in serious disagreement with experiment.
[13]J. Schneider, V. Lepeltier, P. Bonamy, P. Borgeaud, O. Guisan, and P. Sonderegger, Phys. Rev. Letters 23, 1068 (1969).
[14]F. Hayot and B. Kayser, Nuovo Cimento 3A, 45 (1971).

PHYSICAL REVIEW D VOLUME 6, NUMBER 7 1 OCTOBER 1972

Light-Cone Approach to Structure Functions in a Theory of Weak and Electromagnetic Interactions

Riazuddin*
Daresbury Nuclear Physics Laboratory, Daresbury, Nr. Warrington, Lancashire, England

and

Fayyazuddin
Institute of Physics, University of Islamabad, Islamabad, Pakistan
(Received 24 April 1972)

Light-cone algebra is written for the currents in a unified theory of weak and electromagnetic interactions, and is used to obtain predictions for deep-inelastic structure functions of $\nu + N \to \nu +$ anything in terms of those of $e + N + e +$ anything and/or $\nu_l + N \to l +$ anything. The predictions are characteristic of the theory because of the existence in the theory of a neutral strangeness-conserving weak hadronic current which couples with a neutral lepton current, whereas in the conventional theory with only charged currents the process $\nu + N \to \nu +$ anything does not occur in the lowest order.

A number of attempts have been made to unify weak and electromagnetic interactions in a Yang-Mills theory with spontaneous breaking of gauge invariance.[1] In the version studied by Salam and Weinberg these interactions are mediated by two massive charged vector bosons $W_\mu^\pm$, a neutral massive vector boson Z_μ, and the massless photon A_μ. Recently the model has been extended by Weinberg[2] to include the hadrons. In order to eliminate the strangeness-changing neutral current, a four-quark scheme of Glashow, Iliopoulos, and Maiani[3] is used. The symmetries act on two $S_LU(2)$ [$S_LU(2)$ group being generated by a "left-handed isospin"] multiplets

$$\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} q_1 \\ q_2' \end{pmatrix},$$

$$\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} q_4 \\ q_3' \end{pmatrix},$$

with $q_2' = q_2 \cos\theta_C + q_3 \sin\theta_C$, $q_3' = -q_2 \sin\theta_C + q_3 \cos\theta_C$ (θ_C being the Cabibbo angle) and on four right-handed singlets $\frac{1}{2}(1-\gamma_5)q_i$ ($i = 1, \ldots, 4$). Here q_1, q_2, q_3, and q_4 denote the four quarks, the first three forming an SU(3) triplet, while the fourth quark q_4 has the same charge as q_1 but differs from the triplet by one unit of a new quantum number, the "charm" $\mathcal{C}$. In terms of the four quarks, the electromagnetic, weak hadronic charged and neutral currents in Weinberg's model are given by

$$J_\mu^{\text{e.m.}} = i\bar{q}\gamma_\mu Q q, \tag{1a}$$

$$J_\mu^W = i\bar{q}\gamma_\mu(1+\gamma_5)Wq, \tag{1b}$$

$$J_\mu^Z = \tfrac{1}{2}J_\mu^{W(0)} - \tfrac{1}{2}(4\sin^2\theta_W)J_\mu^{\text{e.m.}}, \tag{1c}$$

where q is the column matrix

$$\begin{pmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix},$$

Q is the charge matrix

$$Q = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & (a-1) & 0 \\ 0 & 0 & 0 & (a-1) \end{pmatrix}, \tag{2a}$$

with a, a, $a-1$, and $a-1$ being the charges of the quarks q_4, q_1, q_2, and q_3, respectively. In the fractionally charged quark model $a = \frac{2}{3}$ while $a = 1$ and $a = 0$ provide examples of integrally charged models. In Eq. (1b), W is the matrix

$$W=\begin{pmatrix} 0 & 0 & -\sin\theta_C & \cos\theta_C \\ 0 & 0 & \cos\theta_C & \sin\theta_C \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2b}$$

and $J_\mu^{W(0)}$ is defined by

$$J_\mu^{W(0)} = i\bar{q}\gamma_\mu(1+\gamma_5)W^0 q, \tag{3a}$$

with the matrix W^0 defined as follows:

$$[W, W^+]_- = W^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3b}$$

Note also that

$$[W, W^+]_+ = I, \quad [W^0, W^0]_+ = 2I. \tag{3c}$$

The angle θ_W appearing in Eq. (1c) in Weinberg's model is defined by $\tan\theta_W = g'/g$, where $g/2\sqrt{2}$ and $\frac{1}{2}(g^2+g'^2)^{1/2}$ appear in the interactions of J_μ^W and J_μ^Z with the gauge bosons W and Z,

$$m_Z = \frac{(g^2+g'^2)^{1/2}}{g} m_W, \quad e^2 = \frac{g^2 g'^2}{g^2+g'^2}, \qquad \frac{g^2+g'^2}{16 m_Z^2} = \frac{g^2}{16 m_W^2} = \frac{G_W}{2\sqrt{2}}, \tag{4}$$

where e is the electric charge and G_W is the usual coupling constant of weak interactions.

Note that the existence of the neutral current J_μ^Z would make predictions in processes like (N denoting a nucleon)

$$\nu + N \to \nu + \text{anything},$$

for example. The purpose of this paper is to write the light-cone algebra[4] for the commutators of the currents defined in Eqs. (1) in the free-quark model and to use this algebra for relating structure functions (for the spin-averaged case) of the processes

$$e + N \to e + \text{anything}, \tag{5a}$$

$$\nu_l + N \to l + \text{anything}, \tag{5b}$$

$$\nu + N \to \nu + \text{anything}, \tag{5c}$$

in the scaling limit, supposing that the structure functions of the process (5c) also scale as do those of the processes (5a) and (5b). We obtain various relations. The unequivocal new predictions of the model are the relations between the corresponding structure functions of Eq. (5c) and Eqs. (5a) or (5b); in particular we obtain, independent of θ_W, a lower bound for $F_2^{\nu\nu p}$ in terms of F_2^{eN}, an experimental test of which may be feasible in future. A similar bound has recently been obtained by Budny,[5] who has treated the problem in the parton model.

In order to write the light-cone algebra, let us define the bilocal operators

$$J_\sigma^R(r;S;x,y) = \tfrac{1}{2}[i\bar{q}(x)\gamma_\sigma(1+r\gamma_5)Rq(y) + (x \leftrightarrow y)] = V_\sigma^R(S;x,y) + rA_\sigma^R(S;x,y), \tag{6a}$$

$$J_\sigma^R(r;A;x,y) = \tfrac{1}{2}[i\bar{q}(x)\gamma_\sigma(1+r\gamma_5)Rq(y) - (x \leftrightarrow y)] = V_\sigma^R(A;x,y) + rA_\sigma^R(A;x,y), \tag{6b}$$

where R is any of the matrices W, W^+, W^0, I, etc. introduced above and $r=\pm 1$. The light-cone algebra satisfied by the commutators of the currents defined in Eq. (1) can be easily worked out in the free-quark model and is given below:

$$[J_\mu^W(x), \bar{J}_\nu^W(y)] \underset{(x-y)^2=0}{\sim} 2\{s_{\mu\nu\rho\sigma}[J_\sigma^{W(0)}(1,S;x,y) + J_\sigma^I(1,A;x,y)] + \epsilon_{\mu\nu\rho\sigma}[J_\sigma^{W(0)}(1,A;x,y) + J_\sigma^I(1,S;x,y)]\} \frac{\partial}{\partial x_\rho} D(x-y), \tag{7a}$$

$$[J_\mu^{\text{e.m.}}(x), J_\nu^{\text{e.m.}}(y)] \underset{(x-y)^2=0}{\sim} 2[s_{\mu\nu\rho\sigma}V_\sigma^{Q^2}(A;x,y) + \epsilon_{\mu\nu\rho\sigma}A_\sigma^{Q^2}(S;x,y)] \frac{\partial}{\partial x_\rho} D(x-y), \tag{7b}$$

$$\begin{aligned}[J_\mu^Z(x), J_\nu^Z(y)] \underset{(x-y)^2=0}{\sim} \tfrac{1}{4}2\{ & s_{\mu\nu\rho\sigma}[2J_\sigma^I(1,A;x,y) + 16\sin^4\theta_W V_\sigma^{Q^2}(A;x,y) - 4\sin^2\theta_W J_\sigma^X(1,A;x,y)] \\ & + \epsilon_{\mu\nu\rho\sigma}[2J_\sigma^I(1,S;x,y) - 4\sin^2\theta_W J_\sigma^X(1,S;x,y) \\ & + 16\sin^4\theta_W A_\sigma^{Q^2}(S;x,y)]\} \frac{\partial}{\partial x_\rho} D(x-y),\end{aligned} \tag{7c}$$

where

$$X = [W^0, Q]_+ = 2\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & -(a-1) & 0 \\ 0 & 0 & 0 & -(a-1) \end{pmatrix}, \tag{8a}$$

$$s_{\mu\nu\rho\sigma} = \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma}, \tag{8b}$$

$$D(z) = -\frac{1}{2\pi}\epsilon(z_0)\delta(z^2). \tag{8c}$$

In order to use the algebra, let us define the spin-summed matrix elements of the bilocal operators between the one-nucleon states to be

$$\tfrac{1}{2}(2\pi)^3 \frac{p_0}{m}\langle p|J_\sigma^R(1,S;z,0)|p\rangle = \tilde{G}_R^S(p\cdot z)\frac{p_\sigma}{m}+\cdots = \tfrac{1}{2}(2\pi)^3\frac{p_0}{m}\langle p|V_\sigma^R(S;z,0)|p\rangle, \tag{9a}$$

$$\tfrac{1}{2}(2\pi)^3 \frac{p_0}{m}\langle p|J_\sigma^R(1,A;z,0)|p\rangle = \tilde{G}_R^A(p\cdot z)\frac{p_\sigma}{m}+\cdots = \tfrac{1}{2}(2\pi)^3\frac{p_0}{m}\langle p|V_\sigma^R(A;z,0)|p\rangle, \tag{9b}$$

where z is lightlike, $z^2 \simeq 0$. The omitted terms are proportional to z_μ and play no role in the discussion below. We also define the Fourier transform of $\tilde{G}_R^{S,A}(p\cdot z)$

$$G_R^{S,A}(\xi) = \frac{1}{2\pi}\int d(p\cdot z)e^{i\xi(p\cdot z)}\tilde{G}_R^{S,A}(p\cdot z),$$
$$\tilde{G}_R^{S,A}(p\cdot z) = \int_{-1}^{+1} d\xi\, e^{-i\xi(p\cdot z)}G_R^{S,A}(\xi). \tag{10}$$

Then following the method of Fritzsch and Gell-Mann,[4] it is easy to see that one has the following relations:

$$2\xi F_1(\xi) = F_2(\xi), \text{ for all the processes} \tag{11a}$$
$$F_2^{\bar{\nu}T}(\xi) = 2\xi[G_{W^0}^S(\xi) + G_I^A(\xi)], \tag{11b}$$
$$F_2^{\nu l}(\xi) = 2\xi[-G_{W^0}^S(\xi) + G_I^A(\xi)], \tag{11c}$$
$$F_3^{\bar{\nu}T}(\xi) = -2[G_{W^0}^A(\xi) + G_I^S(\xi)], \tag{11d}$$
$$F_3^{\nu l}(\xi) = -2[-G_{W^0}^A(\xi) + G_I^S(\xi)], \tag{11e}$$
$$F_2^e(\xi) = 2\xi[G_{Q^2}^A(\xi)], \tag{11f}$$
$$F_2^{\nu\nu}(\xi) = \tfrac{1}{4}2\xi[2G_I^A(\xi) + 16\sin^4\theta_W G_{Q^2}^A(\xi) - 4\sin^2\theta_W G_X^A(\xi)], \tag{11g}$$
$$F_3^{\nu\nu}(\xi) = -\tfrac{1}{4}\times 2[2G_I^S(\xi) - 4\sin^2\theta_W G_X^S(\xi)], \tag{11h}$$

where ξ is the scaling variable $\xi = q^2/2m\nu = 1/\omega$ and F_1, F_2, and F_3 are the usual structure functions in the scaling limit. The nonzero value of $F_i^{\nu\nu}$ is the essential novelty of Weinberg's model. If one notes that

$$Q^2 = (a^2 - a + \tfrac{1}{2})I + (a - \tfrac{1}{2})W^0, \tag{12a}$$
$$X = I + 2(a - \tfrac{1}{2})W^0, \tag{12b}$$
$$\langle G_I^{S,A}\rangle_{\text{proton}} = \langle G_I^{S,A}\rangle_{\text{neutron}}, \tag{13}$$

it follows from the above relations that

$$F_{2,3}^{\bar{\nu}Tp}(\xi) + F_{2,3}^{\nu lp}(\xi) = F_{2,3}^{\bar{\nu}Tn}(\xi) + F_{2,3}^{\nu ln}(\xi), \tag{14a}$$
$$[F_2^{ep}(\xi) - F_2^{en}(\xi)] = -(a-\tfrac{1}{2})\xi[F_3^{\nu ln}(\xi) - F_3^{\nu lp}(\xi)]$$
$$= -(a-\tfrac{1}{2})\xi[F_3^{\bar{\nu}Tp}(\xi) - F_3^{\bar{\nu}Tn}(\xi)], \tag{14b}$$
$$[F_2^{\nu\nu p}(\xi) - F_2^{\nu\nu n}(\xi)] = -2\sin^2\theta_W(1-2\sin^2\theta_W)[F_2^{ep}(\xi) - F_2^{en}(\xi)], \tag{15a}$$
$$\xi[F_3^{\nu\nu p}(\xi) - F_3^{\nu\nu n}(\xi)] = -2(a-\tfrac{1}{2})\sin^2\theta_W[F_2^{\nu lp}(\xi) - F_2^{\nu ln}(\xi)], \tag{15b}$$
$$F_2^{\nu\nu}(\xi) = [\tfrac{1}{4} - a(1-a)\sin^2\theta_W][F_2^{\bar{\nu}T}(\xi) + F_2^{\nu l}(\xi)] - 2\sin^2\theta_W(1-2\sin^2\theta_W)F_2^e(\xi), \tag{15c}$$
$$F_3^{\nu\nu}(\xi) = \tfrac{1}{4}\left\{(1-2\sin^2\theta_W)[F_3^{\bar{\nu}T}(\xi) + F_3^{\nu l}(\xi)] - 4(a-\tfrac{1}{2})\sin^2\theta_W\frac{F_2^{\nu l}(\xi) - F_2^{\bar{\nu}T}(\xi)}{\xi}\right\}. \tag{15d}$$

In the relations (14b) and (15b)–(15d) $a=\frac{2}{3}$ or 1, 0 for fractionally or integrally charged quarks. The relation (14b) is the same as in the usual theory[6] while the relations (15) are characteristic of Weinberg's theory. The relations (15) involve the parameter θ_W. We, therefore, look for other relations independent of θ_W.

We shall now confine ourselves to the fractionally charged quarks ($a=\frac{2}{3}$). The cases $a=1$ and $a=0$ can be similarly worked out and we give the results for these in the Appendix. For $a=\frac{2}{3}$, note that we can write the matrices Q^2 and X as

$$Q^2=\tfrac{1}{6}(\tfrac{8}{3}N_4+\tfrac{8}{3}N_1+\tfrac{2}{3}N_2+\tfrac{2}{3}N_3), \tag{16a}$$

$$X=\tfrac{1}{2}(\tfrac{8}{3}N_4+\tfrac{8}{3}N_1+\tfrac{4}{3}N_2+\tfrac{4}{3}N_3), \tag{16b}$$

where N_i ($i=1,\ldots,4$) are 4×4 diagonal matrices with non-negative eigenvalues, e.g.,

$$N_1=\begin{pmatrix}0&&&\\&1&&\\&&0&\\&&&0\end{pmatrix}.$$

They have the property $N_i^2=N_i$ and can be expressed as

$$\begin{aligned}N_4&=\Lambda_{\mathcal{C}}\,,\\N_1&=-\tfrac{1}{2}\Lambda_{\mathcal{C}}+\Lambda_B+\tfrac{1}{2}\Lambda_Y+\Lambda_3\,,\\N_2&=-\tfrac{1}{2}\Lambda_{\mathcal{C}}+\Lambda_B+\tfrac{1}{2}\Lambda_Y-\Lambda_3\,,\\N_3&=(\Lambda_B-\Lambda_Y)\,,\end{aligned} \tag{17}$$

where $\Lambda_{\mathcal{C}}$, Λ_B, Λ_3, and Λ_Y are the 4×4 matrices corresponding to charm ($\mathcal{C}$) ($\mathcal{C}=1$ for q_4 and 0 for q_1, q_2, and q_3), baryon number (B), isospin (T_3), and hypercharge (Y), respectively, e.g.,

$$\Lambda_B=\begin{pmatrix}\frac{1}{3}&&&\\&\frac{1}{3}&&\\&&\frac{1}{3}&\\&&&\frac{1}{3}\end{pmatrix},$$

$$\Lambda_3=\begin{pmatrix}0&&&\\&+\frac{1}{2}&&\\&&-\frac{1}{2}&\\&&&0\end{pmatrix}.$$

It is clear that for the matrix elements G_R^A of the bilocal operators we have

$$\begin{aligned}&\langle G_R^A\rangle_p=\langle G_R^A\rangle_n=c,b,y\quad\text{for } R=\Lambda_{\mathcal{C}},\Lambda_B,\Lambda_Y\,,\\&\langle G_I^A\rangle_p=\langle G_I^A\rangle_n=3b\quad\text{since } I=3\Lambda_B\,,\\&\langle G_R^A\rangle_p=-\langle G_R^A\rangle_n=d\quad\text{for } R=\Lambda_3\,,\end{aligned} \tag{18}$$

where p or n denotes the matrix elements with respect to proton or neutron. If we now define currents $V_\sigma^{N_i}(0)\sim i\bar{q}(0)\gamma_\sigma N_i q(0)$ and the corresponding bilocal operators $V_\sigma^{N_i}(A;z,0)$, it follows in exactly the same manner as in obtaining the relation (11f) that $F_2^{N_i}=2\xi\,[\,G_{N_i}^A(\xi)\,]$. Then from the positivity of $F_2^{N_i}$, it follows that

$$\langle G_{N_i}^A\rangle_{p\text{ or }n}\geq 0\,, \tag{19}$$

which gives on using Eqs. (17) and (18)

$$c\geq 0,\quad u_\pm=-\tfrac{1}{2}c+b\pm d+\tfrac{1}{2}y\geq 0\,,\quad v=(b-y)\geq 0,\quad b\geq 0\,. \tag{20}$$

The positivity conditions (20) are the ones which result from SU(2) invariance of the hadronic vertices. Using the results (16) and (20), we have from Eqs. (11f) and (11g)

$$\frac{F_2^{ep}(\xi)}{2\xi}=\tfrac{1}{6}[\tfrac{8}{3}(u_++c)+\tfrac{2}{3}(u_-+v)]\,, \tag{21a}$$

$$\frac{F_2^{en}(\xi)}{2\xi}=\tfrac{1}{6}[\tfrac{8}{3}(u_-+c)+\tfrac{2}{3}(u_++v)]\,, \tag{21b}$$

$$\frac{F_2^{\nu\nu p}(\xi)}{2\xi}=\tfrac{1}{4}\{6b+\tfrac{16}{9}\sin^4\theta_W\,[\,4(u_++c)+(u_-+v)\,]-\tfrac{8}{3}\sin^2\theta_W\,[\,2(u_++c)+(u_-+v)\,]\}\,, \tag{22a}$$

$$\frac{F_2^{\nu\nu n}(\xi)}{2\xi}=\tfrac{1}{4}\left\{6b+\tfrac{16}{9}\sin^4\theta_W\left[4(u_-+c)+(u_++v)\right]-\tfrac{8}{3}\sin^2\theta_W\left[2(u_-+c)+(u_++v)\right]\right\}. \tag{22b}$$

From Eqs. (21), using the positivity conditions (20), it follows that

$$4\geq\frac{F_2^{en}(\xi)}{F_2^{ep}(\xi)}\geq\frac{1}{4}, \tag{23}$$

which is the same result as in the usual theory.[7] Further,

$$\frac{F_2^{ep}(\xi)}{2\xi}=\tfrac{1}{6}\left[\tfrac{8}{3}(u_++u_-+c+v)-2(u_-+v)\right]$$
$$=\tfrac{1}{6}\left[8b-2(u_-+v)\right]\leq\tfrac{4}{3}b \tag{24a}$$

and

$$F_2^{ep}/2\xi\geq\tfrac{1}{3}b. \tag{24b}$$

Similarly

$$\frac{F_2^{ep}+\tfrac{1}{4}F_2^{en}}{2\xi}\leq\tfrac{5}{3}b,\quad\frac{F_2^{ep}+\tfrac{1}{4}F_2^{en}}{2\xi}\geq\tfrac{5}{12}b. \tag{25}$$

Also from Eqs. (22a) and (20) [the latter also gives $3b-(u_++c)=u_-+v\geq0$]

$$\frac{F_2^{\nu\nu p}(\xi)}{2\xi}=\tfrac{1}{4}\left\{2b+\tfrac{1}{3}(u_-+v)+\tfrac{1}{3}\left[(1-4\sin^2\theta_W)^2(u_++c)+(3-4\sin^2\theta_W)^2b\right]\right\}\geq\tfrac{1}{2}b. \tag{26}$$

If we now use Eq. (24a) we get

$$F_2^{\nu\nu p}(\xi)\geq\tfrac{3}{8}F_2^{ep}(\xi), \tag{27a}$$

and if we use (11a), (11b), and (18) we obtain

$$F_2^{\nu\nu p}(\xi)\geq\tfrac{1}{12}\left[F_2^{\nu l p}(\xi)+F_2^{\bar\nu\bar l p}(\xi)\right]. \tag{27b}$$

Actually one can obtain a stronger bound than that given in the inequality (27a) as follows[8]: From Eqs. (21a), (22a), and the relation $3b=(u_++c)+(u_-+v)$ we have

$$\frac{F_2^{\nu\nu p}(\xi)}{2\xi}-\beta\frac{F_2^{ep}(\xi)}{2\xi}=\tfrac{1}{12}\left\{\tfrac{1}{3}(u_++c)\left[(18-16\beta)-48\sin^2\theta_W+64\sin^4\theta_W\right]\right.$$
$$\left.+\tfrac{1}{3}(u_-+v)\left[(\tfrac{45}{4}-4\beta)-24\sin^2\theta_W+16\sin^4\theta_W\right]+\tfrac{9}{4}(u_-+v)\right\}. \tag{27c}$$

Now from inequalities (20), $(u_++c)\geq0$, $(u_-+v)\geq0$ and their coefficients in the first and second terms on the right-hand side of Eq. (27c) are perfect squares for $\beta=\tfrac{9}{16}$ so that $F_2^{\nu\nu p}-\tfrac{9}{16}F_2^{ep}\geq0$, giving the bound

$$F_2^{\nu\nu p}(\xi)\geq\tfrac{9}{16}F_2^{ep}(\xi). \tag{27d}$$

The inequalities (27b) and (27d) are the primary predictions of Weinberg's model in the light-cone-algebra approach. It may be feasible to test these bounds in the future.

Several sum rules follow from Eqs. (11) if we multiply them with $e^{-i\xi(p\cdot z)}$ and integrate them with respect to ξ from −1 to +1. Then we note that for $z=0$ the bilocal operator $V_\sigma^R(S;z=0,0)$ reduces to the local current operator $V_\sigma^R(0)$ while $V_\sigma^R(A;z=0,0)=0$ so that from Eqs. (10) and (9)

$$\int_{-1}^{+1}d\xi\,G_R^S(\xi)\equiv\tilde G_R^S(0)=\langle F_R\rangle,$$
$$\tilde G^A(0)=0, \tag{28}$$

where F_R denotes a generator of SU(4). Now we can express the matrices I, W^0, and X as

$$I=3\Lambda_B,$$
$$W^0=\Lambda_{\mathfrak{C}}+2\Lambda_3-(\Lambda_B-\Lambda_Y), \tag{29}$$
$$X=\tfrac{1}{3}\Lambda_{\mathfrak{C}}+\tfrac{8}{3}\Lambda_B+\tfrac{2}{3}\Lambda_3+\tfrac{1}{3}\Lambda_Y.$$

Then, using the above results and the crossing property

$$F_{2,3}^{\bar\nu\bar l}(\xi)=F_{2,3}^{\nu l}(-\xi),$$
$$F_{2,3}^{\nu\nu}(\xi)=F_{2,3}^{\nu\nu}(-\xi), \tag{30}$$

the following sum rules follow from Eqs. (11b)–(11e) and (11h), respectively:

$$\int_0^1 [F_2^{\bar{\nu}\bar{l}}(\xi) - F_2^{\nu l}(\xi)]\,\frac{d\xi}{\xi} = 2(\mathcal{C} - B + Y + 2T_3), \tag{31}$$

$$-\int_0^1 [F_3^{\bar{\nu}\bar{l}}(\xi) + F_3^{\nu l}(\xi)]\,d\xi = 6B, \tag{32}$$

$$-\int_0^1 F_3^{\nu\nu}(\xi)\,d\xi = \tfrac{1}{4}[6B - 4\sin^2\theta_W(\tfrac{1}{3}\mathcal{C} + \tfrac{8}{3}B + \tfrac{1}{3}Y + \tfrac{2}{3}T_3)], \tag{33}$$

where $\mathcal{C}$, B, Y, and T_3, respectively, denote the charm, baryon number, hypercharge, and third component of isospin of the target. For proton or neutron target $\mathcal{C} = 0$, $B = 1 = Y$, and $T_3 = \pm\frac{1}{2}$. Note that Eqs. (31) and (32) are, respectively, the Adler and Gross–Llewellyn Smith sum rules [6] in the present theory. We note that in particular from Eq. (33)

$$\int_0^1 d\xi\,[F_3^{\nu\nu p}(\xi) - F_3^{\nu\nu n}(\xi)] = \tfrac{2}{3}\sin^2\theta_W, \tag{34}$$

which gives a means for determining the parameter θ_W experimentally. By using the relation $Q = T_3 + \frac{1}{2}(Y + \mathcal{C})$ where Q here denotes the charge of the hadronic target, one can express the right-hand sides of (31) and (33) in terms of the charge Q and the baryon number B of the target.

The fractionally charged quark model considered in the previous two paragraphs corresponds to a hypercharge assignment to q_4 such that $Q = T_3 + \frac{1}{2}(Y + \mathcal{C})$, where Q here denotes the charge of the hadron. A different hypercharge assignment has been given to q_4 in Ref. 3 which corresponds to $Q = T_3 + \frac{1}{2}Y + \mathcal{C}$. The results for this case trivially follow from the previous considerations by making the replacements $Y \to Y + \mathcal{C}$, $\Lambda_Y \to \Lambda_Y + \Lambda_{\mathcal{C}}$. The only effect is that for this case the sum rules (31) and (33) take the form

$$\int_0^1 [F_2^{\bar{\nu}\bar{l}}(\xi) - F_2^{\nu l}(\xi)]\,\frac{d\xi}{\xi} = 2(2\mathcal{C} - B + Y + 2T_3), \tag{31'}$$

$$-\int_0^1 F_3^{\nu\nu}(\xi)\,d\xi = \tfrac{1}{4}[6B - 4\sin^2\theta_W(\tfrac{2}{3}\mathcal{C} + \tfrac{8}{3}B + \tfrac{1}{3}Y + \tfrac{2}{3}T_3)]. \tag{33'}$$

Finally if we identify in the free-quark model $-\frac{1}{4}(\bar{q}\gamma_\mu \overleftrightarrow{\partial}_\nu q + \mu \leftrightarrow \nu)$ with the energy-momentum tensor, we have [4]

$$\int_{-1}^{+1} \xi G_I^A(\xi)\,d\xi = 1 \quad \text{or} \quad \int_{-1}^{+1} \xi\, b(\xi)\,d\xi = \tfrac{1}{3}. \tag{35}$$

Then from Eqs. (11b), (11c), (24b), and (26), using the crossing property, Eq. (30),

$$\int_0^1 [F_2^{\bar{\nu}\bar{l}}(\xi) + F_2^{\nu l}(\xi)]\,d\xi = 2, \tag{36}$$

$$\int_0^1 F_2^{ep}(\xi)\,d\xi \leq \tfrac{4}{9}, \qquad \int_0^1 F_2^{ep}(\xi)\,d\xi \geq \tfrac{1}{9}, \tag{37}$$

$$\int_0^1 F_2^{\nu\nu p}(\xi)\,d\xi \geq \tfrac{1}{6}. \tag{38}$$

The lower bound provided by Eq. (38) is quite large if we compare with the second of the inequalities (37). The factors on the right-hand sides of the above relations arise in the pure quark model. If there is a gluon present, one would obtain deviation from 2 in the relation (36), for example, and such a deviation would measure the gluon contribution.

Note added in proof. After the paper was submitted for publication, we came across a report by R. Budny and P. N. Scharbach [9] covering essentially the same material.

One of us (R.) would like to thank Professor A. Donnachie for hospitality at the Daresbury Nuclear Physics Laboratory.

APPENDIX

In this appendix we summarize the results for the integrally charged quarks (i.e., for $a = 1$, and $a = 0$) corresponding to the results we obtained for the fractionally charged quarks in the text. For $a = 1$, q_1, q_2, and q_3 have the quantum numbers of p, n, and Λ with zero charm while q_4 has the same charge as p but has charm 1. For $a = 0$, q_1, q_2, and q_3 have the quantum numbers of Ξ^0, Ξ^-, Ω^-, for example, while q_4 has the same charge as Ξ^0 but differs from the triplet by one unit of charm. We first note from Eqs. (12a) and (12b) that for both these cases

$$X = 2Q^2, \tag{A1}$$

so that from (11b), (11c), (11f), and (11g) we have

$$F_2^{\nu\nu}(\xi) = \tfrac{1}{4}[F_2^{\bar{\nu}\bar{l}}(\xi) + F_2^{\nu l}(\xi)] - 2\sin^2\theta_W(1 - 2\sin^2\theta_W)F_2^e(\xi), \tag{A2}$$

which is a particular case of the relation (15c) for $a = 1$ or 0.

Case a = 1.

$$\Lambda_{\mathcal{C}} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$

$$\Lambda_Y = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix}, \quad \Lambda_B = I,$$

$$Q^2 = N_4 + N_1, \tag{A3}$$

$$N_4 = \Lambda_{\mathcal{C}}, \quad N_1 = \tfrac{1}{2}(\Lambda_Y - \Lambda_{\mathcal{C}}) + \Lambda_3, \quad N_3 = \Lambda_B - \Lambda_Y, \tag{A4}$$

so that following the same notation and same argument as in the text

$$c \geq 0, \quad \tfrac{1}{2}(y-c) \pm d \geq 0, \quad b - y \geq 0. \tag{A5}$$

Now from Eqs. (12a) and (12b) for $a=1$

$$I = 2\,Q^2 - W^0. \tag{A6}$$

Thus we can write (11g) as

$$\frac{F_2^{\nu\nu}(\xi)}{2\xi} = \tfrac{1}{4}\left[G^A_{Q^2}(\xi) + (1 - 4\sin^2\theta_W)^2 G^A_{Q^2}(\xi) + G^A_I(\xi) - G^A_{W^0}(\xi) \right]. \tag{A7}$$

Now we can express W^0 as

$$W^0 = \Lambda_{\mathcal{C}} - (\Lambda_B - \Lambda_Y) + 2\,\Lambda_3, \tag{A8}$$

so that

$$\langle G^A_{W^0}\rangle_{p,n} = c - (b-y) \pm 2d \tag{A9a}$$

and

$$\langle G^A_I - G^A_{W^0}\rangle_{p,n} = 2b - c - y \mp 2d = 2(b-y) + 2\left[\tfrac{1}{2}(y-c) \mp d\right] \geq 0, \tag{A9b}$$

on using the positivity conditions (A5). Thus using the positivity condition (A9b) and positivity of $G^A_{Q^2}$, we obtain from Eq. (A7)

$$\frac{F_2^{\nu\nu p,n}}{2\xi} \geq \tfrac{1}{4}\langle G^A_{Q^2}\rangle_{p,n}, \tag{A10a}$$

which gives on using Eq. (11f)

$$F_2^{\nu\nu p,n}(\xi) \geq \tfrac{1}{4} F_2^{ep,en}(\xi). \tag{A10b}$$

The $z=0$ sum rules are obtained by noting the relation (A8), $\Lambda_B = I$ and

$$X = \Lambda_{\mathcal{C}} + \Lambda_Y + 2\,\Lambda_3, \tag{A11}$$

and following the procedure used in the text. Such sum rules are

$$\int_0^1 \left[F_2^{\bar{\nu}\,T}(\xi) - F_2^{\nu\,l}(\xi)\right]\frac{d\xi}{\xi} = 2\left[\mathcal{C} - B + Y + 2T_3\right], \tag{A12}$$

$$-\int_0^1 \left[F_3^{\bar{\nu}\,T}(\xi) + F_3^{\nu\,l}(\xi)\right] = 2B, \tag{A13}$$

$$-\int_0^1 F_3^{\nu\nu}(\xi)d\xi = \tfrac{1}{4}\left[2B - 4\sin^2\theta_W(\mathcal{C} + Y + 2T_3)\right]. \tag{A14}$$

The sum rule (A14) gives, in particular,

$$\int_0^1 \left[F_3^{\nu\nu p}(\xi) - F_3^{\nu\nu n}(\xi)\right] = 2\sin^2\theta_W. \tag{A15}$$

The sum rule (36) of the text, namely,

$$\int_0^1 \left[F_2^{\bar{\nu}\,T}(\xi) + F_2^{\nu\,l}(\xi)\right] d\xi = 2, \tag{A16}$$

holds also in the case $a=1$. Using Eq. (A16), we obtain from (A2)

$$\int_0^1 F_2^{\nu\nu}(\xi)\,d\xi - 2\sin^2\theta_W(2\sin^2\theta_W - 1) \times \int_0^1 F_2^e(\xi)\,d\xi = \tfrac{1}{2}. \tag{A17}$$

Case $a=0$. We quote here the results without giving details which are similar to the case considered above. The bound (A10b) holds here also and so does the sum rule (A13). The sum rules (A12) and (A14), however, are different and are, respectively,

$$\int_0^1 \left[F_2^{\bar{\nu}\,T}(\xi) - F_2^{\nu\,l}(\xi)\right]\frac{d\xi}{\xi} = 2(\mathcal{C} + B + Y + 2T_3), \tag{A18}$$

$$-\int_0^1 F_3^{\nu\nu}(\xi)\,d\xi = \tfrac{1}{4}\left[2B + 4\sin^2\theta_W(\mathcal{C} + Y + 2T_3)\right]. \tag{A19}$$

The sum rule (A19) gives in particular

$$\int_0^1 \left[F_3^{\nu\nu p}(\xi) - F_3^{\nu\nu n}(\xi)\right] d\xi = -2\sin^2\theta_W.$$

The sum rules (A16) and (A17) also hold in this case.

By using the relation $Q = T_3 = \tfrac{1}{2}(Y + \mathcal{C})$, one can express the right-hand sides of the sum rules (A12), (A14), (A18), and (A19) in terms of the charge Q and the baryon number B of the target.

---

*On leave of absence from Institute of Physics, University of Islamabad, Islamabad, Pakistan.

[1]A. Salam and J. C. Ward, Phys. Letters 13, 168 (1964); S. Weinberg, Phys. Rev. Letters 19, 1264 (1967); A. Salam, *Elementary Particle Theory*, edited by N. Svartholm (Almquist and Forlag, Stockholm, 1968),

p. 367; S. Weinberg, Phys. Rev. Letters 27, 1688 (1971), and other references cited therein.

[2]S. Weinberg, Phys. Rev. D 5, 1412 (1972). See also B. W. Lee, NAL Report No. NAL-THY-34, 1972 (unpublished).

[3]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); J. D. Bjorken and S. Glashow, Phys. Letters 11, 255 (1964).

[4]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* Proceedings of the 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2.

[5]R. Budny, Phys. Letters 39B, 553 (1972).

[6]For summaries of the predictions of the usual theory in the present context, see Ref. 4 and C. H. Llewellyn Smith, Phys. Reports 3C, 263 (1972).

[7]O. Nachtmann, Nucl. Phys. B38, 397 (1972).

[8]One of us (R.) would like to thank Dr. R. Budny and Dr. Scharbach for pointing this out to him.

[9]R. Budny and P. N. Scharbach, Phys. Rev. D (to be published).