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[10]We work in the narrow-resonance approximation, where

$$\mathrm{Im} f_{(J\pm 1/2)\mp}(s) = (\pi/q) x_J M_J \Gamma_J \delta(s - M_J{}^2);$$

e.g., the works of Ref. 8 indicate this is satisfactory for our purpose. Our procedure can be repeated with $\mathrm{Im}A^-$ and $\mathrm{Im}B^-$ calculated through phase shifts; whether this is a significant improvement over resonance saturation is rather unclear.

[11]Our resonance contributions to $\mathrm{Im}B^{(-)}$ at $t=0$ (not presented in this paper) have been compared and found in approximate agreement with the corresponding results of Ref. 8; also, they are in agreement with R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1967) (a comparison of the signs of the contributions can easily be made by means of our results of Table I; between $t=0$ and $t=-0.175$ the sign remains unaltered).

[12]Our normalization is as follows:

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) = \frac{8\pi}{s}(|F_+|^2 + |F_-|^2),$$

$$P(\pi^- p \to \pi^0 n) = 2\,\mathrm{Im}(F_+ F_-^*)/(|F_+|^2 + |F_-|^2).$$

The sign of our F_+ (F_-) is the same (opposite) to that of F_{++} (F_{+-}) of HM.

[13]An estimate of the polarization by a somewhat similar approach has been reported by M. S. Chen and F. E. Paige, Phys. Rev. D 5, 2760 (1972); there are important differences in our approach and in our results.

[14]For pion laboratory momentum of 6 GeV with $\sqrt{s_M} = 2.0$ GeV and $\alpha(t_-)$ varying in the range $-0.2 \le \alpha(t_-) \le 0.2$, we find $3.9 \le (d\sigma/dt)(t_-) \le 5.2$ μb/GeV^2 and $0.384 \ge P \ge 0.287$. With $\sqrt{s_M} = 2.5$ and 3.25 the variation is smaller.

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Renormalizable Model of Weak and Electromagnetic Interactions with *CP* Violation*

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A renormalizable gauge-field model of weak and electromagnetic interaction of leptons and hadrons is constructed. The model can explain *CP* violation in hadronic weak processes and the suppression of hadronic neutral currents.

I. INTRODUCTION

Recently, considerable attention has been focused on the problem of constructing models[1] of weak and electromagnetic interaction of leptons using the Higgs-Kibble mechanism for spontaneously broken gauge symmetries. The spontaneity of symmetry breaking enables one to have a massive vector boson mediating the weak processes and a massless one mediating electromagnetic processes, while simultaneously preserving the gauge invariance of the Lagrangian. This gauge freedom can be exploited to show that such models are renormalizable.[2] In order to give a unified theory of weak and electromagnetic interactions various attempts have been made to include hadrons[3] in such a scheme, and the most symmetrical way to do this seems to be to enlarge the hadron spectrum from the SU(3) to the SU(4) group.[4] This model is consistent with the present upper limits on the coupling of $\Delta S=1$ neutral hadronic currents; however, it appears to violate experimental upper limits[5] on the process $\sigma(\nu + p \to \nu + N^{*+})$. The purpose of this note is to suggest the possibility of introducing *CP* violation into the above SU(4) scheme in such a way that the gauge invariance (and, therefore, the renormalizability) of the model is preserved. The resulting model also explains the upper limit on $\sigma(\nu+p\to\nu+N^{*+})$ and seems consistent with present experiments.

II. CONSTRUCTION OF THE MODEL

We consider a theory invariant under the local SU(2)×U(1) gauge group and introduce spontaneous breaking by giving a nonzero vacuum expectation value to some scalar field in the model, as usual.[1] Furthermore, we construct the following objects:

	SU(2) representation	U(1) quantum number	
$Q_1=\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L\cos\theta+\lambda_L\sin\theta\end{pmatrix}$	$I=\frac{1}{2}$	$Y=a$	(1a)
$Q_2=\begin{pmatrix}\mathcal{P}'_L\\ -\mathfrak{N}_L\sin\theta+\lambda_L\cos\theta\end{pmatrix}$	$I=\frac{1}{2}$	$Y=a$	(1b)
$Q_3=\begin{pmatrix}\mathcal{P}'_R\\ \mathfrak{N}_R\cos\phi+i\lambda_R\sin\phi\end{pmatrix}$	$I=\frac{1}{2}$	$Y=a$	(1c)
$Q_4=\mathcal{P}_R$	$I=0$	$Y=a+1$	(1d)
$Q_5=i\sin\phi\,\mathfrak{N}_R+\cos\phi\,\lambda_R$	$I=0$	$Y=a-1$	(1e)

where θ is Cabibbo angle, ϕ is a small number, and $\mathcal{P}',\mathcal{P},\mathfrak{N},\lambda$ are the quark field operators belonging to the basic representation of the SU(4) group. (Note that $\mathcal{P}'$ has a charm quantum number.) I and Y represent the isospin and hypercharge quantum numbers associated with the SU(2)×U(1) group and have nothing to do with the corresponding strong-interaction quantum numbers. We have not specified a because one could assign integral or fractional charges to the quarks; for example, the $(\frac{2}{3},\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$ charge assignment requires $a=\frac{1}{3}$ (since $Q=I_3+\frac{1}{2}Y$). The results that follow are independent of the value one chooses for a and are therefore the same in both fractionally and integrally charged quark models. The assignment of lepton states is the same as in the original model.[1] One can write down a gauge-invariant Lagrangian using these states which after the introduction of spontaneous symmetry breaking gives rise to massive vector bosons. We do not repeat these standard steps.

The semiweak Lagrangian obtained in this model is the following:

$$L_{sw}=(g/2\sqrt{2})(J_\mu+J_{\mu,c}+l_\mu)W^+_\mu+\text{H.c.}+\frac{U_\mu}{2(g^2+g_1^2)^{1/2}}[K_\mu+(g^2+g_1^2)\cos\phi\sin\phi(V^7_\mu-A^7_\mu)]\,, \tag{2}$$

where

$$J_\mu=\cos\theta(V^{1+i2}_\mu+A^{1+i2}_\mu)+\sin\theta(V^{4+i5}_\mu+A^{4+i5}_\mu)\,, \tag{3}$$

$$\begin{aligned}&J_{\mu,c}=(\cos\phi-\sin\theta)V^{\Delta S=0}_{\mu,c}+(\cos\theta+i\sin\phi)V^{\Delta S=-1}_{\mu,c}-(\sin\theta+\cos\phi)A^{\Delta S=0}_{\mu,c}+(\cos\theta-i\sin\phi)A^{\Delta S=-1}_{\mu,c}\,,\\ &l_\mu=(ig/2\sqrt{2})[\bar\nu\gamma_\mu(1+\gamma_5)e+\bar\nu'\gamma_\mu(1+\gamma_5)\mu]\,,\end{aligned} \tag{4}$$

$$\begin{aligned}K_\mu=&\tfrac{1}{6}i(3g^2-g_1^2)\bar{\mathcal{P}}'\lambda_\mu\mathcal{P}'-(\tfrac{2}{3})^{1/2}(g^2+g_1^2)V^0_\mu+\frac{g^2(1+3\sin^2\phi)-3g_1^2\cos^2\phi}{2\sqrt{3}}V^8_\mu+\frac{3(g^2+g_1^2)\cos^2\phi}{2\sqrt{3}}A^8_\mu\\ &+\tfrac{1}{2}[(2+\cos^2\phi)g^2-(1+\sin^2\phi)g_1^2]V^3_\mu+\tfrac{1}{2}[(1+\sin^2\phi)(g^2+g_1^2)]A^3_\mu+\tfrac{1}{2}i(g^2+g_1^2)[\bar\nu\gamma_\mu(1+\gamma_5)\nu-\bar e\gamma_\mu(1+\gamma_5)e]\\ &+2ig_1^2\bar e\gamma_\mu e+(e\to\mu\text{ and }\nu\to\nu')\,.\end{aligned} \tag{5}$$

We have the following definitions:

$$V^a_\mu=\tfrac{1}{2}i\bar q\gamma_\mu\lambda_a q\,,$$
$$A^a_\mu=\tfrac{1}{2}i\bar q\gamma_\mu\gamma_5\lambda_a q\,,$$

$$q=\begin{pmatrix}\mathcal{P}\\ \mathfrak{N}\\ \lambda\end{pmatrix}, \tag{6}$$

$$V_{\mu,c}^{\Delta S=0}=\tfrac{1}{2}i\bar{\mathcal{P}}'\gamma_\mu\mathfrak{N},$$

$$V_{\mu,c}^{\Delta S=1}=\tfrac{1}{2}i\bar{\mathcal{P}}'\gamma_\mu\lambda.$$

g and g_1 are the universal couplings of the isovector and isoscalar gauge fields, respectively, and are related to the Fermi coupling as follows:

$$\frac{G}{\sqrt{2}}=\frac{g^2}{8m_W{}^2}=\frac{g_1{}^2+g^2}{8m_U{}^2}, \tag{7}$$

m_W and m_U being the masses of the charged (W_μ^+) and neutral (U_μ) vector mesons, respectively.

Now, observe that the term $J_\mu^7 U_\mu$ in the interaction Lagrangian has odd-*CP* properties and will cause *CP* violation in weak interaction, and to get the right order of magnitude, we will have to give a small value to ϕ.

III. MAGNITUDE OF ϕ

In this section, we will argue that a value of ϕ of the order of 10^{-4} will give rise to the correct order of magnitude for *CP*-violating processes and will also be consistent with other observations. Let us consider the decay $K_L^0\to 2\pi$. The matrix element for this process is given by

$$M_{K_L^0\to 2\pi}=C\sin\phi\int d^4k\,\frac{\delta_{\mu\nu}+k_\mu k_\nu/m_U{}^2}{k^2+m_U{}^2}\int e^{-ik\cdot x}\,d^4x\langle 2\pi|\,T(K_\nu^h(x)(V_\mu^7-A_\mu^7))|K_L^0\rangle, \tag{8}$$

where C is some coupling constant and K_μ^h is the hadronic neutral current. It is clearly very difficult to estimate this matrix element, because a whole set of hadronic intermediate states will contribute to it. But a rough order-of-magnitude estimate suggests that

$$\frac{M_{K_L^0\to 2\pi}}{M_{K_S^0\to 2\pi}}\simeq\frac{\sin\phi}{\sin\theta}, \tag{9}$$

and this implies that $\phi\approx 4\times 10^{-4}$.

One thing to be noticed now is that the *CP*-violating neutral current term contributes to K_L-K_S mass difference and $K_S^0\to\mu\bar{\mu}$ decay. In the former case one will have contributions from a complete set of intermediate states; however, if one keeps only the vacuum contribution, one gets, using the above value of ϕ,

$$\Delta M_{K_L-K_S}\simeq(G/\sqrt{2})\,f_K{}^2M_K\sin^2\phi$$
$$\simeq 6\times 10^{-6}\ \text{eV}. \tag{10}$$

Even though this suggests that the order of magnitude suggested for ϕ is consistent with observation, one must remember that we have evaluated the contribution of the vacuum state only; for example, when one evaluates the π^0 intermediate state, this tends to cancel a part of the vacuum contribution to $\Delta M_{K_L-K_S}$; therefore the value of ϕ could really be slightly higher (between 10^{-3} and 10^{-4} eV).

Moreover, we note that the neutral-current term does not contribute to $K_L^0\to\mu\bar{\mu}$ decay, whereas it does contribute to $K_S^0\to\mu\bar{\mu}$ decay, and the Hamiltonian causing this decay is

$$H_{K_S^0\to\mu\bar{\mu}}=i(G/\sqrt{2})\cos\phi\sin\phi\,A_\mu^7\,\bar{\mu}\gamma_\mu\gamma_5\mu. \tag{11}$$

From this we obtain

$$\frac{\Gamma(K_S^0\to\mu\bar{\mu})}{\Gamma(K_S^0\to\text{all})}\simeq 0.43\times 10^{-2}\left(\frac{\sin\phi}{\sin\theta}\right)^2$$
$$\simeq 10^{-7}\text{ to }10^{-8}. \tag{12}$$

From this result, we conclude that the $K_L^0\to\mu\bar{\mu}$ puzzle[6] cannot be resolved within this scheme unless the value of $\phi\approx 3\times 10^{-3}$, and one will have to resort to some other exotic mechanism[7] to understand this.

IV. OTHER CONSEQUENCES

(a) The new Hamiltonian gives rise to $K^+\to\pi^+\nu\bar{\nu}$ decay to lowest order in the Fermi coupling constant due to the presence of the V_μ^7 term, and we get

$$\Gamma^{(-)}(K^+\to\pi^+\nu\bar{\nu})=\left(\frac{\sin\phi}{\sin\theta}\right)^2\Gamma(K^+\to\pi^0e^+\nu), \tag{13}$$

so

$$\frac{\Gamma^{(-)}(K^+\to\pi^+\nu\bar{\nu})}{\Gamma(K^+\to\text{all})}\simeq\frac{1}{20}\left(\frac{\sin\phi}{\sin\theta}\right)^2$$
$$\simeq 2\times 10^{-7}\text{ if }\phi\sim 4\times 10^{-4}$$
$$\simeq 10^{-6}\text{ if }\phi\simeq 10^{-3}\,. \tag{14}$$

Experimentally, the upper limit on the above ratio is 1.2×10^{-6}; therefore our prediction is consistent with present experimental limit.

(b) The present model gives rise to $\nu+p\to\nu+p$ and $\nu+p\to\nu+n+\pi^+$ processes to first order in the Fermi coupling constant. The experimental limits on the first process are not so good. However, the limit on the second process is around 8% of $\sigma(\nu+p\to\mu^-+p+\pi^+)$. Making a theoretical estimate of this process is rather complicated in such models because of V-A interference and the appearance of form factors. However, using certain simplifying assumptions, and assuming N^* dominance, Weinberg[3] has evaluated the ratio

$$\alpha=\frac{\sigma(\nu+p\to\nu+n+\pi^+)}{\sigma(\nu+p\to\mu^-+p+\pi^+)}$$
$$\simeq\frac{1}{3}\frac{\sigma(\nu+p\to\nu+N^{*+})}{\sigma(\nu+p\to\mu^-+N^{*++})} \tag{15}$$

within the SU(4) scheme but with a different assignment of SU(2)×U(1) multiplets from ours: The assignment is that Q_1 and Q_2 form $I=\frac{1}{2}$ multiplets, where $\mathcal{P}_R$, $\mathcal{P}'_R$, $\mathfrak{N}_R$, and λ_R are singlets. In such a model, the weak neutral hadron current contributing to α is

$$\tfrac{1}{2}(g^2+g_1^2)^{1/2}\,U_\mu\left(V_\mu^3+A_\mu^3-\frac{2g_1^{\,2}}{g^2+g_1^{\,2}}V_\mu^3\right). \tag{16}$$

The upper limits on $\nu_e+e\to\nu_e+e$ seem to indicate that

$$g_1^{\,2}\ll(g^2+g_1^{\,2})\,. \tag{17}$$

Also, for small momentum transfers, only the space components of V_μ^3 contribute. Therefore, if one neglects the second term, one finds using SU(2) properties of currents that $\alpha\simeq\frac{1}{9}$. However, in our model (assuming $\phi\approx 0$) the corresponding interaction turns out to be

$$\frac{U_\mu}{2(g^2+g_1^{\,2})^{1/2}}\left[\tfrac{1}{2}(3g^2-g_1^{\,2})V_\mu^3+\tfrac{1}{2}(g^2+g_1^{\,2})A_\mu^3\right], \tag{18}$$

which can be rewritten as

$$\tfrac{1}{4}\,U_\mu(g^2+g_1^{\,2})^{1/2}\left((V_\mu^3+A_\mu^3)+2\,\frac{g^2-g_1^{\,2}}{g^2+g_1^{\,2}}V_\mu^3\right). \tag{19}$$

Now, if we neglect the second term, we get

$$\alpha\simeq\tfrac{1}{36}\simeq 3\%\,, \tag{20}$$

which is four times lower than Weinberg's estimate. Even though the assumptions made in both these models are quite plausible, we really do not know the effect of the vector part; however, in our particular case, we do not believe that the vector part will change the result so much as to contradict experiments. So, we believe that our model provides a rather satisfactory explanation of the suppression of $\Delta S=0$ neutral hadron currents also.

(c) The neutron dipole moment in this model will be of order 10^{-23} e cm. Moreover, the decay process $K^+\to\pi^+e\bar{e}$ will be CP-conserving to lowest order. In the case of $K_L^0\to\pi^0e\bar{e}$, the CP-conserving amplitude is of order $G\alpha^2$, whereas the CP-violating contribution in our model is $G\sin\phi$, and therefore both amplitudes are of the same order in strength and one will observe gross CP violation in this process. This can be contrasted with Okubo's[8] model of CP violation, where the $K^+\to\pi^+e\bar{e}$ mode has both kinds of amplitudes of the same order, whereas the $K_L^0\to\pi^0e\bar{e}$ is dominantly CP-violating. These processes could therefore be used to distinguish our model from Okubo's model. Moreover, the $K_L^0\to 2\pi$ amplitude will have both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ parts.

(d) Our model does not provide any explanation for the observed $\Delta I=\frac{1}{2}$ selection rule in nonleptonic decays.

(e) We further note that the order of magnitude of the CP violation remains the same when ϕ is replaced by $\pi/2-\phi$, but, as is clear from Eq. (5), in this case one will get $\alpha\simeq\frac{1}{9}$, in disagreement with experiment. It would, therefore, seem preferable from the experimental point of view to have a value of ϕ near zero rather than $\pi/2$.

(f) Finally, it seems that the γ_5 anomalies[9] will persist in our model and will tend to destroy the renormalizability. However, this can be taken care of by introducing a pair of heavy leptons with right-handed coupling to the isovector gauge fields and setting $a=1$ in Eq. (1). (See below.)

V. MORE ABOUT THE MODEL

It may appear from the construction of the representations in Eq. (1) that CP violation may be removed from the theory completely by making a transformation $\lambda_R\to -i\lambda_R$ and $Q_5\to iQ_5$. However, if there is a diagonal mass matrix for the quarks in the Lagrangian, the above transformation will make it CP-violating, and therefore, in presence of the mass term, we cannot make the above change of variables to remove CP violation. The purpose of this section is to indicate how the mass terms can be constructed. For this purpose, we require two multiplets of Higgs scalar bosons:

	SU(2) representation	U(1) quantum number
$T=\begin{pmatrix} T^+ \\ T^0 \end{pmatrix}$	$I=\frac{1}{2}$	$Y=1$
$\vec{\pi}=\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$	$I=1$	$Y=0$

(21)

We will choose the following gauge-invariant potential for the T's and π's:

$$\mathfrak{L}_{\rm I} = \mu_1^2 T^\dagger T - h_1^2 (T^\dagger T)^2 + \mu_2^2 \vec{\pi}\cdot\vec{\pi} - h_2^2(\vec{\pi}\cdot\vec{\pi})^2 + f\, T^\dagger \vec{\tau}\, T\cdot\vec{\pi} + f'(T^\dagger T)(\vec{\pi}\cdot\vec{\pi})\,. \tag{22}$$

It is then possible to give a nonzero vacuum expectation value to T^0 and π^0, i.e.,

$$\langle T^0\rangle = \kappa, \quad \langle \pi^0\rangle = \rho\,. \tag{23}$$

We have therefore two Higgs bosons; however, the number of massless scalar particles is still three, and therefore when we work in the unitary gauge we do not have any zero-mass particles. We then choose the following interaction between quarks (Q's), the scalar triplet ($\vec{\pi}$), and the scalar doublet T:

$$\begin{aligned}\mathfrak{L}_{\rm II} = {} & \frac{a_1}{\kappa}\overline{Q}_1 Q_5 T + \frac{a_2}{\kappa}\overline{Q}_2 Q_5 T + \tfrac{1}{2} i a_3 \overline{Q}_3 Q_2 - \frac{i a_3}{2\rho}\overline{Q}_3 \vec{\tau} Q_2\cdot\vec{\pi} + \tfrac{1}{2} i a_4 \overline{Q}_3 Q_1 - \frac{i a_4}{2\rho}\overline{Q}_3\vec{\tau} Q_1\cdot\vec{\pi} \\ & + \frac{i a_1}{\kappa}\overline{Q}_1 Q_5 T + \frac{i}{\kappa} b_2 \overline{Q}_2 Q_5 T + b_3 \overline{Q}_3 Q_2 + \frac{1}{\rho} b_4 \overline{Q}_3 \vec{\tau} \overline{Q}_2\cdot\vec{\pi} + \tfrac{1}{2} b_5 \overline{Q}_3 Q_1 - \frac{b_5}{2\rho}\overline{Q}_3\vec{\tau} Q_1\cdot\vec{\pi} + \frac{b_6}{2\kappa}\overline{Q}_1 Q_4 \cdot \begin{pmatrix} T_0 \\ -T^- \end{pmatrix} + \text{H.c.}\end{aligned} \tag{24}$$

Using Eq. (23) in Eq. (24), we get a lot of bilinear terms in the quark fields, and one can choose the a_i's and b_i's in such a way that all nondiagonal and parity-violating bilinear terms cancel, leaving a diagonal mass matrix as desired. Therefore, the CP violation in our model is *genuine.*

Next we turn to the question of canceling hadronic and leptonic γ_5 anomalies. As stated earlier, to achieve this we have to introduce two heavy leptons (X^0, X^-) which have the following transformation properties:

$$\begin{aligned} & X_R = \begin{pmatrix} X_R^0 \\ X_R^- \end{pmatrix}, \quad I=\tfrac{1}{2}, \quad Y=-1 \\ & X_L^-, \quad I=0, \quad Y=-2 \\ & X_L^0, \quad I=0, \quad Y=0\,. \end{aligned} \tag{25}$$

Then the anomalies arising out of the Q_3 loop will be canceled by the corresponding anomalies due to the X_R loop. The electron and muon doublets of Weinberg[1] cancel the hadronic anomalies arising out of the Q_1 and Q_2 loops and the theory is free of γ_5 anomalies. Moreover, if we add to the Lagrangian the interactions

$$\mathfrak{L}_{\rm III} = d_1 X_R^\dagger X_L^- T + d_2 X_R^\dagger X_L^0 \begin{pmatrix} T_0^\dagger \\ -T^- \end{pmatrix} + \text{H.c.}, \tag{26}$$

then this gives mass to the X leptons, and they can be made heavy by choosing d_1 and d_2 suitably.

In summary, we would like to say that we have been able to construct a renormalizable model of weak and electromagnetic interactions of leptons and hadrons with CP violation. The model is an extension of the original Weinberg-Salam model. We require three heavy vector bosons $(W_\mu^\pm, U_\mu)$; four heavy leptons $(X^+, X^0, X^{0\dagger}, X^-)$ apart from electron, neutrino, and muons; four heavy scalar bosons $(\sigma, \pi^\pm, \pi^{0\prime})$; and an extra charm quark $\mathcal{P}'$. Some of the consequences of the model are also given.

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Simplified Regge Analysis of Backward π^+p Scattering*

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Polarization and differential-cross-section data for backward π^+p scattering are explained in terms of a model which includes the N_α Regge pole plus a background term which one may interpret as either a secondary pole or cut. This model, although simplifying the number of exchanges, provides an excellent fit to the data. The dip-bump structure and polarization are related. The rise of the differential cross section after the dip is shown in this model to be due in part to the background term rather than solely the recovery of the N_α Regge-pole term from zero due to the vanishing of $\alpha-\frac{1}{2}$ near $u=-0.14$ (GeV/c)^2.

Backward π^+p scattering has proven to be a difficult reaction to explain in terms of simple Regge-pole exchanges. The earliest models[1] in which N_α and Δ are exchanged were found to be inadequate. Further attempts which added either a cut[2] or an additional pole[3] to the N_α and Δ have not met with much more success either. The most serious aspect of these failures has been their consistent inability[4] to explain the polarization.[5,6] We return to this reaction with a simple model which includes not only the N_α Regge pole, but also includes a background term which we have parametrized as a fixed pole. This term probably represents a cut contribution, but in the interest of limiting the number of free parameters we have approximated it by a pole.

This approach is similar to the one used in our recent analysis[7] of π^-p charge-exchange (CEX) scattering in which we showed that both the differential cross section[8] and polarization data[9] could be explained with a ρ Regge pole plus a fixed-pole non-spin-flip background term. We were also able to relate the dip-bump structure and the polarization. For $|t|$ less than 0.6 (GeV/c)^2 we assumed that the single ρ Regge pole dominates and that for larger $|t|$ the background term dominates. The rise of the cross section after the dip at $t=-0.6$ (GeV/c)^2 is due to the explicit t dependence of the background and not because the α_ρ factor was recovering from zero. The parameters of the ρ pole and background were determined by fitting the differential-cross-section data. This enabled us to determine the polarization without recourse to parameter adjustment. Our excellent agreement with the π^-p CEX data indicated the validity of our assumptions and the possibility of their extension to other reactions. As we noted earlier, the absence of a dip and any significant amount of polarization for the reaction $\pi^-p\to\eta n$ is consistent with our scheme but is not really a valid test of our ideas.

We feel an attempt to fit the backward π^+p scattering would provide a nontrivial test of our model because of the fact that this reaction is even more complicated than π^-p CEX and also because it has so far eluded a successful analysis. We were also pleased by the prospect of extending the model to a reaction involving the exchange of baryon trajectories instead of meson trajectories.

We express[10] the differential cross section and polarization in terms of the s-channel helicity amplitudes H_{++} and H_{+-}: