157, 1376 (1967).

[12]Such a phenomenological interaction could be induced, however, by baryon loop graphs and the current-current weak interaction. This anomalous, strange axial-vector-current divergence effect is being calculated.

[13]See, for example, L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Letters 21, 707 (1968).

[14]For simplicity we disregard ω-ϕ mixing effects.

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Veneziano-like Representation for the Pion Form Factor

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A simple Veneziano-like representation for the pion form factor is related to the two-pion unitarity equation and the Veneziano formula for the π-π scattering amplitude. Some further approximations to the unitarity equation are also discussed.

Recently there has been considerable interest in describing the hadron electromagnetic form factors in terms of Veneziano-like expressions. In particular, for the pion form factor it has been suggested by Jengo and Remeddi and by Suura[1] that

$$F_\pi(s) \sim \frac{\Gamma(1-\alpha_\rho(s))}{\Gamma(n-\alpha_\rho(s))}. \tag{1}$$

A way to introduce a Veneziano-like formalism for the form factor is to adopt a model in which the photon interacts with hadrons through pair creation, say of partons. The form factor then arises from the strong final-state interactions. For the pion form factor, for example, keeping only the 2-pion intermediate states, one may use the Veneziano formula for the $\pi\pi$ scattering amplitude and expect to obtain a formula equivalent to (1) from the resulting Omnès equation.[2] We proceed to discuss this approach.

Let

$$h_1(s) = \frac{N_1(s)}{D_1(s)}.$$

Here $h_1(s)$ is the π-π scattering amplitude in the $I = J = 1$ state. $N_1(s)$ and $1/D_1(s)$ have the usual left-hand and right-hand cuts. Then, with the above approximation, the pion form factor is given by

$$F(s) \sim \frac{1}{D_1(s)}. \tag{2}$$

Now in the Veneziano model, the amplitude $h_1(s)$ is

$$h_1(s) = \left(\frac{s-4\mu^2}{s}\right)^{1/2} \frac{1}{32\pi}\frac{1}{2}\int_{-1}^{+1} dz\, P_1(z)[V(s,t)-V(s,u)]$$

$$= \left(\frac{s-4\mu^2}{s}\right)^{1/2} \frac{1}{32\pi}\beta \int_{-1}^{+1} dz\, z \frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))},$$

$$z = \left(1+\frac{2t}{s-4\mu^2}\right).$$

It is clear from the above expression that right-hand singularities in $h_1(s)$ can only come from the function $\Gamma(1-\alpha_\rho(s))$.

Thus

$$\frac{1}{D_1(s)} \sim F(s) \sim \Gamma(1-\alpha_\rho(s))$$

in the Veneziano model. We may of course multiply $F(s)$ by an entire function to have proper asymptotic behavior. Thus

$$F(s) = \frac{\Gamma(1-\alpha_\rho(s))}{\Gamma(n-\alpha_\rho(s))}$$

is one of the possible solutions of the Omnès equation.

In equating $F(s)$ to $1/D_1(s)$, one assumes two things:

(i) Neglect of all intermediate states except 2π from the spectral function of the form factor.

(ii) The π-π scattering amplitude $h_1(s)$ satisfies *elastic* unitarity,

$$\mathrm{Im} h_1(s) = |h_1(s)|^2\,,$$

i.e., $h_1(s) = e^{i\delta}\sin\delta$, δ real, so that

$$\mathrm{Im} F(s) = F(s) h_1^*(s)\,\theta(s-4\mu^2)\,. \qquad (3)$$

If we relax condition (ii), i.e., take into account the contribution of other states in $\pi\pi$ scattering as is the case when we use the Veneziano model, δ complex, (3) should be replaced by $\mathrm{Im}F(s) = \mathrm{Re}(F(s)h_1^*(s))$, since the spectral function should be real. We no longer have the equality of the phases of the form factor and the partial-wave amplitude, and (2) does not hold.

The expression (1) cannot, however, be confronted directly with data in the timelike region until the width of the resonances have been incorporated in it. One way to shift the poles of $F(s)$ to the second sheet is to use Martin's prescription for smoothing out $F(s)$ (Refs. 3, 4), i.e.,

$$F(s) = \int_{\lambda=0}^{\lambda_m} d\lambda\, \phi(\lambda) \frac{\Gamma(1-\alpha(\lambda s))}{\Gamma(n-\alpha(\lambda s))}.$$

The smoothing function $\phi(\lambda)$ is chosen to give the proper threshold behavior for the p-wave amplitude and suitable positions for the poles on the second sheet. As shown in (3),

$$\phi(\lambda) \sim \frac{(\lambda-\lambda_m)^{3/2}}{(\lambda-1)^2+\gamma^2}\,, \quad \gamma = \frac{\Gamma_\rho}{m_\rho}$$

gives a reasonable fit to the data.

It is also of some interest to discuss a different distortion of the unitarity equation. We put $F(s) \approx 1$ on the right-hand side in (3):

$$\mathrm{Im}F(s) \approx h_1^*\,\theta(s-4\mu^2)\,.$$

Note that in Veneziano model, since $h_1(s)$ is real, the "approximation" preserves the reality of the spectral function. A dispersion relation for $F(s)$ with one subtraction gives the following:

$$F(s) = 1 + \frac{s}{\pi}\int_{4\mu^2}^{\infty} \frac{h_1(s')\,ds'}{s'(s'-s)}\,,$$

where

$$h_1(s) = \left(\frac{s-4\mu^2}{s}\right)^{1/2} \frac{1}{32\pi}\beta \int_{-1}^{+1} z\,dz\, V(s,t)\,.$$

Let us now take $\mu_\pi = 0$ for simplicity. Then

$$h_1(s) = \frac{\beta}{32\pi}\int_{-s}^{0} dt \left(\frac{2}{s}+\frac{4t}{s^2}\right) \sum_n \frac{f_n(t)}{s-s_n}.$$

The series $\sum_n$ converges for $\mathrm{Re}\alpha(t) < 0$.

This gives

$$F(s) = 1 + \frac{\beta}{4\pi^2 s}\sum_{n=0}^{\infty}\int_{-\infty}^{0} dt \frac{f_n(t)}{s-s_n}\left[(\ln(1+s/t))(\tfrac{1}{2}+t/s)-1\right] + \frac{cs}{\pi}\,,$$

where c is a constant, or

$$F(s) = 1 + \frac{\beta}{4\pi^2 s}\int_0^{\infty} dx\, V(-x,s)\left[(\ln(1-s/x))(\tfrac{1}{2}-x/s)-1\right] + \frac{cs}{\pi}\,,$$

which is identical to the formula obtained by Gerstein *et al.*[5] The difficulties associated with this approach, for example the improper asymptotic behavior, emergence of vector dominance through second-order poles, etc., can be traced to the drastic assumptions made above.

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