Estimate of $\pi^- p \to \pi^0 n$ Amplitudes in Terms of Direct-Channel Resonances and Dual-Absorptive Requirements

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The real parts of the s-channel helicity amplitudes, the differential cross section, and the polarization for $\pi^- p \to \pi^0 n$ are estimated at $t = -0.175$ and $-0.5\ \text{GeV}^2$ by means of dispersion relations, s-channel resonances, and dual absorptive requirements. The results are in satisfactory agreement with experiment and with recent amplitude analyses.

I. INTRODUCTION

Important experimental work on pion-nucleon scattering has recently made possible model-independent amplitude analyses such as that of Halzen and Michael (HM).[1] So far, however, very little has been accomplished towards a theoretical understanding of the results of this analysis.

As a small step in this direction we present an estimate of the real parts of the helicity-nonflip (F_+) and -flip (F_-) amplitudes and of the differential cross section and polarization[2] for $\pi^- p \to \pi^0 n$ at two characteristic values of the momentum transfer $\sqrt{-t}$. This estimate is based on

(a) the requirements of the dual-absorptive (DA) model of Harari,[3]

$$\operatorname{Im} F_\pm(\nu, t_\pm) = 0\,, \tag{1}$$

where $\nu = (s-u)/4M$, M is the nucleon mass, and $t_+ \simeq -0.175\ \text{GeV}^2$, $t_- \simeq -0.5\ \text{GeV}^2$ [the requirements (1) receive definite support from the HM and other πN amplitude analyses[4,5]];

(b) the resonance saturation of the imaginary parts of the $\pi^- p \to \pi^0 n$ amplitudes at low energies; and

(c) fixed-t dispersion relations.

It will become clear that our procedure is almost model-independent.

II. ESTIMATE AT $t = t_+$

The amplitude F_+ is given by

$$\frac{4\pi\sqrt{s}}{M}\left[\tfrac{1}{2}(1+z_s)\right]^{-1/2} F_+(\nu, t) = A^{(-)}(\nu, t) + \left(\nu - \frac{t}{4M}\right) B^{(-)}(\nu, t)\,, \tag{2}$$

where z_s is the cosine of the s-channel c.m. scattering angle and $A^{(-)}, B^{(-)}$ the $\pi N \to \pi N$ invariant amplitudes, which satisfy well-known dispersion relations.[6] The important observation is that at sufficiently high energy ($\nu \gg |t|/4M$) the right-hand side of (2) becomes approximately equal to

$$A^{(-)}(\nu, t) + \nu B^{(-)}(\nu, t)\,,$$

and that this quantity has definite $s \leftrightarrow u$ crossing symmetry; hence a fixed-t dispersion relation for $A^{(-)} + \nu B^{(-)}$ can be written. At high energy ($z_s \simeq 1$) we obtain

$$\frac{4\pi\sqrt{s}}{M} \operatorname{Re} F_+(\nu, t) \simeq \frac{g^2 \nu_B}{2M}\left(\frac{1}{\nu_B - \nu} - \frac{1}{\nu_B + \nu}\right) + \frac{1}{\pi} \mathrm{P}\int_{\nu_0}^{\infty} d\nu' \operatorname{Im}\left[A^{(-)}(\nu', t) + \left(\nu' - \frac{t}{4M}\right) B^{(-)}(\nu', t)\right]\left(\frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu}\right)$$
$$+ \frac{t}{4M}\,\frac{1}{\pi} \mathrm{P}\int_{\nu_0}^{\infty} d\nu' \operatorname{Im} B^{(-)}(\nu', t)\left(\frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu}\right), \tag{3}$$

where $g^2/4\pi = 14.6$ and

$$\nu_B = -\frac{\mu^2}{2M} + \frac{t}{4M}\,, \quad \nu_0 = \mu + \frac{t}{4M}\,;$$

μ is the pion mass. Now, at $t = t_+$ the DA requirement $\operatorname{Im} F_+(\nu, t_+) = 0$ via Eq. (2) completely eliminates the first integral of (3).

In the remaining integral the high-energy part ($\nu > \nu_M$, ν_M to be specified later) is parametrized as follows:

$$\mathrm{Im}F_-(\nu, t_+) = \frac{M}{4\pi\sqrt{s}}\sqrt{-t_+}\;\beta\nu^{\alpha(t_+)-1} \quad (\nu > \nu_M)\,, \tag{4}$$

where β is constant (at $t = t_+$) and $\alpha(t)$ is the "effective" Regge exponent. In view of the DA requirement (1) and the relation

$$\frac{16\pi s}{M}\left(\frac{1-z_s}{2}\right)^{-1/2} F_-(\nu, t) = \frac{4M\nu + 4M^2 - t}{M} A^{(-)}(\nu, t) + (4M\nu + 4\mu^2 - t)B^{(-)}(\nu, t)\,, \tag{5}$$

we obtain at large ν ($1 - z_s \simeq -2t/s$)

$$\mathrm{Im}A^{(-)}(\nu, t_+) = -\nu\,\mathrm{Im}B^{(-)}(\nu, t_+) = 2M\beta\nu^{\alpha(t_+)}\,.$$

Defining

$$D_B(\nu) \equiv \frac{1}{\pi}\int_{\nu_0}^{\nu_M} d\nu'\,\mathrm{Im}B^{(-)}\left(\frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu}\right) \tag{6}$$

and

$$I(\nu, \alpha) \equiv \frac{1}{\pi}\int_{\nu_M}^{\infty} d\nu'\,\nu'^{\alpha}\left(\frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu}\right)\,, \tag{7}$$

we finally obtain

$$\frac{4\pi\sqrt{s}}{M}\,\mathrm{Re}F_+(\nu, t_+) = \frac{g^2\nu_B}{2M}\left(\frac{1}{\nu_B - \nu} - \frac{1}{\nu_B + \nu}\right) + \frac{t_+}{4M} D_B(\nu) - \tfrac{1}{2}t_+\beta\, I(\nu, \alpha(t_+) - 1)\,. \tag{8}$$

By a similar procedure Eq. (5) gives at large ν

$$\frac{4\pi s^{3/2}}{M}\,\mathrm{Re}F_-(\nu, t_+) = \sqrt{-t_+}\left[4\pi\sqrt{s}\;\mathrm{Re}F_+(\nu, t_+) + \nu D_A(\nu) + 2M\beta\nu I(\nu, \alpha(t_+))\right]\,, \tag{9}$$

where $D_A(\nu)$ is given by (6a) with $B^{(-)}$ replaced by $A^{(-)}$.

The integrals $D_A(\nu)$ and $D_B(\nu)$ ($\equiv$ low-energy parts) will be calculated by saturating $\mathrm{Im}A^{(-)}$ and $\mathrm{Im}B^{(-)}$ by the known πN resonances.[7] The contribution of a resonance of spin J, mass M_J, width Γ_J, and elasticity x_J to the partial wave $f_{(J\pm 1/2)\mp}$ is

$$f_{(J\pm1/2)\mp}(s) = -\frac{1}{q}\,\frac{x_J M_J \Gamma_J}{s - M_J^2 + iM_J\Gamma_J}\,.$$

q is the c.m. momentum. Then $\mathrm{Im}A^{(-)}$ and $\mathrm{Im}B^{(-)}$ are calculated using well-known partial-wave expansions.[8-10]

Through the various πN amplitude analyses that have reached us, the value $t = t_+$ of the DA requirement (1) varies between -0.15 and -0.2 GeV^2; we take $t_+ = -0.175$ GeV^2. It is known that the basic features of the helicity-flip amplitude $F_-(\nu, t)$ are reasonably well accounted for by a single ρ trajectory; we thus take $\alpha(t_+) = 0.473 + 0.9t_+ = 0.316$. We fix the constant β of Eq. (4) by requiring that at 6 GeV $\mathrm{Im}F_- = -0.29$, which is (within error bars) in agreement with both HM and Kelly.[4] The resonance parameters x_J, Γ_J of our calculation are given in Table I and are *always* taken to be the average of the values in the Particle Data Group tables.[7] We carry calculations with three different values of ν_M corresponding to $\sqrt{s_M} = 2.0$, 2.5, and 3.25 GeV (Table II); of course, in every case the integrals D_A and D_B contain the contributions of the resonances with $M_J < \sqrt{s_M}$.[11]

The resulting values of $\mathrm{Re}F_\pm$ for pion laboratory momentum of 6 GeV and at $t = t_+$ are given in Table II; to facilitate comparison we also list the corresponding values of the HM and Kelly analyses, after taking account of our sign conventions.[12] In general, our estimates can be considered as satisfactory. They can be brought into better agreement with either HM or Kelly by changing x_j and Γ_j (within the limits of the tables[7]) or by introducing extra resonances reported in certain phase-shift analyses (especially for $\sqrt{s_M} > 2$ GeV); however, we feel this serves no purpose. The following remarks are in order:

(i) For $\sqrt{s_M} \lesssim 2$ GeV the πN resonances are fairly well established and known to provide a good description of the absorptive parts of the $\pi^- p \to \pi^0 n$ amplitudes. Thus our calculation with $\sqrt{s_M} = 2$ GeV is probably the most reliable. It turns out that the case $\sqrt{s_M} = 2$ GeV leads to the over-all best results

TABLE I. Resonance contributions to the absorptive and dispersive parts of $A^{(-)}$ and $B^{(-)}$ at $P_{\mathrm{lab}}=6$ GeV/c.

Resonance	Γ_J (MeV)	x_J	$t=t_+$ $-3\,\mathrm{Im}A^{(-)}/4\pi^2$ (mb$^{1/2}$)	$t=t_+$ $-3\,\mathrm{Im}B^{(-)}/4\pi^2$ (mb$^{1/2}$/GeV)	D_B/ν	D_A	$t=t_-$ $-3\,\mathrm{Im}A^{(-)}/4\pi^2$ (mb$^{1/2}$)	$t=t_-$ $-3\,\mathrm{Im}B^{(-)}/4\pi^2$ (mb$^{1/2}$/GeV)	G	D_A
N_α (938)	···	···	···	−4.64	···	···	···	−4.64	0.5	···
N_α (1688)	142	0.6	2.29	−2.29	−0.3	1.77	−0.643	1.1	0.133	−0.5
N_α (2220)	295	0.15	18.19	−8.63	−0.137	1.68	−2.95	2.94	0.095	−0.274
Δ_δ (1236)	116	0.99	3.24	−14.70	−1.78	2.44	−0.767	−15.936	−0.418	−0.586
Δ_δ (1950)	180	0.45	2.72	−1.7	−0.23	2.184	0.156	−0.455	−0.087	0.125
Δ_δ (2420)	310	0.11	1.797	−0.686	−0.1	1.67	−0.212	−0.043	−0.0168	−0.196
Δ_δ (2850) [a]	400	0.03	0.855	−0.237	−0.05	1.01	−0.23	0.025	0.01996	−0.296
Δ_δ (3230) [a]	440	0.005	0.187	−0.041	−0.019	0.499	−0.08	0.0112	0.025	−0.215
Δ (1650)	165	0.28	0.798	0.3	0.0055	0.085	0.798	0.3	0.0048	0.087
Δ (1670)	237.5	0.15	−7.19	8.43	0.116	−0.586	−2.6	2.15	0.0266	−0.2157
Δ (1890)	242.5	0.17	−11.14	8.798	0.13	−0.979	−2.1	0.46	0.00898	−0.186
Δ (1910)	325	0.25	−2.5	2.57	0.075	−0.43	−2.5	2.577	0.102	−0.436
N_γ (1520)	128	0.5	1.45	−2.13	0.274	1.1	−0.28	0.836	0.0686	−0.218
N_γ (2190)	294	0.25	2.96	−1.58	0.224	2.5	0.49	−0.042	−0.01248	0.424
N_γ (2650) [a]	360	0.075	1.68	−0.54	−0.0967	1.78	−0.0125	0.0936	0.052	−0.007
N_γ (3030) [a]	400	0.0625	0.20	−0.045	−0.0125	0.334	−0.04	0.0168	0.02	−0.066
N (1470)	282	0.6	3.7	−6.96	−0.527	1.666	3.7	−6.96	−0.292	1.685
N' (1535)	105	0.35	−0.77	0.312	−0.005	−0.0717	−0.77	−0.312	−0.003	−0.0724
N (1670)	140	0.4	−8.32	8	0.173	−1.073	−0.848	2.826	0.055	−0.11
N'' (1700)	250	0.65	−0.8	−0.3	−0.0187	−0.297	−0.805	−0.3	−0.018	−0.299
N''' (1780)	250	0.3	2.7	−3.22	−0.089	0.446	2.7	−3.22	−0.098	0.45
N (1860)	255	0.25	−4.6	2.07	0.047	−0.63	−3.737	2.377	0.0686	−0.516

[a] Assigned to the indicated Regge recurrence.

TABLE II. Re$F_\pm$, $d\sigma/dt$, and P calculated with $\alpha(t_-)=0$.

	ReF_+ ($\sqrt{\text{mb}}$)					ReF_- ($\sqrt{\text{mb}}$)				
	Calculation with $\sqrt{s_M}$ =					Calculation with $\sqrt{s_M}$ =				
t	2.0	2.5	3.25	HM	Kelly	2.0	2.5	3.25	HM	Kelly
t_+	0.047	0.047	0.041	~0.07 ±0.12	0.027±0.007	−0.131	−0.133	−0.232	~ −0.292±0.05	−0.10±0.033
t_-	−0.017	−0.016	−0.0346	−0.036±0.019	0.017±0.011	−0.0085	−0.0078	−0.0121	−0.0284±0.007	−0.0226±0.007

	Calculation with $\sqrt{s_M}$ =2.0 GeV					
	6 GeV/c		8 GeV/c		11.2 GeV/c	
t	$\frac{d\sigma}{dt}\left(\frac{\mu\text{b}}{\text{GeV}^2}\right)$	P	$\frac{d\sigma}{dt}\left(\frac{\mu\text{b}}{\text{GeV}^2}\right)$	P	$\frac{d\sigma}{dt}\left(\frac{\mu\text{b}}{\text{GeV}^2}\right)$	P
t_+	207	0.262	145	0.186	95	0.124
t_-	4.3	0.344	2.3	0.286	1.2	0.224

(for $t=t_+$ and $t=t_-$, see below).

(ii) Assuming that $F_-(\nu, t\simeq t_+)$ is dominated by a single ρ Regge trajectory, we have

$$\frac{\mathrm{Re}F_-(\nu, t_+)}{\mathrm{Im}F_-(\nu, t_+)} = \tan[\tfrac{1}{2}\pi\alpha(t_+)] \simeq 0.5\,.$$

With $\mathrm{Im}F_- = -0.29$ our calculation at 6 GeV gives $\mathrm{Re}F_-/\mathrm{Im}F_- = 0.451$, 0.458, and 0.712, corresponding to $\sqrt{s_M} = 2.0$, 2.5, and 3.25 GeV.

Our values of $\mathrm{Re}F_\pm(t_+)$ together with $\mathrm{Im}F_- = -0.29$ can be used to calculate the $\pi^- p \to \pi^0 n$ differential cross section $d\sigma/dt$ and polarization P.[13] Our results are in agreement with experiment[2] (see Table II for results with $\sqrt{s_M} = 2.0$ GeV; for other s_M they are comparable). It is to be noted that our approach leads to sizable polarizations at $t=t_+$ (and at $t=t_-$; see below and Table II).

Finally, we have extended our calculation of $\mathrm{Re}F_\pm$, $d\sigma/dt$, and P to 8 and 11.2 GeV. $\mathrm{Im}F(\nu, t_+)$ is calculated from Eq. (4) with β and $\alpha(t_+)$ the same as at 6 GeV. Our results (Table II) are also satisfactory.

III. ESTIMATE AT $t=t_-$

In view of Eq. (5) the DA requirement $\mathrm{Im}F_-(\nu, t_-) = 0$ implies that at large ν ($\gtrsim \nu_M$)

$$\mathrm{Im}[(\nu+M)A^{(-)}(\nu, t_-) + M\nu B^{(-)}(\nu, t_-)] = 0\,. \tag{10}$$

It is known that the helicity-nonflip amplitude cannot be well accounted for by a single Regge exchange. Nevertheless for our purpose we shall also write

$$\mathrm{Im}F_+(\nu, t_-) = \frac{M}{4\pi\sqrt{s}}\,\gamma\nu^{\alpha(t_-)} \quad (\nu \gtrsim \nu_M)\,,$$

γ = constant (at $t=t_-$), and shall vary the effective exponent $\alpha(t_-)$ over a wide range of values. With this parametrization Eq. (10) gives

$$-\frac{1}{M}\,\mathrm{Im}A^{(-)}(\nu, t_-) = \frac{1}{\nu+M}\,\mathrm{Im}B^{(-)}(\nu, t_-)$$
$$= \gamma\nu^{\alpha(t_-)-1} \quad (\nu \gtrsim \nu_M)\,. \tag{11}$$

To calculate $\mathrm{Re}F_\pm(\nu, t_-)$ we use the expressions (2) and (5) for $\nu \gg |t|/4M$, μ^2/M, together with the dispersion relations for $\mathrm{Re}A^{(-)}(\nu, t_-)$ and $\mathrm{Re}B^{(-)}(\nu, t_-)$.[6] Again the dispersion integrals are split into a low-energy ($\nu_0 < \nu < \nu_M$) and a high-energy ($\nu_M < \nu < \infty$) piece; and in the latter we use (11). The final result is

$$\frac{4\pi\sqrt{s}}{M}\,\mathrm{Re}F_+(\nu, t_-) = D_A(\nu) + \nu G(\nu) + \gamma I(\nu, \alpha(t_-))\,,$$

$$\frac{4\pi s^{3/2}}{M}\,\mathrm{Re}F_-(\nu, t_-) = \sqrt{-t}\,\{(\nu+M)D_A(\nu) + M\nu G(\nu) + M\gamma[I(\nu, \alpha(t_-)) - \nu I(\nu, \alpha(t_-)-1)]\}\,,$$

where $D_A(\nu)$, $I(\nu, \alpha)$ as in Eqs. (6) and (7) and

$$G(\nu) \equiv \frac{g^2}{2M}\left(\frac{1}{\nu_B-\nu} + \frac{1}{\nu_B+\nu}\right) + \frac{1}{\pi}\int_{\nu_0}^{\nu_M} d\nu'\,\mathrm{Im}B^{(-)}(\nu', t_-)\left(\frac{1}{\nu'-\nu} + \frac{1}{\nu'+\nu}\right)\,.$$

Again the integrals in D_A and G are calculated by saturating $\mathrm{Im}A^{(-)}$ and $\mathrm{Im}B^{(-)}$ with the same resonances (Table I).

In accord with HM we take $t_- = -0.5\ \mathrm{GeV}^2$. The constant γ is fixed so that at 6 GeV, $\mathrm{Im}F_+(t_-) = -0.0425$ in accordance with both HM and Kelly. We have varied $\alpha(t_-)$ in the range $-0.2 \leqslant \alpha(t_-) \leqslant 0.2$. For all our s_M, $\mathrm{Re}F_-$ varies by less than 6%. $\mathrm{Re}F_+$ is more sensitive: For $\sqrt{s_M} = 2.0$ the variation is $-0.0071 \geqslant \mathrm{Re}F_+ \geqslant -0.027$, correspondingly; for $\sqrt{s_M} = 2.5$, $-0.008 \geqslant \mathrm{Re}F_+ \geqslant -0.025$; for $\sqrt{s_M} = 3.25$, $-0.024 \geqslant \mathrm{Re}F_+ \geqslant -0.042$.[14] In Table II we present results for $\alpha(t_-) = 0$. Then $\mathrm{Re}F_+$ agrees in sign and magnitude with HM. Our $\mathrm{Re}F_-$ is somewhat too small in absolute value, but still acceptable.

The calculated polarization P at $t=t_-$ (Table II) is at the lower limit of the experimental value $P_{\mathrm{exp}}(6\ \mathrm{GeV}, t_-) = 0.62 \pm 0.21$.[1] At all energies (and for all s_M) our $P(t_-)$ is larger than $P(t_+)$, in accord with experiment. However, our calculated $(d\sigma/dt)(t_-)$ are somewhat smaller than the experimental. Again improvements are possible by changing x_j, Γ_j, and/or the input value of $\mathrm{Im}\,F_+(t_-)$; however, we do not pursue this point.

At 8 GeV with $\sqrt{s_M} = 2.0$ we also obtain $\mathrm{Re}F_+(t_+) = -1.025\ \sqrt{\mu\mathrm{b}}$; this is in agreement with the corresponding amplitude of the Ringland-Roy analysis.[5] Also, we obtain $\mathrm{Re}F_+(t_-) = 0.0098\ \sqrt{\mu\mathrm{b}}$, which is within their error bars. With $\sqrt{s_M} = 2.5$ and 3.25 our results are very similar.

Finally Table II contains our prediction for $P(t_-)$ at 11.2 GeV.

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[10]We work in the narrow-resonance approximation, where

$$\mathrm{Im} f_{(J\pm 1/2)\mp}(s) = (\pi/q) x_J M_J \Gamma_J \delta(s - M_J{}^2);$$

e.g., the works of Ref. 8 indicate this is satisfactory for our purpose. Our procedure can be repeated with $\mathrm{Im}A^-$ and $\mathrm{Im}B^-$ calculated through phase shifts; whether this is a significant improvement over resonance saturation is rather unclear.

[11]Our resonance contributions to $\mathrm{Im}B^{(-)}$ at $t=0$ (not presented in this paper) have been compared and found in approximate agreement with the corresponding results of Ref. 8; also, they are in agreement with R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1967) (a comparison of the signs of the contributions can easily be made by means of our results of Table I; between $t=0$ and $t=-0.175$ the sign remains unaltered).

[12]Our normalization is as follows:

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) = \frac{8\pi}{s}(|F_+|^2 + |F_-|^2),$$

$$P(\pi^- p \to \pi^0 n) = 2\,\mathrm{Im}(F_+ F_-^*)/(|F_+|^2 + |F_-|^2).$$

The sign of our F_+ (F_-) is the same (opposite) to that of F_{++} (F_{+-}) of HM.

[13]An estimate of the polarization by a somewhat similar approach has been reported by M. S. Chen and F. E. Paige, Phys. Rev. D 5, 2760 (1972); there are important differences in our approach and in our results.

[14]For pion laboratory momentum of 6 GeV with $\sqrt{s_M} = 2.0$ GeV and $\alpha(t_-)$ varying in the range $-0.2 \le \alpha(t_-) \le 0.2$, we find $3.9 \le (d\sigma/dt)(t_-) \le 5.2$ μb/GeV^2 and $0.384 \ge P \ge 0.287$. With $\sqrt{s_M} = 2.5$ and 3.25 the variation is smaller.

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Renormalizable Model of Weak and Electromagnetic Interactions with *CP* Violation*

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A renormalizable gauge-field model of weak and electromagnetic interaction of leptons and hadrons is constructed. The model can explain *CP* violation in hadronic weak processes and the suppression of hadronic neutral currents.

I. INTRODUCTION

Recently, considerable attention has been focused on the problem of constructing models[1] of weak and electromagnetic interaction of leptons using the Higgs-Kibble mechanism for spontaneously broken gauge symmetries. The spontaneity of symmetry breaking enables one to have a massive vector boson mediating the weak processes and a massless one mediating electromagnetic processes, while simultaneously preserving the gauge invariance of the Lagrangian. This gauge freedom can be exploited to show that such models are renormalizable.[2] In order to give a unified theory of weak and electromagnetic interactions various attempts have been made to include hadrons[3] in such a scheme, and the most symmetrical way to do this seems to be to enlarge the hadron spectrum from the SU(3) to the SU(4) group.[4] This model is consistent with the present upper limits on the coupling of $\Delta S = 1$ neutral hadronic currents; however, it appears to violate experimental upper limits[5] on the process $\sigma(\nu + p \to \nu + N^{*+})$. The purpose of this note is to