The σ Term in KN Scattering

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The magnitude of the nucleon expectation value of the σ commutator for KN scattering has been calculated using KN phase shifts and fixed-t dispersion relations. The result agrees with most of the recent calculations for πN scattering, but is smaller than the estimate by Cheng and Dashen, and is in favor of the $(3,\bar{3})+(\bar{3},3)$ model for chiral symmetry breaking.

In this note we examine the hypothesis of a $(3,\bar{3})$ breaking[1,2] of chiral SU(3)×SU(3) symmetry via the σ term in KN scattering. Although this theory is elegant and physically plausible, as yet there seems to be no firm experimental evidence in favor of it. Recent calculations[3-6] of the σ commutator (i.e., the equal-time commutator of the axial-vector current with its divergence) in πN scattering approximately agree with the original estimate of von Hippel and Kim[7] and therefore give support to the notion that chiral SU(2)×SU(2) is in fact a better symmetry than SU(3). However, Cheng and Dashen[8] obtained a value for the σ term roughly three times larger, indicating that the $(3,\bar{3})+(\bar{3},3)$ model might not be correct (at least with its conventional interpretation). Thus, the whole question appears to be still open. A reliable evaluation of the σ term takes on further importance, as it may be useful in providing an understanding of the mechanism by which scale invariance is broken.[9]

The basic idea of chiral symmetry is that the strong Hamiltonian density can be meaningfully written as $\mathcal{H}=\mathcal{H}_0+\epsilon\mathcal{H}'$, where $\mathcal{H}_0$ is the SU(3)×SU(3)-invariant part and the symmetry-breaking term $\epsilon\mathcal{H}'$ is in some sense small (we introduced the "small" scale parameter ϵ as a formal device for keeping track of powers of symmetry breaking). It has become useful to look at $\mathcal{H}'$ itself as a sum of two terms: one breaks the chiral symmetry SU(3)×SU(3) and SU(3) as well but conserves SU(2)×SU(2), and the other one breaks SU(2)×SU(2). There are two interesting cases: (i) SU(2)×SU(2) is a much better symmetry than SU(3), which is suggested but not required by the smallness of the pion mass, and (ii) SU(2)×SU(2) and SU(3) breakings are comparable in magnitude.

In the model of Gell-Mann, Oakes, and Renner[1] (GMOR) and Glashow and Weinberg[2] it is suggested that the SU(3) singlet and octet parts of $\epsilon\mathcal{H}'$ belong to the same $(3,\bar{3})+(\bar{3},3)$ representation of SU(3)×SU(3) with the specific form: $\epsilon\mathcal{H}'=u_0+cu_8$; u_0 and u_8 belong to a set of scalars u_a and pseudoscalars v_a $(a=0,\ldots,8)$ which transforms as the $(3,\bar{3})+(\bar{3},3)$ representation of SU(3)×SU(3). Fitting the pseudoscalar meson masses gives $c=-1.25$, indicating the closeness of this theory to the SU(2)×SU(2) symmetric limit ($c=-\sqrt{2}$). Therefore the $(3,\bar{3})+(\bar{3},3)$ model definitely implies case (i). Important tests for this theory of chiral symmetry breaking come from low-energy theorems of meson-baryon scattering. These can be expressed via the σ terms, defined in our case by

$$\sigma_{NN}^{KK}=\langle N|[F_b^5,[F_a^5,\epsilon\mathcal{H}']]|N\rangle\,, \tag{1}$$

where $a,b=4,5$ and F_a^5 denotes the axial-vector charge. Since the σ term is directly proportional to the symmetry-violating part of the total Hamiltonian, the value of this matrix element might tell us something interesting about symmetry breaking. In the GMOR model Eq. (1) can be written as

$$\sigma_{NN}^{KK}=\tfrac{1}{3}(\sqrt{2}-\tfrac{1}{2}c)\langle N|\sqrt{2}\,u_0+\tfrac{1}{2}\sqrt{3}\,u_3-\tfrac{1}{2}u_8|N\rangle\,. \tag{2}$$

The matrix elements of u_i $(i=1,\ldots,8)$ between baryons are fixed by the measured octet mass splittings. This yields

$$\sigma_{NN}^{KK}\approx-40\text{ MeV}+0.96\langle N|u_0|N\rangle\,. \tag{3}$$

The matrix element $\langle N|u_0|N\rangle$ is not known, but the naive guess would be that its magnitude is similar to that of $\langle N|u_8|N\rangle$. The reason for this is that SU(3) mass splittings are always of the same order as the masses of the pseudoscalar octet. This observation suggests that the strengths of the two symmetry-violating terms are comparable. Since u_0 breaks SU(3)×SU(3) and u_8 breaks SU(3) as well as SU(3)×SU(3), we cannot allow $\langle N|u_0|N\rangle$ to be different by as much as an order of magnitude, say, from $\langle N|u_8|N\rangle$ and still have the two symmetries broken by a comparable amount.[10] Therefore in the $(3,\bar{3})+(\bar{3},3)$ model one estimates that

$$|\sigma_{NN}^{KK}|\approx100\text{ to }200\text{ MeV}. \tag{4}$$

This estimate seems to be more reliable than in the case of πN scattering where $\sigma_{NN}^{\pi\pi}$ is proportional to $(\sqrt{2}+c)$ which is very sensitive to slight variations of the negative number c.

Unfortunately, objects like matrix elements of the σ commutator cannot be measured directly, but can be obtained by extrapolation from on-shell scattering amplitudes, provided that effects of second order in $\epsilon\mathcal{H}'$ can be neglected. A brief derivation of this connection goes as follows: Consider the process $K(q)+N(p)\rightarrow K(q')+N(p')$ with four-momenta of the particles indicated in parentheses. Adopting for the crossing-even T matrix the conventional decomposition $T^+=A^++\frac{1}{2}(\not{q}+\not{q}')B^+$ and taking the appropriate nucleon spin average, this transforms into

$$T^+=A^++\nu B^+,$$

where

$$T^+=T^+(\nu,\nu_B,q^2,q'^2),$$
$$\nu=(p+p')\cdot(q+q')/4m_N,$$
$$\nu_B=-q\cdot q'/2m_N$$
$$=(t-2m_K^2)/4m_N.$$

(We follow closely the notation of Ref. 11.) Partial conservation of axial-vector current (PCAC) for kaons[12] leads to the consistency conditions

$$T^+(0,0,m_K^2,0)=T^+(0,0,0,m_K^2)=0, \tag{5}$$

and the low-energy theorem reads

$$T^+(0,0,0,0)=-F_K^{-2}\sigma_{NN}^{KK}, \tag{6}$$

where the kaon decay constant F_K is given by $F_K/F_\pi=1.26$ with $F_\pi=96$ MeV, and σ_{NN}^{KK} is defined as in Eq. (1). Unfortunately, Eq. (6) is not very useful since it involves an off-shell amplitude. Similar to the method of Cheng and Dashen,[8] the amplitudes with either one or two kaons on the mass shell may be expanded in powers of m_K^2 (which is of order ϵ), and one obtains, using Eqs. (5) and (6),

$$T^+(0,0)=F_K^{-2}\sigma_{NN}^{KK}+O(\epsilon^2), \tag{7}$$

where we no longer display the q^2,q'^2 dependence of the on-shell amplitude $T(\nu,\nu_B)$. If we agree to neglect[13] the $O(\epsilon^2)$ term in Eq. (7), the determination of σ_{NN}^{KK} from experiment is now completely unambiguous.

To reach the unphysical (but on-mass-shell) point $\nu=\nu_B=0$ one has to use a fixed-t dispersion relation. Making one subtraction at threshold (ν_0) of the conventional dispersion relation[11] for the crossing even amplitude of KN scattering, and then setting $\nu=\nu_B=0$, gives the expression we wish to evaluate:

$$T^+(0,0)=\mathrm{Re}\,T^+(\nu_0,0)-\nu_0^2\sum_{y=\Lambda,\Sigma}\frac{R_y}{\Delta_y(\Delta_y^2-\nu_0^2)}-\frac{2\nu_0^2}{\pi}P\int_{\nu_0}^{\infty}d\nu'\,\frac{\mathrm{Im}\,T^+(\nu',0)}{\nu'(\nu'^2-\nu_0^2)}-\frac{\nu_0^2}{\pi}P\int_{\nu_\Lambda}^{\nu_0}d\nu'\,\frac{\mathrm{Im}\,T_-(\nu',0)}{\nu'(\nu'^2-\nu_0^2)}. \tag{8}$$

In the last integral, $T_-(\nu,\nu_B)$ denotes the K^-N amplitude and $\nu_0=m_K+m_K^2/2m_N$, $\Delta_y=(m_y^2-m_N^2)/2m_N$, and

$$\nu_\Lambda=[(m_\Lambda+m_\pi)^2-m_N^2]/2m_N,$$
$$R_y=\frac{g_y^2}{4m_N^2}(m_y-m_N)^2,$$

where g_y^2 is the rationalized pseudoscalar coupling constant for KyN ($y=\Lambda,\Sigma$). Using existing results of KN phase-shift analyses, we have made a rather thorough evaluation of the dispersion integrals. One has to combine different phase-shift solutions for the various $K^\pm N$ dispersion integrals. It is very difficult to establish the errors exactly. We have used four different sets of phase-shift solutions for the physical integrals in Eq. (8). The variation in the outputs should give us some idea of the uncertainties in the final result. These four sets are taken as following:

(A) For K^+p scattering below 2.5-GeV/c incident kaon momentum we have used the CERN phase-shift analysis of Albrow *et al.*[14] (solution γ), and for K^-p below 0.6 GeV/c we used the K-matrix analysis of Kim[15] and between 0.6 and 1.2 GeV/c the solution of Armenteros *et al.*[16];

(B) for K^+p the same as in (A), and for K^-p below 1.2 GeV/c we used the multichannel analysis of Kim[17];

(C) for K^+p below 2.5 GeV/c we used the CERN analysis[14] (solution α) and for K^-p the same as in (B);

(D) for K^+p below 0.7 GeV/c we used the CERN analysis[14] (solution α) and from 0.7 to 2.5 GeV/c we used the analysis of Miller *et al.*,[18] and for K^-p the same as in (B).

No partial-wave analyses exist above 2.5 and 1.2 GeV/c for K^+p and K^-p scattering, respectively. Therefore, above those two momenta we have been forced to use a model. It is convenient for us to use the Regge-pole model of Dass *et al.*,[19] extrapolated to lower energies. Because of the subtracted form of our dispersion relation Eq. (8), the high-energy parts of the integrals are strongly suppressed. Therefore, possible errors introduced by extrapolating a Regge-pole model not only to lower energies but also to $t=2m_K^2$ are small; these high-energy tails typically contribute at most 15%. The s-wave unphysical integrals [the last term in Eq. (8)] were evaluated using the K-matrix solution of Martin and Sakitt,[20] continued below the $\bar{K}N$

threshold. The p-wave unphysical regions were assumed to be dominated by the $Y_1^*(1385)$ resonance, and here the narrow width approximation was used. The $\bar{K}Y_1^*N$ coupling[21] was taken to be $g^2/4\pi = 1.2 \pm 0.6$ (see, e.g., Ref. 11). The KN coupling constants in the Born terms of Eq. (8) were taken to be $g_{K\Lambda N}{}^2/4\pi = 5.0 \pm 1.9$ and $g_{K\Sigma N}{}^2/4\pi = 1.0 \pm 1.5$, which are the values of Ref. 20 and cover practically all values presently known.[22] Finally, we determined the threshold subtraction constant in Eq. (8) using s- and p-wave scattering lengths.[11] In cases where errors of the phase-shift analyses were not available, we assigned to each partial wave an arbitrary 30% error (this constitutes approximately an upper limit of the errors in present phase-shift analyses).

With these input data we now can calculate $T^+(0,0)$ and we obtained, for the four sets of phase-shift solutions, the following values [in units $(\mathrm{GeV})^{-1}$]:

$$\begin{aligned} &(\mathrm{A})\ \ 39.6 \pm 11.6,\\ &(\mathrm{B})\ \ 35.2 \pm 11.0,\\ &(\mathrm{C})\ \ 35.0 \pm 10.7,\\ &(\mathrm{D})\ \ 36.9 \pm 11.2. \end{aligned} \tag{9}$$

Together with Eq. (7) we therefore obtain an average value for σ_{NN}^{KK} of

$$\sigma_{NN}^{KK} = (540 \pm 160)\ \mathrm{MeV}. \tag{10}$$

In spite of the difficulties in determining the errors as mentioned above, the quoted error is a reasonable estimate. Although the point $t = 2m_K{}^2$ is relatively far away from the physical region, Eq. (10) shows that the errors in the partial waves are still kept within tolerable limits when extrapolated to the unphysical region. Because of this rather long-range extrapolation one might argue that our results strongly depend on the extrapolation procedure used. This is, however, not the case as one can see from the partial-wave decomposition of the amplitudes: At low energies the main contributions are coming from s- and p-waves; the $s_{1/2}$ and $p_{1/2}$ contributions are independent of the extrapolation procedure, whereas the $p_{3/2}$ term depends only linearly on the c.m. scattering angle. It also should be kept in mind that the subtraction constant $\mathrm{Re}T^+(\nu_0, 0)$ is not uniquely determined since the sign of the real part of the $p_{3/2}$ K^-p scattering length in the isospin $I = 1$ channel is not uniquely determined by experimental data. However, changing the sign of this scattering length only decreases the value of σ_{NN}^{KK} given in Eq. (10). It is clear that more (accurate) data are required to resolve this point.

Finally, let us briefly compare our results with those obtained from πN scattering. In the $(3,\bar{3}) + (\bar{3},3)$ model [Eq. (3)] we obtain for the nucleon expectation value of u_0, using Eq. (10),

$$\langle N|u_0|N\rangle \approx (600 \pm 200)\ \mathrm{MeV}. \tag{11}$$

In this model the σ term for πN scattering is given by

$$\sigma_{NN}^{\pi\pi} = \tfrac{1}{3}(\sqrt{2} + c)\langle N|\sqrt{2}\, u_0 + u_8|N\rangle.$$

Using Eq. (11) we predict that $\sigma_{NN}^{\pi\pi} \approx (55 \pm 15)$ MeV. This is slightly larger than we would expect from a purely theoretical point of view ($|\sigma_{NN}^{\pi\pi}| \sim 10$ to 20 MeV), but agrees with all the estimates obtained recently.[3-6,23] All these values are somewhat larger but close to the original calculation of von Hippel and Kim[7] but disagree with the rather large value obtained by Cheng and Dashen[8] who found $\sigma_{NN}^{\pi\pi} \approx 110$ MeV which requires $\langle N|u_0|N\rangle$ to be approximately 1350 MeV, predicting 1260 MeV for σ_{NN}^{KK}. While this large value would be very welcome in the discussion of broken scale invariance,[9,10] it clearly upsets the general philosophy of the $(3,\bar{3}) + (\bar{3},3)$ symmetry-breaking model. However, most of the calculations done up to now and also the present estimate are in favor of the GMOR model and disagree with the value found by Cheng and Dashen.

Although our result is somewhat larger than the theoretical estimate in Eq. (4), in conclusion we can say that, within the quoted uncertainties, it is in favor of the $(3,\bar{3}) + (\bar{3},3)$ breaking of chiral symmetries. However, it could turn out that further admixtures in the Hamiltonian [presumably contributions which transform according to a $(1,8) + (8,1)$ representation] might be required in order to explain these slightly larger σ terms, if analyses of more accurate future experiments confirm such values.

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[10]There could be a possible enhancement of $\langle N|u_0|N\rangle$ with respect to $\langle N|u_8|N\rangle$, which comes about if one assumes u_0 to be coupled to the Goldstone boson of a further symmetry (scale invariance) which would be broken also by $u_0 + cu_8$ [see, e.g., G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Letters 35B, 415 (1971)]. Although this is an attractive possibility, there is no further hard experimental evidence for such a so-called "dilaton," and for the moment we have no *a priori* reason to assume such a situation.

[11]B. R. Martin, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1970), Vol. 55, p. 73.

[12]Although the PCAC hypothesis for K mesons appears to be uncertain, there is no definite evidence known against it; rather the recent estimates of kaon Yukawa coupling constants [J. K. Kim, Phys. Rev. Letters 19, 1079 (1967); C. H. Chan and F. T. Meiere, *ibid.* 20, 568 (1968)] are compatible with generalized Goldberger-Treiman relations [H. T. Nieh, Phys. Rev. Letters 20, 1254 (1968); R. Dashen and M. Weinstein, Phys. Rev. 188, 2330 (1969)].

[13]*A priori* it is not clear that terms like $m_K{}^4$ can be safely neglected. Work is in progress investigating such higher-order contributions using mass dispersion relations and methods proposed by Altarelli *et al.* (Ref. 10).

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[21]This coupling is defined by

$$\mathcal{L} = (g/m_N)\tfrac{1}{2}(\partial_\mu \bar{\psi}\phi - \bar{\psi}\partial_\mu\phi)\gamma_5\Psi^\mu + \text{H.c.},$$

where ψ is the nucleon field, ϕ the kaon field, and Ψ^μ the Y_1^* (Rarita-Schwinger) field.

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[23]For comparison these values for $\sigma_{NN}^{\pi\pi}$ are: 60 MeV (Ref. 3); 40 MeV (Ref. 4); (80 ± 30) MeV (Ref. 5); 34 MeV (Ref. 6).

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Possible Anomalous Interaction in Muon-Proton Scattering at Low Energies*

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We have investigated the possibility of an anomalous effect in muon-proton scattering due to the exchange of a scalar meson. This model for an anomalous effect differs from most others in that the effect cannot be described simply as a muon form factor depending on momentum transfer, but is strongly energy-dependent. The effect is largest at muon energies of a few hundred MeV and vanishingly small at the high energies of present experiments. It would thus be an appropriate experiment for the high-intensity, low-energy muon beams possible at meson facilities such as the Los Alamos Meson Physics Facility (LAMPF). We have also investigated limits on such an interaction obtained from muon $g-2$ and muonic x-ray measurements. For a sizable range of scalar couplings and masses a 5% effect in scattering appears to be easily possible without conflicting with other information.

I. INTRODUCTION

The question of the difference between muon and electron has been a long-standing puzzle. In spite of a large number of precise experimental tests no real differences have been found other than the mass difference and effects directly traceable to it.[1,2] There have been, however, several recent experiments[3-6] measuring muon-proton scattering, both elastic[3-5] and inelastic,[6] at high energies and large momentum transfers which contain some hints of possible deviations from results predicted on the basis of electron-proton scattering. At present such deviations appear to be most easily explained in terms of normalization uncertainties between the e-p and μ-p experiments, although a possible interpretation of the differences could be the presence of an anomalous muon interaction.

As the new meson facilities become operational, and it thus becomes possible to produce intense low-energy muon beams, a new realm of experiments becomes feasible. That is, one can then do