frac{d^3N^*}{d^3p^*}\right|_i=\int_{-1}^{1} d\lambda\, F(\lambda)q(\lambda,\gamma_0)L(\lambda,\gamma_0)\, f_i(p_i',T(\lambda,\gamma_0))\,, \tag{6}$$

which corresponds to (2) except for the function $q(\lambda,\gamma_0)$ which expresses the increase of multiplicity due to the decay chain of fireballs. This function is of the form

$$q(\lambda,\gamma_0)=c\frac{\gamma_0}{|\lambda|\gamma_0+1}\,. \tag{7}$$

It has been shown[9,10] that (6) and (7) lead to a flat nonvanishing central plateau or to nonvanishing pionization in agreement with recent intersecting storage ring data. In Fig. 2 such rapidity distributions $d^2N^*/dY^*dp_\perp{}^2$ are plotted. Asymptotically this model predicts flat distributions in $\xi=\ln\tan\frac{1}{2}\theta^*$, with possibly two not-very-pronounced peaks in the fragmentation regions.

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Connection Between Nonlinearity of the Pomeranchuk Trajectory and an Intercept Below 1

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An order-of-magnitude relation is suggested between the curvature of the Pomeranchuk trajectory and the displacement of its intercept below 1.

Experimental evidence has recently been reported for curvature of the Pomeranchuk trajectory.[1] For t in the range $0.05<|t|<0.10\ \text{GeV}^2$, the slope has an average value of $0.37\pm0.08\ \text{GeV}^{-2}$, while for $0.10<|t|<0.30\ \text{GeV}^2$ the average slope is $0.10\pm0.06\ \text{GeV}^2$. Such behavior has been qualitatively anticipated from the multiperipheral model as a consequence of interaction between the leading pole and the leading branch point.[2] We here present a simplified description of this pole–branch-point interaction which allows an immediate order-of-magnitude estimate of the displacement of the Pomeranchukon intercept below 1. We avoid the detailed model-dependent considerations of Ref. 2 which tend to obscure the essential elements of the mechanism.

The source both of the curvature of $\alpha_P(t)$ and of the displacement of $\alpha_P(0)$ below 1 is the Finkelstein-Kajantie requirement of a nonvanishing interval between pole and branch point.[3] The argument of these authors establishes such a gap only at $t=0$, but the multiperipheral model extends their argument to make plausible that the pole and branch point are not allowed to intersect for any real negative t.[2] The magnitude of the separation between pole and branch point is model-dependent, but at $t=0$ the branch-point position is related to that of the pole by the formula

$$\alpha_c(0)=2\alpha_P(0)-1\,, \tag{1}$$

so the gap width

$$\Delta\equiv\alpha_P(0)-\alpha_c(0) \tag{2}$$

is also equal to $1-\alpha_P(0)$, the displacement below 1 of the Pomeranchukon intercept. How do we infer curvature of the trajectory?

We may infer curvature from the circumstance that if the pole trajectory were linear, the branch-point trajectory would also be linear and with half the slope, because

$$\alpha_c(t)=2\alpha_P(\tfrac{1}{4}t)-1\,. \tag{3}$$

Since the branch point lies beneath the pole at $t=0$, an intersection at some negative value of t would be inevitable. To avoid intersection with the branch point the trajectory must develop positive curvature.

We are now in a position to make an order-of-magnitude estimate. With no curvature, and a slope α_P', intersection would occur at

$$\bar{t}=\frac{2\Delta}{\alpha_P'(0)}\,. \tag{4}$$

To avoid intersection the trajectory slope must decrease by about a factor of two in going from $t=0$ to $t=\bar{t}$. The recently acquired CERN Intersecting Storage Rings (ISR) data[1] suggest that the order of magnitude of $\bar{t}$ is $0.1\ \text{GeV}^2$, while $\alpha_P'(0)\approx0.4\ \text{GeV}^{-2}$. Thus from formula (4) we estimate

$$\Delta=\tfrac{1}{2}\alpha_P'(0)\bar{t}\approx0.02\,. \tag{5}$$