[7]F. von Hippel and J. K. Kim, Phys. Rev. Letters 22, 740 (1969); Phys. Rev. D 1, 151 (1970).

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[9]H. Fritzsch and M. Gell-Mann, in report presented to 1971 Coral Gables Conference on Fundamental Interactions at High Energy (unpublished); Caltech Report No. CALT-68-297 (unpublished).

[10]There could be a possible enhancement of $\langle N|u_0|N\rangle$ with respect to $\langle N|u_8|N\rangle$, which comes about if one assumes u_0 to be coupled to the Goldstone boson of a further symmetry (scale invariance) which would be broken also by $u_0 + cu_8$ [see, e.g., G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Letters 35B, 415 (1971)]. Although this is an attractive possibility, there is no further hard experimental evidence for such a so-called "dilaton," and for the moment we have no *a priori* reason to assume such a situation.

[11]B. R. Martin, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1970), Vol. 55, p. 73.

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[13]*A priori* it is not clear that terms like $m_K{}^4$ can be safely neglected. Work is in progress investigating such higher-order contributions using mass dispersion relations and methods proposed by Altarelli *et al.* (Ref. 10).

[14]M. G. Albrow *et al.*, Nucl. Phys. B30, 273 (1971).

[15]J. K. Kim, Phys. Rev. Letters 19, 1074 (1967).

[16]R. Armenteros *et al.*, Nucl. Phys. B8, 195 (1968).

[17]J. K. Kim, Phys. Rev. Letters 27, 356 (1971).

[18]R. C. Miller *et al.*, Nucl. Phys. B37, 401 (1972).

[19]G. V. Dass, C. Michael, and R. J. N. Phillips, Nucl. Phys. B9, 549 (1969).

[20]B. R. Martin and M. Sakitt, Phys. Rev. 183, 1345 (1969).

[21]This coupling is defined by

$$\mathcal{L} = (g/m_N)\tfrac{1}{2}(\partial_\mu \bar{\psi}\phi - \bar{\psi}\partial_\mu\phi)\gamma_5\Psi^\mu + \text{H.c.},$$

where ψ is the nucleon field, ϕ the kaon field, and Ψ^μ the Y_1^* (Rarita-Schwinger) field.

[22]G. Ebel *et al.*, Nucl. Phys. B33, 317 (1971).

[23]For comparison these values for $\sigma_{NN}^{\pi\pi}$ are: 60 MeV (Ref. 3); 40 MeV (Ref. 4); (80 ± 30) MeV (Ref. 5); 34 MeV (Ref. 6).

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Possible Anomalous Interaction in Muon-Proton Scattering at Low Energies*

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We have investigated the possibility of an anomalous effect in muon-proton scattering due to the exchange of a scalar meson. This model for an anomalous effect differs from most others in that the effect cannot be described simply as a muon form factor depending on momentum transfer, but is strongly energy-dependent. The effect is largest at muon energies of a few hundred MeV and vanishingly small at the high energies of present experiments. It would thus be an appropriate experiment for the high-intensity, low-energy muon beams possible at meson facilities such as the Los Alamos Meson Physics Facility (LAMPF). We have also investigated limits on such an interaction obtained from muon $g-2$ and muonic x-ray measurements. For a sizable range of scalar couplings and masses a 5% effect in scattering appears to be easily possible without conflicting with other information.

I. INTRODUCTION

The question of the difference between muon and electron has been a long-standing puzzle. In spite of a large number of precise experimental tests no real differences have been found other than the mass difference and effects directly traceable to it.[1,2] There have been, however, several recent experiments[3-6] measuring muon-proton scattering, both elastic[3-5] and inelastic,[6] at high energies and large momentum transfers which contain some hints of possible deviations from results predicted on the basis of electron-proton scattering. At present such deviations appear to be most easily explained in terms of normalization uncertainties between the e-p and μ-p experiments, although a possible interpretation of the differences could be the presence of an anomalous muon interaction.

As the new meson facilities become operational, and it thus becomes possible to produce intense low-energy muon beams, a new realm of experiments becomes feasible. That is, one can then do very high-precision muon scattering, at least in the low-energy (few hundred MeV) and low-momentum-transfer region. Such experiments have in fact been proposed for the Los Alamos Meson Physics Facility[7] (LAMPF) and should allow one to check the equivalence of μ-p and e-p scattering to a new level of precision.

It thus becomes important to ask whether or not there exist any models of an anomalous muon interaction which predict effects large enough to be observable at low energies, without violating limits set by high-energy, high-momentum-transfer experiments. Perhaps the central result of this paper is that there exists at least one such model, namely, an anomalous μ-p interaction mediated by the exchange of a scalar meson, which predicts effects of reasonable size at low energies, but so small as to be totally unobservable at the energies of existing experiments. Such scalar-meson-exchange models were suggested long ago[8] and variants of such models have since been considered by a number of authors[9,10] in a number of different contexts.

Before looking at the details of such a model we should review briefly the present status of μ-p scattering experiments and the standard techniques used for comparing μ-p and e-p results. As noted before there have been both elastic μ-p experiments[3-5] (at muon energies of 6, 11, and 17 GeV) and inelastic ones[6] (at energies of 10 and 12 GeV). For comparison with e-p scattering, results are usually presented as a ratio of the experimental cross section $[d\sigma(s)/dt]_{\text{exp}}$, where s is the square of the total center-of-mass energy and t is the square of the four-momentum transfer, to $[d\sigma(s)/dt]_{\text{Rosenbluth}}$. The latter is the cross section which would be obtained in the one-photon-exchange approximation (Rosenbluth formula) using the nucleon form factors F_1 and F_2 (or the appropriate generalizations, W_1 and W_2, for inelastic scattering) as obtained from electron scattering. This ratio is usually written as a function of t as

$$R \equiv \left[\frac{d\sigma(s)}{dt}\right]_{\text{exp}} \bigg/ \left[\frac{d\sigma(s)}{dt}\right]_{\text{Rosenbluth}} = [G_\mu(t)]^2, \tag{1.1}$$

where $G_\mu(t)$ is interpreted as a muon form factor, i.e., a form factor at the $\gamma\mu\mu$ vertex. This representation for R is particularly convenient as it summarizes the kind of effect one would observe as a result of a number of different physical mechanisms violating the usual ideas of quantum electrodynamics and producing an anomalous muon interaction. For example various photon-propagator modifications, a photon mass, the exchange of a vector meson, a finite muon size or other vertex modification all lead to a ratio R which is a function of t alone and is given in terms of a $G_\mu(t)$ which formally can be interpreted as a muon form factor.[1,2] Usually $G_\mu(t)$ is written as[1,2]

$$G_\mu(t) = \left(1 - \frac{t}{\Lambda^2}\right)^{-1}, \tag{1.2}$$

where $t < 0$ for scattering and Λ^2 is a parameter which can be bounded using experimental information on R. Implicit in this parametrization is the assumption that the effect is expected to be largest at large t and hence most easily observable at high energies.

The difficulty with Eq. (1.2) however is that it predicts a fairly rapid variation of R with t. This is in disagreement with the high-energy experiments which show an effect more or less independent of t. (The μ-p cross section is consistently about 8% below the e-p results.[3-6]) Thus motivated, Perl[1] has suggested a different parametrization for $G_\mu(t)$,

$$G_\mu(t) = 1 - b\frac{t}{t - \Lambda^2}, \quad 0 \leq b \leq 1. \tag{1.3}$$

For large t, $|t| \gg \Lambda^2$, and b of the order of 0.02–0.04, this gives $R = (1-b)^2$ which is indeed independent of t and looks like a few percent normalization uncertainty. Even for $|t| \lesssim \Lambda^2$, which probably corresponds more closely to present experiments, $G_\mu(t)$ is a sufficiently slowly varying function of t that within statistics it would probably appear as a constant.

The interesting point about the scalar-meson-exchange model is that it leads to a parametrization of the form of Eq. (1.3), with one extremely important difference, namely, in this model b is a function of both s and t. Furthermore, for large s, $b \sim 1/s$. This means that at the relatively high energies of present experiments b, and thus the anomalous effect, is vanishingly small, much smaller than at the relatively low energies accessible to meson facilities.

In Sec. II we describe the scalar-exchange model and the detailed calculations leading to estimates of the size of the anomalous scattering. Section III is devoted to a number of comments regarding possible variations of the model, other exchanges, quantum number restrictions, and the effects of muon spin. In Sec. IV we investigate possible limits on such a scalar interaction obtained from other processes, in particular the very accurate measurements of $g-2$ for the muon and information on energy levels in muonic atoms.

II. CALCULATIONS AND RESULTS

To begin, we postulate the existence of a scalar meson, which we shall call a σ meson, which couples both to nucleons and to muons. The possible existence of such a scalar meson is of course motivated by similar mesons which have been suggested, for example, in analyses of pion-nucleon, nucleon-nucleon, and pion-pion interactions,[11] in σ models of various kinds,[9,10] and in work of Gammel.[9] We should emphasize most strongly however that, for our purposes, we treat the mass and coupling constants of the σ as free parameters and simply ask what ranges of parameters will produce a measurable effect. Thus the scalar meson we consider may or may not be related to the scalar mesons which have arisen in other contexts.

Given the existence of such a scalar meson, one must add to the usual one-photon-exchange description of μ-p elastic scattering [Fig. 1(a)] a contribution from the one-σ-exchange diagram, Fig. 1(b). This leads to a matrix element for $\mu(k_1)+p(p_1)\rightarrow\mu(k_2)+p(p_2)$,[12]

$$M_{fi}=-\frac{ie^2}{t}F_\mu(t)\bar{u}(k_2)\gamma_\mu u(k_1)\bar{u}(p_2)\left[F_1(t)\gamma^\mu+\frac{i\kappa}{2m}F_2(t)\sigma^{\mu\nu}k_\nu\right]u(p_1)-\frac{ig_{\sigma\mu}F_{\sigma\mu}(t)g_{\sigma N}F_{\sigma N}(t)}{t-m_\sigma^2}\,\bar{u}(k_2)u(k_1)\bar{u}(p_2)u(p_1),\tag{2.1}$$

where $k^\mu=(p_2-p_1)^\mu$, $k^2=t$, $e^2/4\pi=\alpha=\frac{1}{137}$, $\kappa=1.79$ is the proton anomalous magnetic moment, and m and m_σ are, respectively, the proton and σ masses. The coupling constants at the $\sigma\mu$ and σN vertices are $g_{\sigma\mu}$ and $g_{\sigma N}$, while the corresponding form factors at these vertices are $F_{\sigma\mu}(t)$ and $F_{\sigma N}(t)$. $F_\mu(t)$ is a possible muon electromagnetic form factor and $F_1(t)$ and $F_2(t)$ are the usual nucleon electromagnetic form factors. For convenience we define a coupling strength $\lambda\equiv g_{\sigma\mu}g_{\sigma N}/e^2$.

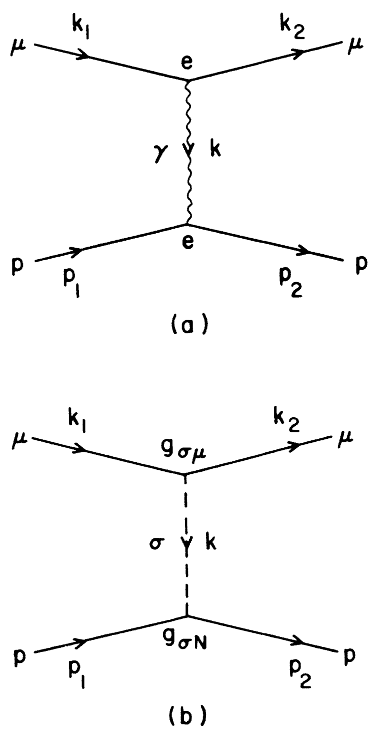

FIG. 1. Contributions to muon-proton scattering: (a) one-photon-exchange diagram; (b) one-σ-exchange diagram.

Thus $\lambda=1$, which will be used in most numerical examples, corresponds to a σ-meson coupling effectively of electromagnetic strength. Our normalization is such that

$$\frac{d\sigma}{dt}=\frac{1}{\pi\lambda(s,m^2,m_l^2)}\,\frac{m^2m_l^2}{4}\sum_{\text{spins}}|M_{fi}|^2,\tag{2.2}$$

where m_l is the lepton mass, $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz$, and where the factor $\frac{1}{4}$ comes from an average over initial proton and muon spins.

The nucleon electromagnetic form factors $F_1(t)$ and $F_2(t)$ are given by the standard expressions

$$F_1(t)=\frac{G_E(t)+\tau G_M(t)}{1+\tau},\quad \kappa F_2(t)=\frac{G_M(t)-G_E(t)}{1+\tau},\tag{2.3}$$

where $\tau=-t/4m^2$. For numerical work we used the scaling hypothesis and the dipole form

$$\begin{aligned}&G_E^p(t)=G_E(t),\\&G_M^p(t)=(1+\kappa^p)G_E(t),\\&G_E^n(t)=0,\\&G_M^n(t)=\kappa^n G_E(t),\\&G_E(t)=\left(1-\frac{t}{0.71\ (\text{GeV}/c)^2}\right)^{-2}.\end{aligned}\tag{2.4}$$

The muon electromagnetic form factor $F_\mu(t)$ was taken to be unity, as we are really investigating a different kind of anomalous interaction here than that used to generate a $F_\mu(t)$. The real problems of course are the unknown form factors $F_{\sigma\mu}(t)$ and $F_{\sigma N}(t)$ or, more precisely, the combination $F_{\sigma\mu}(t)F_{\sigma N}(t)$, which is all that appears. Clearly some sort of form factor must be included. Otherwise the ratio of the one-σ-exchange to the one-photon-exchange contribution becomes extremely large for large t because the one-photon-exchange contribution is strongly suppressed at large t by its form factors. In the absence of concrete information on these form factors we examine two alternatives:

Model A. $F_{\sigma\mu}(t) \sim F_{\sigma N}(t) \sim G_E(t)$. This is a fairly conservative guess and corresponds to saying that there should be a form factor at every vertex and that all form factors are more or less like the electromagnetic dipole.

Model B. $F_{\sigma\mu}(t) F_{\sigma N}(t) \sim G_E(t)$. This is a somewhat less conservative hypothesis and leads to a larger anomalous effect. This model would be realized in the case that the photon and σ meson were assumed to couple in the same way, i.e., to a proton with spatial structure, but to a pointlike muon.

Given these preliminaries it is a straightforward procedure to square the matrix element of Eq. (2.1) and obtain the cross section. As noted above, the relevant quantity is R, the ratio of the cross section with both one-photon and one-σ exchanges included to the one-photon-exchange cross section. We find for R

$$R = 1 + \lambda \mathfrak{F}(t) \frac{t}{t - m_\sigma^2} \frac{4mm_l}{\Sigma}(s - m^2 - m_l^2 - 2m^2\tau) + \lambda^2[\mathfrak{F}(t)]^2 \left(\frac{t}{t - m_\sigma^2}\right)^2 \frac{4m^2}{\Sigma}(1+\tau)(m^2\tau + m_l^2), \tag{2.5}$$

where

$$\Sigma = \frac{1}{G_E^2(t)}\left(\frac{G_E^2(t) + \tau G_M^2(t)}{1+\tau}[\lambda(s, m^2, m_l^2) + 4m^2 m_l^2 - 4m^2\tau(s - m_l^2)] + 4m^2\tau G_M^2(t)(2m^2\tau - m_l^2)\right) \tag{2.6}$$

is related to the Rosenbluth formula for nonzero lepton mass by

$$\left(\frac{d\sigma}{dt}\right)_{\text{Rosenbluth}} = \frac{4\pi\alpha^2}{\lambda(s, m^2, m_l^2)} \frac{[F_\mu(t)]^2}{t^2}[G_E(t)]^2\Sigma, \tag{2.7}$$

and where we have lumped the form factors into a function $\mathfrak{F}(t)$ given by

$$\mathfrak{F}(t) = \frac{F_{\sigma\mu}(t) F_{\sigma N}(t)}{F_\mu(t) G_E(t)}. \tag{2.8}$$

This expression for R contains two contributions, a λ term from the one-photon–one-σ interference and a λ^2 term from the square of the one-σ-exchange graph. The ratio of λ^2 to λ contributions is

$$\lambda \mathfrak{F}(t) \frac{t}{t - m_\sigma^2} \frac{m}{m_l} \frac{(1+\tau)(m^2\tau + m_l^2)}{(s - m^2 - m_l^2 - 2m^2\tau)}. \tag{2.9}$$

For large s this goes as $-\lambda t \mathfrak{F}(0) m m_l / m_\sigma^2 s$ as $t \to 0$, and as $\lambda \mathfrak{F}(-s) s^2 / 8mm_l(s + m_\sigma^2)$ as $t \to -s$. Thus for sufficiently small t the λ term dominates and the λ^2 term can be neglected, provided of course that λ is not large. For large t (i.e., at large angles), however, the λ^2 term dominates unless $\mathfrak{F}(-s)$ falls sufficiently rapidly with s (or unless λ is very small). For our model A, $\mathfrak{F}(-s) \sim s^{-2}$ and for model B, $\mathfrak{F}(-s) \sim 1$; and we in fact found that for model A the λ^2 term was quite small, but for model B it was comparable to the λ term even at relatively low energies. Hence we keep both λ and λ^2 terms in further discussion.

A number of qualitative features of this kind of anomalous muon interaction are immediately evident from the formula for R. In the first place, observe that the sign of the effective $\sigma\mu$-σN coupling λ is unknown. Thus the anomalous effect, $R - 1$, may be positive or negative or in fact may even change sign as a function of s or t for those values of the parameters for which the λ and λ^2 terms are comparable. This is in contrast to the usual parametrization, Eqs. (1.1) and (1.2), which predict $R < 1$ for all values of s and t.

For fixed s, the over-all t dependence of R is determined by two factors, the $t/(t - m_\sigma^2)$ factor from the ratio of σ to photon propagators and the form-factor combination $\mathfrak{F}(t)$. The propagator factor ranges from zero at $t = 0$ to

$$[1 + m_\sigma^2 s \lambda^{-1}(s, m^2, m_l^2)]^{-1} \underset{s\to\infty}{\sim} (1 + m_\sigma^2/s)^{-1}$$

at the maximum value of $|t|$. This factor is always less than unity and is largest for large $|t|$. The form factor term $\mathfrak{F}(t)$ enters only for model A which has $\mathfrak{F}(t) \sim G_E(t)$ and produces a drastic suppression of the anomalous effect at large t.

Another important feature can be seen by looking at R for fixed t as $s \to \infty$. Since $\Sigma \sim s^2$ the λ term in R goes as s^{-1} while the λ^2 term goes as s^{-2}. This is simply a result of the fact that we are comparing the amplitudes for spin-zero and spin-one exchange which in field theory have leading terms differing by one power of s. More formally consider R in the limit $s \gg |t|, m^2, m_l^2$ and $|t| \sim m^2 \gg m_l^2$. This corresponds to the conditions of the experiments so far performed, i.e., the energy and momentum transfer are large, but the scattering angle and hence t is still small compared to the maximum value allowable. In this limit

$$R = 1 + \frac{\lambda \mathfrak{F}(t) t}{t - m_\sigma^2}\left(\frac{4m^2 - t}{4m^2 - t(1+\kappa)^2}\right)\frac{4mm_l}{s} + \lambda^2\left(\frac{\mathfrak{F}(t) t}{t - m_\sigma^2}\right)^2\left(\frac{4m^2 - t}{4m^2 - t(1+\kappa)^2}\right)\frac{t(t - 4m^2)}{4s^2}, \tag{2.10}$$

which clearly shows the s dependence. This means that the anomalous effect vanishes as $1/s$ and thus may be expected to be much smaller at the very high energies of present experiments than at the much lower energies accessible to the meson facilities.

Before looking at detailed numerical results we should give one further special case of Eq. (2.5), namely R evaluated at 180°. This formula can be easily obtained by substituting for t in Eq. (2.5) its maximum value, $t_{\max} = -\lambda(s, m^2, m_l^2)/s$. In the limit of large s we obtain

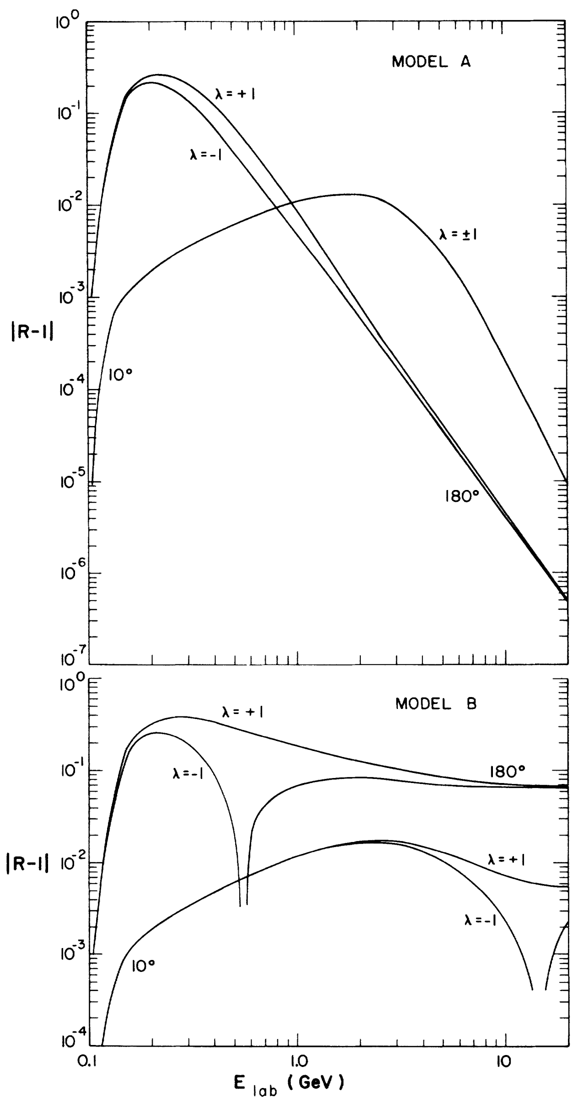

FIG. 2. The magnitude of the anomalous effect, $|R-1|$, as a function of the laboratory energy of the muon, $E_{\rm lab} = (p_{\rm lab}{}^2 + m_l{}^2)^{1/2}$, for models A and B and for laboratory scattering angles 180° and 10°. We took $\lambda = \pm 1$ and $m_\sigma = 700$ MeV.

$$R = 1 + \lambda \frac{\mathfrak{F}(-s)}{s + m_\sigma{}^2} \frac{4mm_l}{(1+\kappa)^2} + \lambda^2 \left(\frac{\mathfrak{F}(-s)s}{s + m_\sigma{}^2} \right)^2 \frac{1}{2(1+\kappa)^2} . \tag{2.11}$$

Since the λ term goes as $1/s$ while the λ^2 term goes as unity for large s, it is clear from this formula that, as we noted above, the λ^2 term will eventually dominate the λ term, unless $\mathfrak{F}(-s)$ falls sufficiently rapidly with s.

Thus, armed with a qualitative understanding of the kind of features to expect, we now look at some detailed numerical results as obtained from Eq. (2.5). For purpose of example we take $|\lambda| = 1$ and $m_\sigma = 700$ MeV. In Fig. 2 we show the magnitude of the anomalous effect, $|R-1|$, plotted versus the total laboratory energy of the muon,

$$E_{\rm lab} = (p_{\rm lab}{}^2 + m_l{}^2)^{1/2},$$

for models A and B and for two different laboratory scattering angles 180° and 10°. At low energies one can hope to be able to actually reach 180°, whereas 10° is an angle comparable to or slightly larger than the maximum angle attained in the existing high-energy experiments.

From these figures we see that for both models A and B the effect peaks in the muon energy range 200–250 MeV, reaching, for $\lambda = 1$ and $m_\sigma = 700$ MeV at least, the fairly sizable values of 26% and 38%, respectively. As the energy increases, $|R-1|$ drops rapidly, especially for model A where the form factor $\mathfrak{F}(t)$ is very important. To actually compare current high-energy experiments with the proposed low-energy one, we should compare the 10° curves at ~10 GeV with the 180° curves at ~200 MeV. We thus see that the low-energy experiment is 2 to 3 orders of magnitude more sensitive to this kind of anomalous effect than the high-energy experiments. One could of course do better at high energies by going to larger angles, provided, as in model B, that form factors are not too important. However, because of the extremely strong forward peaking of the Coulomb cross section, the magnitude of the cross section to be measured soon becomes so small that the measurement becomes impossible.

Finally observe that we have plotted results for both $\lambda = \pm 1$. The difference between the two curves measures the importance of the λ^2 contribution. For model A the λ term dominates and thus $R-1$ has the same sign as λ. For model B, however, as the energy increases the λ^2 term becomes comparable to the λ term and then eventually dominates. If $\lambda < 0$ the λ and λ^2 contributions destructively interfere and eventually lead to a zero in $R-1$, the position of which depends in a very complicated way on $\lambda, m_\sigma, t, E_{\rm lab}$, and the particular choice of form factors.

Figure 3 shows $|R-1|$ versus the laboratory scattering angle of the muon for several different incident energies. In general these curves are relatively flat for large angles which indicates that the 180° results we have given should be representative of results over a wide range of angles in the backward hemisphere. Again a cusp appears for those circumstances when the λ and λ^2 terms destructively interfere. To compare high- and low-energy experiments we must compare the 11 GeV curve essentially at the intercept with the left axis with the 250 MeV curve over the broad range $0 > \cos\theta_\mu^{\mathrm{lab}} > -1$. Again we see the 2 and 3 orders of magnitude advantage of the low-energy experiment.

Finally we have given in Fig. 4 the anomalous effect versus m_σ. For large m_σ we have $|R-1| \sim m_\sigma^{-2}$, coming essentially from the propagator, while for m_σ in the few hundred MeV range $|R-1|$ is much more nearly independent of m_σ, depending somewhat on the energy.

To summarize, we have seen that at least for $\lambda \gtrsim 1$ we can get a sizable anomalous effect from the scalar-exchange model at muon energies in the 200–250 MeV range. At these energies the effect would be several orders of magnitude larger than at the GeV energies of existing experiments and should be easily observable using the high intensity muon beams possible at meson facilities, which should allow experiments with accuracies of a few percent.[7] Low energies however have another very important advantage, as can be seen from Figs. 2–4. That is, at low energies the results are relatively insensitive to the parameters. In particular, form factors do not make a drastic difference and the λ term dominates, so that one does not have to worry about $|R-1|$ changing sign. Thus a low-energy experiment, even one with a null result, can be fairly unambiguously interpreted in

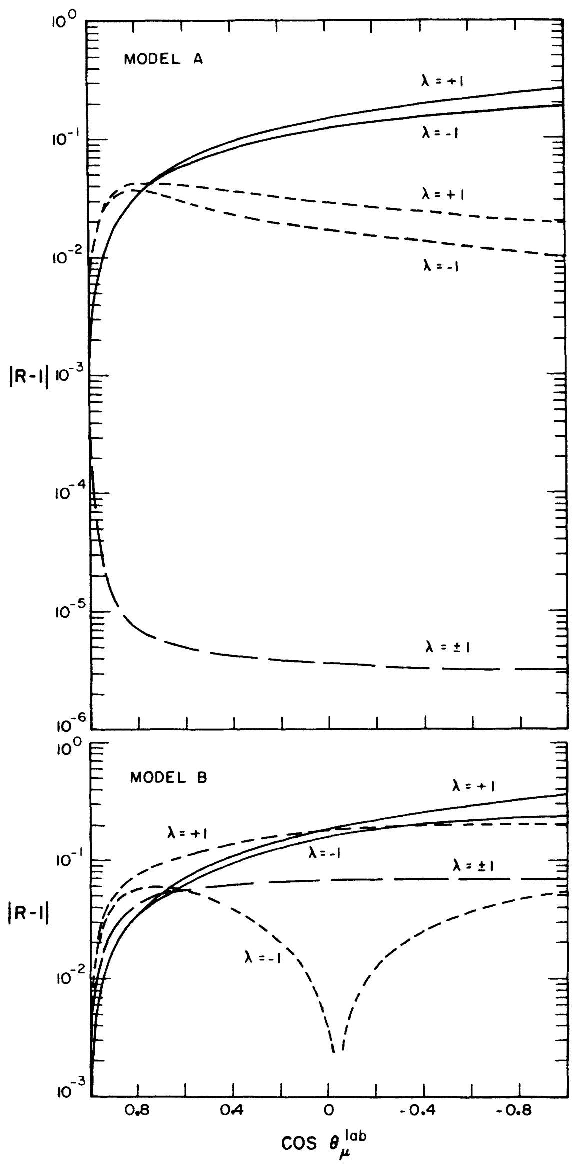

FIG. 3. The magnitude of the anomalous effect, $|R-1|$, as a function of the laboratory scattering angle of the muon for models A and B. We took $\lambda = \pm1$ and $m_\sigma = 700$ MeV. The solid, short-dashed, and long-dashed curves correspond, respectively, to incident muon energies in the laboratory of $E_{\mathrm{lab}} = 250$ MeV, 800 MeV, and 11 GeV.

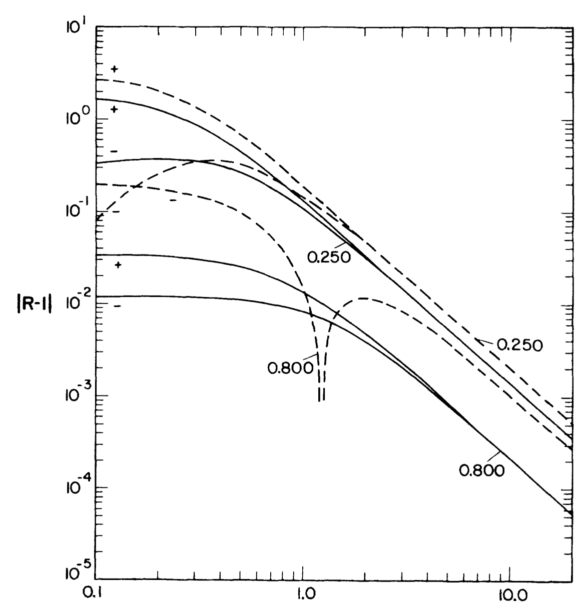

FIG. 4. The magnitude of the anomalous effect, $|R-1|$, as a function of the mass m_σ of the scalar meson for incident muon energies $E_{\mathrm{lab}} = 250$ MeV and 800 MeV and for models A (solid curves) and B (dashed curves). The curves marked + or − correspond to $\lambda = \pm1$.

terms of values of, or limits on, the parameters λ and m_σ. In contrast a null result in a high-energy experiment, even if one had been successful in attaining high accuracy at large angles, would be ambiguous because of the extreme sensitivity to form factors and because of the sizable regions depending on λ, m_σ, and t, where the λ and λ^2 terms destructively interfere so that no effect would be expected.

It is interesting to ask whether or not the apparent effect, $R-1 \simeq -0.08$, observed in high-energy experiments[3-6] can be explained via this scalar-exchange model. Because of the suppression at large s we need a large λ to get an 8% effect at the energies of interest ($\gtrsim$10 GeV). To get a negative effect, however, λ must be negative and sufficiently small that the λ^2 term is negligible. Thus it turns out that at these energies it is rather difficult to get $R-1$ both negative and as large as 8% except at very small t (which also suppresses the λ^2 term). Hence, attributing to normalization uncertainties all or most of the effect observed in existing experiments still seems to be the most attractive possibility.

Finally we should emphasize that experiments at high and low energies are really complementary. If there exists an anomalous effect depending only on momentum transfer as in Eq. (1.1), then regardless of the particular choice of Eq. (1.2) or Eq. (1.3) or something else for the form factor, one would expect to see such an effect most easily at high energies, which allow large momentum transfers. On the other hand, an anomalous effect due to scalar exchange, as has been discussed here, could be seen only at relatively low energies. This means that one can rule out an anomalous effect in muon-proton scattering only by obtaining a null result in *both* high- and low-energy experiments.

III. COMMENTS AND GENERALIZATIONS

Having given some representative numerical results for $\lambda = \pm 1$, our next major task is to investigate what limits on λ have been set by other experiments. Before doing that, however, we digress to make a number of observations and comments regarding the calculation so far and to discuss possible generalizations of the model considered here.

(1) We have averaged over muon spins, i.e., assumed an unpolarized muon beam, in calculating the cross section, whereas in practice, available muon beams are usually strongly polarized. One can show however by explicit calculation that when no other spins are measured the cross section for polarized beams is the same as that for unpolarized ones. Alternatively we simply note that the possible terms which can be formed from only one spin all violate parity or time reversal, both of which are conserved in the scalar exchange model.

(2) If an anomalous effect is actually observed, then one can perhaps learn about the quantum numbers of the exchanged meson by comparing cross sections on neutrons and protons or by comparing μ^+ and μ^- scattering. Specifically if one *assumes* that the σ meson couples strongly to the nucleons so that isospin is conserved, then the coupling to the neutron will have the same or different sign than the coupling to the proton, depending on the isospin of the σ. For "free" neutrons the λ term vanishes since, as can be seen from Eqs. (2.5)–(2.8), it is proportional to G_E^n which is zero. Thus for "free" neutrons one could never learn the sign of the coupling to neutrons. In nuclei, however, interferences among contributions of various nucleons can give terms linear in the neutron-σ coupling. Thus in heavy nuclei one might expect to see significant differences depending on the isospin of the σ. In particular if $I_\sigma = 0$, so that neutron and proton couplings have the same sign, one could expect the effect to be enhanced by roughly the number of nucleons, whereas for $I_\sigma = 1$, so that neutron and proton couplings have opposite signs, there presumably would be large cancellations. Similarly if the σ-μ coupling is charge-conjugation-invariant, then a comparison of μ^+ and μ^- scattering determines the charge conjugation properties of the σ.

(3) One could consider possible generalizations of this model. For example suppose the anomalous interaction is produced by vector-meson, instead of scalar-meson, exchange.[13,14] This would, however, lead to a ratio R of the form of Eq. (1.1) which is independent of s, since both photon and vector meson are spin-one. Thus we would lose the distinctive $1/s$ behavior of the scalar-exchange effect which makes low-energy experiments important. Another possibility would be a pseudoscalar meson. One can show, however, simply by looking at traces, that if the meson couples to muons or nucleons with a simple γ_5 coupling, then the interference with the one-photon-exchange term vanishes. Thus the entire effect would come from the square of the one-meson-exchange diagram which, as we have seen, is usually not very important in the kinematic regions of interest here. Thus it appears that the scalar-meson exchange we have considered would be the most probable candidate for producing an anomalous effect observable at meson facility energies.

(4) We have emphasized muon-proton scattering only; one could also consider muon-nucleus scattering.[15] There is a great deal of information on

electron-nucleus scattering, and a comparison with muon scattering on the same nucleus would be a valid test of muon-electron universality. Even though an effect observed on nuclei would be extremely difficult to interpret theoretically, nuclear targets might be advantageous because they present certain possibilities for enhancing the magnitude of the effect. For example by neglecting all nuclear structure effects and just naively counting nucleons we find the effective λ for nuclear scattering to be $\lambda(A/Z)$ if $I_\sigma = 0$ or $\lambda(N-Z)/Z$ if $I_\sigma = 1$, where A, N, Z are, respectively, the number of nucleons, neutrons, or protons. This gives an enhancement of a factor of 2 for $I_\sigma = 0$, but of course a strong suppression if $I_\sigma = 1$.

One might also consider scattering from a spin-zero nucleus, as there is then no magnetic scattering and we know from electron scattering experience that the elastic Coulomb scattering vanishes in the backward direction, which should thus lead to an enhancement in the relative size of the anomalous effect. However, "vanishes" really means "is proportional to the lepton mass," and since the muon mass is roughly comparable to the energies of interest we might not expect a large effect. We actually calculated the ratio $|R-1|$ for muon scattering on a spin-zero "proton." The maximum effect is roughly a factor of 2 larger than for a spin-one-half proton, but occurs at a higher energy, ~500 MeV, and is thus just out of reach of the meson facilities as now planned. At 200 MeV $|R-1|$ is essentially the same for the two cases. Of course in all of these considerations one must include nuclear structure effects, which may be quite important.

(5) The original motivation for a scalar meson was to explain the difference between muon and electron, and so it presumably coupled only to muons. If such a meson exists, however, one cannot rule out *a priori* the possibility that it may also couple to the electron and perhaps produce an observable effect in electron-proton scattering. Since the λ term in Eq. (2.5) is proportional to m_l one might guess that the effect is small in e-p scattering. However, near threshold, $\Sigma \sim m_l$ as well. We in fact found that for $\lambda = 1$, $m_\sigma = 700$ MeV one could obtain a maximum effect of about 1% for electron energies of the order of 20–25 MeV. In the electron case, of course, scattering from a spin-zero nucleus would lead to a relative anomalous effect greatly enhanced in the backward direction.

IV. LIMITS ON λ FROM OTHER EXPERIMENTS

So far we have given results for $\lambda = 1$, i.e., for an effective anomalous interaction of roughly electromagnetic strength, which was simply a convenient choice which produces an observable effect. The ratio $R-1$ simply scales with λ in the regions where the interference term dominates or can be calculated from Eq. (2.5) for other values of λ in regions where the λ^2 terms are also important. If such an anomalous interaction exists, however, it should produce effects in other places and the extent that such effects have not been seen sets limits on the value of λ. We have in mind particularly the $g-2$ value for the muon and the transition energies in muonic atoms, both of which have been measured quite accurately and both of which are reasonably well described by standard quantum electrodynamics.

Consider first the muon $g-2$ value. The leading contribution to $\kappa_\mu \equiv \frac{1}{2}(g-2)$ is $\alpha/2\pi$ and comes from the diagram Fig. 5(a) involving one-photon exchange, although calculations through sixth order in e have been made.[16] If a σ-μ coupling exists however a diagram such as Fig. 5(b) with one-σ exchange could contribute as well. The contribution of such a diagram has been given by Brodsky and de Rafael[14] and is

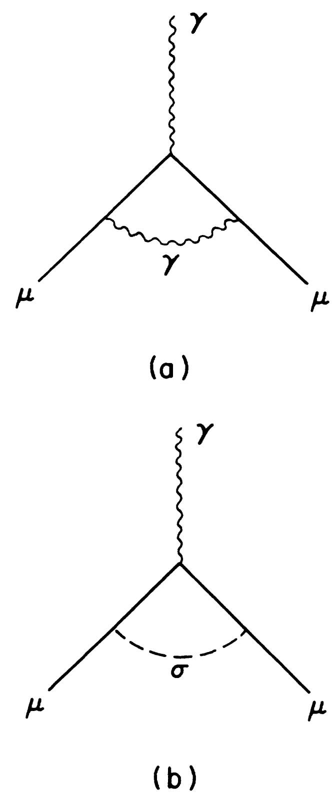

FIG. 5. Leading contributions to the muon $g-2$ value: (a) one-photon-exchange diagram; (b) one-σ-exchange diagram.

$$\frac{\delta\kappa_\mu}{\kappa_\mu}=\frac{g_{\sigma\mu}^2}{e^2}\int_0^1\frac{z^2(2-z)dz}{z^2+(1-z)m_\sigma^2/m_\mu^2}$$
$$\cong\frac{g_{\sigma\mu}^2}{e^2}\,\frac{m_\mu^2}{m_\sigma^2}\left(\ln\frac{m_\sigma^2}{m_\mu^2}-\frac{7}{6}\right)+O\left(\frac{m_\mu^4}{m_\sigma^4}\right). \qquad (4.1)$$

The current experimental value[17] $\kappa_\mu^{\rm exp}=(1\,166\,160\pm 310)\times 10^{-9}$ differs from the best theoretical value[16] $\kappa_\mu^{\rm th}=(1\,165\,848\pm 46)\times 10^{-9}$ by only 312×10^{-9} or about 1 experimental standard deviation. Taking this as the upper limit for an anomalous contribution to $\delta\kappa_\mu$ we obtain, for example, $g_{\sigma\mu}< 0.06e$ for $m_\sigma=700$ MeV and $g_{\sigma\mu}< 0.20e$ for $m_\sigma=3$ GeV. Thus the excellent agreement between experimental and theoretical values of $g-2$ sets very stringent limits on the strength of a possible σ-μ coupling. This however does not rule out a possible effect in muon-proton scattering but simply requires that in order to get $\lambda=g_{\sigma\mu}g_{\sigma N}/e^2\simeq 1$ we must have $g_{\sigma N}\gg e$, i.e., the σ-nucleon coupling must be more like a strong than an electromagnetic coupling. This of course fits in nicely with the kinds of scalar-meson-coupling strengths favored in many theories[9-11] which consider the σ as a strongly interacting particle.

To obtain information about the product coupling λ, which is the relevant quantity for muon-proton scattering, we can look at data obtained from measurement of muonic x-ray transitions in heavy elements, in particular, lead. These transition energies, which are typically of the order of several MeV, have been measured extremely accurately, in many cases to uncertainties of only fractions of a keV.[18] To understand these energies theoretically, one assumes a particular nuclear charge distribution $\rho(r)$ and solves the Dirac equation for a muon bound in the Coulomb potential generated by this distribution. The parameters of the charge distribution are then adjusted to obtain a good fit to the transition energies. A number of corrections must be considered[19] including vacuum polarization, which is the largest, and effects of nuclear polarization which are perhaps the most uncertain, involving quoted uncertainties of a few keV.[20] It should be clear from the very nature of this process, since it involves a certain amount of parameter fitting, that one cannot obtain strict limits on the presence of an anomalous interaction which have the same degree of validity as the limits obtained from the $g-2$ value. What one can do however is to say that all experimental information can be fitted to a high degree of accuracy with certain simple charge distributions involving only a few parameters, and thus to this level of accuracy an anomalous interaction is not needed, even though it may be allowed if one assumes a more complicated charge distribution. Currently an accuracy of 5–10 keV in the energy of the $1s$ level in the lead isotopes is claimed for these fits.[18,21] We should perhaps note in passing however that Dixit *et al.*[22] have recently reported extremely accurate measurements of a number of high-level transitions, which should be essentially independent of the nuclear charge distributions, and have found relatively large discrepancies when compared with theory. Thus conceivably our theoretical understanding of muonic x rays may not be quite as good as generally believed.

In any case, however, if a scalar meson coupling to both muons and nucleons exists, then there would be an effective one-σ-exchange potential which would produce a shift in the energy levels and hence in the x-ray transition energies. It is thus of interest to determine the range of m_σ and λ which would produce energy shifts comparable to present uncertainties in fitting the transition energies, even though we cannot consider such values as unequivocal limits on the presence of an anomalous interaction. To actually carry out such a calculation one should really add the one-σ-exchange potential, properly averaged over the nuclear charge distribution, to the Coulomb potential and solve the Dirac equation using the combined potential. Such a calculation is beyond the scope of this paper. We can however make a very simple perturbation theory estimate of the energy shift to be expected, which we now proceed to do.

To begin we assume, in analogy with the usual procedure for treating the Coulomb interaction, that the muon can be considered as moving in an average potential generated by the sum of the two-body muon-σ-nucleon potentials. Thus we write for this average potential

$$V(\vec{r}_\mu)=\int d^3r\,\rho(\vec{r})V_2(\vec{r}_\mu-\vec{r})$$
$$=\int d^3r\,d^3\Delta\rho(\vec{r})e^{i\vec{\Delta}\cdot(\vec{r}_\mu-\vec{r})}V_2(\vec{\Delta}), \qquad (4.2)$$

where $V_2(\vec{r}_\mu-\vec{r})$ is the two-body muon-nucleon potential generated by σ exchange and $V_2(\vec{\Delta})$ is its Fourier transform. Assuming spherical symmetry and carrying out the angular integrations gives us for the potential

$$V(r_\mu)=\frac{(4\pi)^2}{r_\mu}\int\int d\Delta r\,dr\,V_2(\Delta)\rho(r)\sin\Delta r\sin\Delta r_\mu. \qquad (4.3)$$

The energy shift produced by this potential is thus

$$\Delta E=\int d^3r_\mu|\psi_\mu(\vec{r}_\mu)|^2V(\vec{r}_\mu), \qquad (4.4)$$

where $\psi_\mu(\vec{r}_\mu)$ is the unperturbed muon wave function. The remaining task is to calculate the Fourier transform $V_2(\Delta)$, which we do following the method of Partovi and Lomon.[23] Thus we begin with the momentum-space representation of one-σ exchange as given in the matrix element of Eq. (2.1) and make a reduction to two-component form. The result is expressed in the center-of-mass system in terms of $t=(p_2-p_1)^2=-\vec{\Delta}^2$ and $\vec{Q}^2=(\vec{p}_1+\vec{p}_2)^2$, and in accordance with the arguments of Partovi and Lomon[23] the terms in $\vec{Q}^2$ are dropped. We also neglect all spin-dependent terms. This gives

$$V_2(\Delta)=\frac{-\lambda\alpha}{2\pi^2(mm_\mu)^{1/2}}\frac{(\frac{1}{4}\vec{\Delta}^2+m_\mu^2)^{1/4}(\frac{1}{4}\vec{\Delta}^2+m^2)^{1/4}}{\vec{\Delta}^2+m_\sigma^2}\times F_{\sigma N}(-\vec{\Delta}^2)F_{\sigma\mu}(-\vec{\Delta}^2)\,. \tag{4.5}$$

It is not exactly clear how to handle the form factors, as the nucleon form factor, which measures the spatial extent of the nucleon, has already been included when we assume a smoothed-out charge distribution $\rho(r)$. For numerical calculations we replaced $F_{\sigma N}F_{\sigma\mu}$ by $\mathfrak{F}(-\vec{\Delta}^2)$ which in some sense measures the difference between the form factors for the electromagnetic and σ-exchange processes. The choice is unimportant however as $\mathfrak{F}(t)=1$ or $G_E(t)$ lead to ΔE's which differ only for large m_σ and then only by 5% or so.

In order to obtain a simple estimate for ΔE for the 1s level in lead we made a number of approximations. In particular we have assumed a uniform charge distribution $\rho(r)$ of radius $R_0\simeq 7$ fm,[24] have assumed that $\psi_\mu(r_\mu)\simeq$ constant over the nuclear volume,[24] and have cut off the integral for ΔE at $r_\mu=R_0$. This gives

$$\Delta E=\frac{-9\lambda\alpha A^{\mathrm{eff}}C}{\pi(mm_\mu)^{1/2}R_0^2}\int_0^\infty\frac{dx}{x^4}(\sin x-x\cos x)^2\frac{(x^2+4m^2R_0^2)^{1/4}(x^2+4m_\mu^2R_0^2)^{1/4}}{x^2+m_\sigma^2R_0^2}\,, \tag{4.6}$$

where $C\simeq\frac{1}{2}$ (Ref. 24) is the integral of $|\psi_\mu(r_\mu)|^2$ over the nucleus and A^{eff} is the effective number of nucleons, presumably $N+Z$ if the σ coupling has the same sign for neutrons and protons or $N-Z$ if the sign is different.

We show some of these results in Fig. 6 where

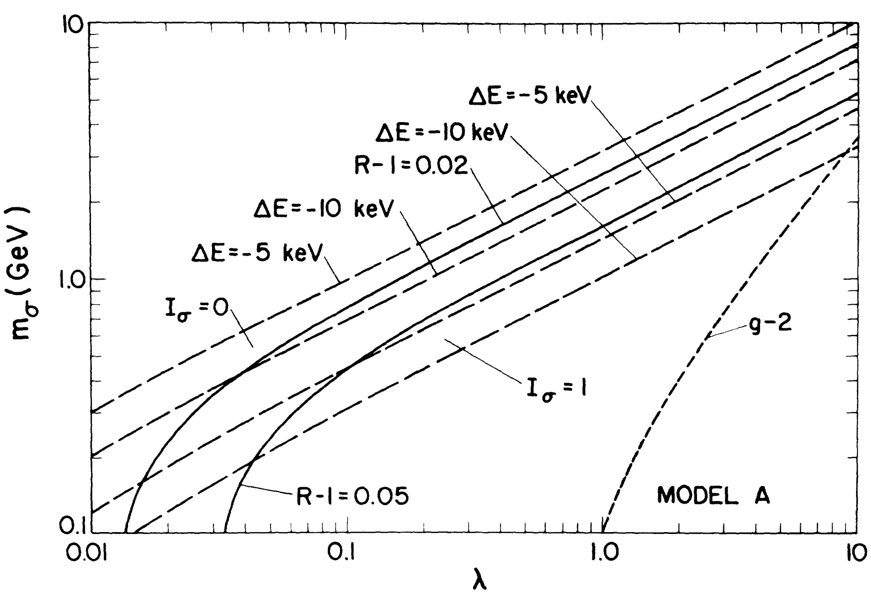

FIG. 6. Values of λ and m_σ which produce a given effect in muon-proton scattering and in the muonic 1s level in lead. The region *below* the curves marked 0.05 and 0.02 corresponds to those values of λ and m_σ leading to an anomalous effect in muon-proton scattering *greater* than 5% and 2%, respectively, for $E_{\mathrm{lab}}=200$ MeV. The region *above* a given ΔE curve corresponds to values of λ and m_σ producing an energy shift in the 1s level in lead in magnitude *less* than ΔE. Curves for somewhat larger values of ΔE, which as discussed in the text are not ruled out by experiment, can be easily obtained using the relation $\Delta E\sim\lambda/m_\sigma^2$. Values of λ and m_σ in the region *above* the curve marked $g-2$ lead to a change in the muon anomalous magnetic moment κ_μ of *less* than the current experimental uncertainty under the assumption $g_{\sigma N}^2/4\pi\simeq 14.5$. We have used model A and assumed $\lambda>0$ so that $\Delta E<0$ and $R-1>0$, although $\lambda<0$ gives almost identical curves, as does model B.

we have plotted in the $\lambda - m_\sigma$ plane lines corresponding to constant ΔE or a constant value of $|R-1|$ in muon-proton scattering. We have taken $\lambda > 0$ which gives $\Delta E < 0$ and $R-1 > 0$, although essentially identical curves hold for $\lambda < 0$. The region *above* the line $\Delta E = -10$ keV corresponds, for example, to those values of λ and m_σ which produce *less* than 10 keV energy shift in the 1*s* state in lead. The region *below* the line $|R-1| = 5\%$ corresponds to a *greater* than 5% effect in the scattering. We see that for $A^{\text{eff}} = N - Z$ there is a sizable region of overlap where a >5% effect could be observed in μ-p scattering while producing a $|\Delta E|$ < 10 keV. On the other hand, for $A^{\text{eff}} = N + Z$ one needs a 2–3% scattering experiment to get much of an overlap. Finally, the region *above* the curve marked $g-2$ is that allowed by the muon $g-2$ results under the assumption that $g_{\sigma N}{}^2/4\pi \simeq 14.5$.

We should reemphasize that the curves in Fig. 6 cannot be considered as strict limits, derived from muonic atoms, on a possible anomalous interaction. This is the case for a number of reasons. In the first place we have made a number of fairly crude approximations, e.g., uniform charge distribution, to actually calculate ΔE. There is also some sensitivity to parameters in that for large m_σ, $\Delta E \sim \lambda A^{\text{eff}} C / m_\sigma{}^2 R_0{}^3$, and so depends particularly on the ratio $C/R_0{}^3$. In addition there is the more subtle approximation that we can consider the muon as moving in an average central potential. This amounts to neglecting nuclear polarization corrections which are small for the Coulomb case but unknown for our case. More important however is the difficulty of principle mentioned before, namely, that we must fit a certain number of parameters of the charge distribution. Thus in principle we can choose a different charge distribution, for example, $\rho(r) \to \rho(r)[1 + \lambda \sum a_i f_i(r)]$, and solve the Dirac equation for the combined Coulomb and σ potentials. Given enough parameters a_i the finite number of transition energies can be fit as well as before for arbitrary strength of the anomalous interaction. Of course electron scattering information puts certain constraints on the charge distribution and those, together with the fact that the standard simple distribution fits the data so well and that the charge distribution must be "reasonable," probably mean that it would be impossible to accommodate a large anomalous interaction. However at the level of a few times the present uncertainty of 10 keV it would seem fairly difficult to rule out an anomalous interaction without much more detailed calculations than those made here.

Thus in summary we see that the range of coupling strengths and σ masses required to produce an easily observable (5%) effect in muon-proton scattering leads also, within fairly large uncertainties, to energy shifts in muonic transitions comparable to current discrepancies. Thus at present it would seem that muonic atom studies neither require nor prohibit an anomalous interaction at the level we have been considering.

The muon anomalous magnetic moment and muon transition energies which we have just discussed certainly provide the most stringent tests for the presence of an anomalous muon-proton interaction, simply because our knowledge, both experimental and theoretical, of these processes is so precise. In addition, effects should in principle be present in other reactions, a few of which we want to mention in these final paragraphs. None of these, however, seem to rule out an interaction of the type and magnitude considered here.

After elastic scattering, the next most complicated reactions involving only muons, photons, and nucleons are photoproduction or electroproduction of lepton pairs or bremsstrahlung by leptons, all in the Coulomb field of a nucleus.[2] The experiments done with muons[2] have so far been at very high energies and/or small momentum transfers where the particular kind of anomalous effect we have discussed would presumably be suppressed by energy or propagator factors. They therefore would not be expected to give much information about a scalar-exchange anomalous interaction. Clearly, however, if the discrepancy in elastic scattering proves to be real and if one wants to consider the scalar-exchange model seriously, one must do a complete analysis of these reactions to see exactly which kinematic regions lead to the largest effect and exactly how large that effect can be. Such a detailed analysis is much more complicated than the present one because of the extra kinematic variables and so does not appear to be particularly profitable at present.

In principle anomalous effects could also be present in decay processes involving muon pairs, e.g., $K \to \mu^+\mu^-$ or $\eta \to \mu^+\mu^-$. Such processes involve couplings other than the simple σ-nucleon coupling and thus are not necessarily directly related to the considerations here. One can make a connection via particular models however. Several such models have been recently considered by Chen, Kawarabayashi, and Shaw[9] and by Barshay.[9] In particular, one model due to Barshay[9] involves scalar mesons. He proposes to explain the $K_L \to \mu^+\mu^-$ puzzle[25] by first assuming CP violation, which leads to a large amplitude for $K_S \to \mu^+\mu^-$,[26] and then assuming that this large amplitude is generated by a 2π intermediate state. This leads to an effective coupling of two pions to a muon, which, for an $I = 0$ scalar meson which couples to two pions, gives an effective σ-muon coupling. With $g_{\sigma N}$

$\simeq g_{\pi N}$, $m_\sigma \simeq 750$ MeV, and the σ width about 300 MeV, Barshay obtains an effective value of $\lambda \simeq 0.07$, which may however be enhanced by form factor effects. From Fig. 6 we see that this model then leads to an effect in scattering consistent with, or somewhat smaller than, that we have been considering.

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Parton Dual-Resonance Model for the Lepton-Hadronic and the Colliding-Beam Inclusive Reactions*

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A unified representation of νW_2 and W_1 for both electroproduction, $e^- + N \to e^- +$ anything, and the colliding-beam reaction $e^+ + e^- \to \bar{N} +$ anything is presented. We then generalize the model to the case of detecting one more hadronic final-state particle, and obtain formulas for *all four* structure functions for both the reaction $l + h_1 \to l + h_2 +$ anything and the reaction $l + \bar{l} \to h_1 + h_2 +$ anything. The explicit formulas further unify the generalized Bjorken scaling law with the Feynman scaling law. We then discuss the fragmentation of the target hadron, the fragmentation of the heavy virtual photon, the two triple-Reggeon limits, the two pionization (nucleonization) limits, the "four-Reggeon" limit, the "fixed-angle" limits, and the generalized threshold behaviors of Bloom and Gilman. The pionization region again shows a universal cutoff of $\exp(-4p_\perp^2)$ in the transverse momentum, and predicts the average multiplicity distribution $\langle n \rangle = a + b \ln s$, where s is the square of missing masses. Formulas for all structure functions for further generalization to an arbitrary number of detected final-state particles are also given.

I. SUMMARY OF THE PHYSICS AND THE MATHEMATICS OF THE PARTON DUAL-RESONANCE MODEL

In the construction of the parton dual-resonance model[1] for electroproduction, $e^- + N \to e^- +$ anything, and the colliding-beam reaction $e^+ + e^- \to \bar{N}$ + anything, we have made the following physical assumptions:

(a) The hadron is made of tightly bound partons.

(b) The parton is characterized by its pointlike coupling with the heavy virtual photon.

(c) The high-energy parton decays into low-energy partons by bremsstrahlung through a parton-parton interaction.

(d) The parton which absorbs the high-energy virtual photon must suffer very strong final-state interactions with the remaining partons inside the hadron, so that no parton can be observed experimentally.

Assumption (d) takes into account the final-state interaction among the partons and resolves the puzzle why the parton is not observed experimentally. By virtue of assumptions (b) and (d) a heavy virtual photon is naturally pictured as a parton-antiparton pair *whenever* it participates in the strong-interaction processes.

The idea of the parton, in this model, is defined to be the *unobserved* field that mediates the electromagnetic interaction with the strong interaction. Being a mediator, the parton possesses both the properties of the electromagnetic interaction and the strong interaction. The electromagnetic properties that the parton possesses are (i) the fundamental coupling to the heavy virtual photon is pointlike and three-legged, (ii) the parton is an unobserved field-theoretical particle having an off-mass-shell Feynman propagator. The strong-interaction properties that the parton has are best stated by saying that the parton leg can be regarded as one of the legs in the n-point Veneziano formula, i.e., two partons (or a parton-antiparton pair) can form a tower of resonances in the same sense as in the ordinary dual-resonance model. [This is due to the physical assumption (c).]

The physical picture of this model can be visualized as follows. To the heavy virtual photon's eyes, the target hadron is a complicated, extended object, composed of infinitely many tightly bound partons, and so the heavy virtual photon interacts at a point constituent (the parton) inside the hadron. After the interaction, the constituent absorbs