ity implies that, given b, the width of the $k_\perp$ distribution about $k_\perp = 0$ cannot become too small without a compensating decrease in the width of the $k_L/\sqrt{s}$ distribution about $k_L/\sqrt{s} \sim 0$.

To summarize, the inequalities (7), (8), and (11) provide rather striking restrictions on models dealing with high-energy behavior of elastic and inclusive reactions. Although the inequalities are in fact satisfied on currently fashionable models, this is so only to within $\ln s$ factors. The inequalities, therefore, impose severe limitations on the kinds of model variations that can be entertained.

We wish to thank Professor N. Khuri for raising the question how the ideas of Ref. 2 could be expressed for inclusive cross sections. One of us (G.T.) wishes to thank Professor A. Pais and the members of the Physics Department of the Rockefeller University for their hospitality.

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[4]See A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing* (Springer, New York, 1969).

[5]The estimate of $M_l(\theta)$ for $\cos\theta \neq \pm 1$ was obtained from a WKB analysis of the Legendre equation.

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Elastic Neutrino-Proton Scattering with Strong W-Boson Interactions*

Thomas Appelquist and Terrence Goldman
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
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We estimate a lower bound on the high-energy behavior of elastic neutrino-proton scattering in a theory in which W bosons have strong pairwise interactions with hadrons. The W-pair-exchange process gives a first-order weak contribution to this cross section which grows as the square of the neutrino energy.

The possibility that the hypothetical intermediate boson of weak interactions might interact strongly with ordinary hadrons[1] has some experimental and theoretical appeal. Consequences of such models, in which the emission or absorption of a single W by hadrons is semiweak while the pairwise interaction of W's with hadrons is strong, have been examined by several authors.[2] An intriguing possibility is that this mechanism might explain the failure of high-energy cosmic-ray muons to obey the $\sec\theta$ law.[3,4] An analysis by Bjorken, Pakvasa, Simmons, and Tuan[5] concludes that a large production cross section ($\gtrsim 0.3$ mb) of the W pair in primary cosmic-ray events at energies of 10^{12}–10^{13} eV is required to explain the effect. There is, in fact, some evidence[4] for a threshold at an energy of about 1.9×10^{12} eV corresponding to a mass $m_W \cong 45$ GeV. Whether the W mass is really this large or not is uncertain. However, it has been argued by Kabir and Kamal[6] that the low muon fluxes found in deep-mine experiments[7] already imply a lower bound of about 10 GeV on the mass of a strongly interacting W. This is because the neutrino-induced muon flux due to the process shown in Fig. 1 exceeds the experimental value unless the threshold for the process is sufficiently high.

Because of the high W mass, it is very difficult to observe the effects of its possible strong interactions in an accelerator experiment.[8,9] The pro-

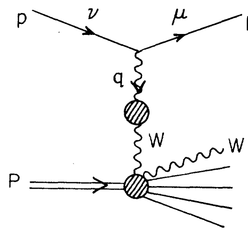

FIG. 1. Semiweak contribution to inelastic neutrino-proton scattering.

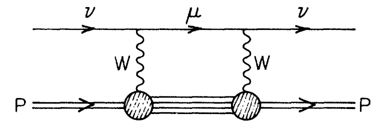

FIG. 2. W-pair contributions to elastic neutrino-proton scattering.

duction threshold is very high and it is useful to look for processes in which the W enters only virtually. In this note we estimate a lower bound on the first-order weak contribution to the forward amplitude for the elastic process $\nu p \to \nu p$ due to W pair exchange as shown in Fig. 2.

This mechanism gives a first-order weak contribution to the forward elastic νp cross section which increases like $E_\nu{}^2$ as $E_\nu \to \infty$ whereas an ordinary neutral W exchange would lead to a constant forward cross section. This rapid growth leads, for example to the approximate lower bound

$$\frac{(d\sigma/dt)(\nu_\mu p \to \nu_\mu p)|_{t=0}}{(d\sigma/dt)(\nu_\mu n \to \mu^- p)|_{t=0}} \gtrsim 10$$

at $E_\nu \approx 50$ GeV.

We first calculate the inelastic neutrino scattering process shown in Fig. 1 and then put this into a dispersion relation for forward neutrino-proton elastic scattering. Referring to the labeling of Fig. 1, we define s' to be the mass of the final hadronic state and $t = -q^2 = -(p - p')^2$. Then

$$\frac{d\sigma^{\nu N}}{dt\,ds'} = \frac{G M_W{}^2}{\sqrt{2}} \frac{1}{16\pi^2} \frac{M}{(s-M^2)^2} M^{\mu\nu} \sum_{\text{lepton spin}} \bar{u}(p')\gamma_\mu(1-\gamma_5)u(p)\bar{u}(p)\gamma_\nu(1-\gamma_5)u(p'), \tag{1}$$

where $s = (p+P)^2$ and, averaging over nucleon spin,

$$M_{\mu\nu} = \overline{\sum_{\text{nucleon spin}}} \sum_{n} \langle P|\, W_\mu(0)\,|n\rangle\langle n|\, W_\nu(0)\,|P\rangle\,(2\pi)^4\delta^4(P+q-P_n). \tag{2}$$

We take the leptons to be massless and normalized such that $\sum_{r=1}^{2} u_r(p)\bar{u}_r(p) = \not{p}$. $W^\mu(x)$ is the interpolating field of the W boson. It is convenient to express the cross section (1) in terms of off-mass-shell W-absorption cross sections. The total W nucleon cross section for a W of polarization ϵ is

$$\sigma_\epsilon(s', t) = \frac{M}{2[(q\cdot p)^2 + tM^2]^{1/2}}\,(t + M_W{}^2)^2 \epsilon_\mu^* M^{\mu\nu}\epsilon_\nu. \tag{3}$$

This relation can be turned around to express $M^{\mu\nu}$ in terms of the cross sections $\sigma_L, \sigma_R, \sigma_S$ for absorption of left-handed, right-handed, and longitudinally polarized W's. We make the additional assumption that the strong interactions of the W are parity conserving, so that

$$\sigma_L = \sigma_R \equiv \sigma_T. \tag{4}$$

Then

$$M^{\mu\nu} = -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{t}\right)\frac{2[(q\cdot P)^2 + tM^2]^{1/2}}{(t+M_W{}^2)^2}\,\sigma_T^{WN} + \left(P^\mu + \frac{(P\cdot q)q^\mu}{t}\right)\left(P^\nu + \frac{(P\cdot q)q^\nu}{t}\right)\frac{2[(q\cdot P)^2 + tM^2]^{1/2}t}{(t+M_W{}^2)^2[t + (q\cdot P)^2/M^2]}\,(\sigma_S^{WN} + \sigma_T^{WN}). \tag{5}$$

Putting this into (1) and doing the lepton traces, we obtain

$$\frac{d\sigma^{\nu N}}{dt\,ds'} = \frac{\sqrt{2}\,G M_W{}^2}{4\pi^2(s-M^2)^2}\,\frac{t}{(t+M_W{}^2)^2}\,[\,(s'+t-M^2)^2 + 4M^2t\,]^{-1/2}\sigma_T^{WN}(s', t) \times \{(s'+t-M^2)^2 + 2(s-M^2)(s-s'-t) + 2tM^2 + 2R[\,(s-M^2)(s-s') - ts\,]\}, \tag{6}$$

where $R=\sigma_S^{WN}(s',t)/\sigma_T^{WN}(s',t)$.

The integral of (6) over the kinematically allowed region $M_W{}^2 \leqslant s' \leqslant s$, $0 \leqslant t \leqslant s-s'$ is $\sigma_{\text{total}}^{\nu N}$ to order G which appears in the optical theorem

$$\operatorname{Im} \overline{T}^{\nu N}(s,0)=\sigma_{\text{total}}^{\nu N}(s)\,, \tag{7}$$

where $\overline{T}^{\nu N}(s,t)$ is the ν-N elastic scattering amplitude averaged over nucleon spin. Following Refs. 5 and 9 we write a dispersion relation for $\overline{T}^{\nu N}(s,0)$:

$$\overline{T}^{\nu N}(s,0)=\frac{1}{\pi}\int_{M_W{}^2}^{\infty} ds'\left(\frac{\sigma_{\text{tot}}^{\nu N}(s')}{s'-s}-\frac{\sigma_{\text{tot}}^{\bar{\nu} N}(s')}{s'+s-2M^2}\right) \tag{8}$$

and explicitly make one subtraction:

$$\overline{T}^{\nu N}(s,0)=\frac{1}{\pi}\int_{M_W{}^2}^{\infty} ds'\left(\frac{\sigma_{\text{tot}}^{\nu N}(s')}{s'}-\frac{\sigma_{\text{tot}}^{\bar{\nu} N}(s')}{s'-2M^2}\right)+\frac{s}{\pi}\int_{M_W{}^2}^{\infty} ds'\left(\frac{\sigma_{\text{tot}}^{\nu N}(s')}{s'(s'-s)}+\frac{\sigma_{\text{tot}}^{\bar{\nu} N}(s')}{(s'+s-2M^2)(s'-2M^2)}\right)=T_0+T_1(s)\,. \tag{9}$$

The threshold is set at $M_W{}^2$ since we are only interested in the strong-W contribution.

The current upper limit on elastic νp scattering is[10]

$$\frac{\sigma(\nu_\mu p\to\nu_\mu p)}{\sigma(\nu_\mu n\to\mu^- p)}\leqslant 0.12\pm 0.06$$

for $E_\nu \leqslant 5$ GeV ($s \lesssim 10M^2$). This indicates that the constant term T_0 is small[11] ($\lesssim G$) so we estimate the linearly growing term $T_1(s)$ for $M^2 \ll s < M_W{}^2$. In this region

$$T_1(s)\approx\frac{s}{\pi}\int_{M_W{}^2}^{\infty}\frac{ds'}{s'^2}\left[\sigma^{\nu N}(s')+\sigma^{\bar{\nu} N}(s')\right]\,.$$

We now integrate (6) over the kinematically allowed region with $\sigma_T^{WN}(s',t)=$ constant, and $R=0$ and use the result in (10). As $R\geqslant 0$, this consistently underestimates $T_1(s)$. We have further assumed that σ_T^{WN} has no strong t dependence.[12]

Performing the t integration, we obtain

$$\begin{aligned}T_1(s)\geqslant \frac{\sqrt{2}\,Gs\sigma_T M_W^2}{4\pi^3}\int_{M_W{}^2}^{\infty} ds''\int_{s''}^{\infty} ds'(s')^{-4}\Bigg\{&\frac{(s'-2s''+2M_W{}^2)(s'-s'')}{s'-s''+M_W^2}+(s''-2M_W{}^2)\ln\left(\frac{s'-s''+M_W^2}{M_W^2}\right)\\&-2s'\left[\ln\left(\frac{s'-s''+M_W{}^2}{M_W^2}\right)-\frac{s'-s''}{s'-s''+M_W^2}\right]\\&+2s'^2\left[\frac{s''}{(s''-M_W{}^2)^2}\ln\left(\frac{s''(s'-s''+M_W{}^2)}{s'M_W^2}\right)+\frac{s''-s'}{(s''-M_W{}^2)(s'-s''+M_W{}^2)}\right]\Bigg\}\,,\end{aligned}$$

where $\sigma_T=\sigma_T^{W^+N}+\sigma_T^{W^-N}$. The s' and s'' integrals have been interchanged to simplify the remaining integrations. Completing these, we obtain

$$T_1(s)\geqslant\frac{\sqrt{2}\,Gs}{6\pi^3}\,\sigma_T\,. \tag{11}$$

This gives a lower bound on the forward elastic νp scattering cross section when s becomes high enough so that T_1 dominates T_0:

$$\left.\frac{d\sigma^{\nu N}}{dt}(s)\right|_{t=0}\gtrsim\frac{1}{16\pi}\left(\frac{(Gs)^2\sigma_T{}^2}{18\pi^6}\right)\,. \tag{12}$$

Note that the result is independent of M_W^2 and rises as the square of the c.m. energy $s=2ME_\nu$. This can be compared with the corresponding charge-exchange forward cross section due to ordinary single W exchange,

$$\left.\frac{d\sigma^{\nu n\to\mu^- p}}{dt}\right|_{t=0}=\frac{G^2}{2\pi}\left[1+\left(\frac{G_A}{G}\right)^2\right]\,, \tag{13}$$

which is independent of E_ν. Combining (12) and (13) (with $G_A/G \approx 1$),

$$\frac{(d\sigma/dt)(\nu_\mu p \to \nu_\mu p)|_{t=0}}{(d\sigma/dt)(\nu_\mu n \to \mu^- p)|_{t=0}} \gtrsim \frac{(\sigma_T M E_\nu)^2}{9\pi^6}, \qquad (14)$$

which gives the values quoted above assuming typical hadronic total cross sections of $\sigma^{W^+N} \approx \sigma^{W^-N} \approx 1$ mb. At the high energies of interest here, we expect both scattering processes to be forward peaked with the scale set by a typical hadronic mass. Then (14) also gives a lower bound on the ratio of total elastic cross sections:

$$\frac{\sigma(\nu_\mu p \to \nu_\mu p)}{\sigma(\nu_\mu n \to \mu^- p)} \gtrsim \frac{(\sigma_T M E_\nu)^2}{9\pi^6}. \qquad (15)$$

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