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Octet Pomeranchukon Component and Absorption in Hypercharge-Exchange Reactions*

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General features of absorption are used to derive simple relations for hypercharge-exchange differential cross sections at small t, with the Pomeranchukon permitted to have an octet as well as a singlet part. The data for these reactions are then used to extract, as a function of s and t, the ratio of the octet absorption term to the sum of the Regge-pole and singlet absorption terms. Four independent determinations yield a unique ratio which is essentially independent of s and exhibits a very slow t variation. It is noted that these features, as well as the sign and magnitude of the ratio, are in excellent agreement with f-dominated Pomeranchukon predictions.

I. INTRODUCTION

It is well known from the high-energy inequality of πp and Kp total cross sections that the Pomeranchukon cannot be a pure unitary singlet. An octet component of the Pomeranchukon is an obvious candidate for the major nonsinglet part and, as we discuss below, several models for the Pomeranchukon exhibit such a component.

The presence of an octet component has consequences for Regge-pole absorption. We use the term "absorption" in the present paper to denote any of several prescriptions for reducing the s-channel lower partial waves of a Regge-pole amplitude. Only the most general features of absorption will be required to derive the consequences of the presence of "octet absorption" in addition to "singlet absorption." Specifically, we assume that at high energies (i) *the Pomeranchukon factorizes and has both singlet and octet components in the t channel,* (ii) *the K^* and K^{**} Regge poles are exchange-degenerate,* and (iii) *the phase of the absorbing amplitude is, for small t, approximately opposite to that of the Regge-pole amplitude.*[1,2] These assumptions are clearly of an approximate nature, and we correspondingly consider general features of differential cross sections in limited kinematic regions, and not more sensitive features such as polarizations and crossing properties.

Omitting spin indices for the moment, we can thus write an amplitude for an inelastic reaction $ab \to cd$ as

$$A(s,t)=A_R(s,t)+A_{RP(1)}(s,t)+A_{RP(8)}(s,t) \quad (1)$$

at high energies. The three terms on the right-hand side represent the Regge-pole term, the singlet absorption term, and the octet absorption term, respectively. Equivalently, Eq. (1) can be written as

$$A(s,t)=A_R(s,t)[C_1(s,t)+(\gamma_a\gamma_b+\gamma_c\gamma_d)C_2(s,t)], \quad (2)$$

where

$$C_1(s,t)\equiv 1+[A_{RP(1)}(s,t)/A_R(s,t)],$$

$$C_2(s,t)\equiv A_{RP(8)}/(\gamma_a\gamma_b+\gamma_c\gamma_d)A_R(s,t),$$

and γ_a is the Clebsch-Gordan coefficient for d-type coupling of particle a to $P(8)$.[3] [$P(8)$ transforms as the isosinglet member of an octet.] From assumption (iii), $C_1(s,t)$ and $C_2(s,t)$ are real functions of s and t (the over-all phase is the pole phase contained in A_R). Our simplifying assumptions limit the range of validity of Eq. (2) to large s and small t; the limits are hard to estimate but $P_{\text{lab}} \gtrsim 8$ GeV/c and $|t| \lesssim 0.2$ (GeV/c)^2 are safe limits and perhaps, as the data suggest, somewhat too restrictive.

From the simple form of Eq. (2) it is easy to write relations for charge-exchange (CEX) and hypercharge-exchange (HCEX) reactions. Since meson-baryon CEX reactions are predominantly spin-flip-dominated and since it has been observed[4] that cut corrections are relatively less important for spin-flip amplitudes, we concentrate here on the HCEX reactions. For sufficiently small $|t|$, $d\sigma/dt$ is dominated by the A' contribution for these reactions. From Eq. (2) we can write the following relations for the prominent HCEX reactions. We choose to present these relations in the limit of SU(3) for the Regge-pole vertices in order to avoid a proliferation of notation, although our main results are independent of this limit. The reader can easily see which reactions are related by isospin and which are not.

$$\begin{aligned}
A'(\pi^+p\to K^+\Sigma^+)&=2(2f-1)(C_1-\tfrac{4}{3}C_2)(-\zeta_-+\zeta_+),\\
A'(K^-p\to \pi^-\Sigma^+)&=2(2f-1)(C_1+\tfrac{5}{3}C_2)(\zeta_-+\zeta_+),\\
A'(\pi^-p\to K^0\Lambda)&=(\tfrac{2}{3})^{1/2}(2f+1)(C_1-\tfrac{4}{3}C_2)(-\zeta_-+\zeta_+),\\
A'(K^-n\to \pi^-\Lambda)&=(\tfrac{2}{3})^{1/2}(2f+1)(C_1-C_2)(\zeta_-+\zeta_+),\\
A'(\pi^-p\to K^0\Sigma^0)&=\sqrt{2}(2f-1)(C_1-\tfrac{4}{3}C_2)(-\zeta_-+\zeta_+),\\
A'(K^-p\to \eta\Lambda)&=\tfrac{1}{3}(2f+1)(C_1+\tfrac{5}{3}C_2)(3\zeta_--\zeta_+),\\
A'(K^-n\to \eta\Sigma^-)&=(\tfrac{2}{3})^{1/2}(2f-1)(C_1-C_2)(3\zeta_--\zeta_+),
\end{aligned} \tag{3}$$

where

$$\zeta_\pm\equiv\beta(t)(1\pm e^{-i\pi\alpha(t)})s^{\alpha(t)}$$

and C_1, C_2 are defined as in Eq. (2).

From these relations, we have the following rules for cross-section ratios:

$$\frac{d\sigma(\pi^+p\to K^+\Sigma^+)}{d\sigma(K^-p\to\pi^-\Sigma^+)}=\left(\frac{C_1-\frac{4}{3}C_2}{C_1+\frac{5}{3}C_2}\right)^2, \tag{4a}$$

$$\frac{d\sigma(\pi^-p\to K^0\Lambda)}{d\sigma(K^-n\to\pi^-\Lambda)}=\left(\frac{C_1-\frac{4}{3}C_2}{C_1-C_2}\right)^2, \tag{4b}$$

$$\frac{d\sigma(K^-n\to\eta\Sigma^-)}{d\sigma(K^-p\to\eta\Lambda)}=6\left(\frac{2f-1}{2f+1}\right)^2\left(\frac{C_1-C_2}{C_1+\frac{5}{3}C_2}\right)^2, \tag{4c}$$

$$\frac{d\sigma(\pi^-p\to K^0\Lambda/\Sigma^0)}{d\sigma(K^-p\to\pi^-\Sigma^+)}=\left(\frac{(2f+1)^2}{6(2f-1)^2}+\frac{1}{2}\right)\left(\frac{C_1-\frac{4}{3}C_2}{C_1+\frac{5}{3}C_2}\right)^2, \tag{4d}$$

where C_1 and C_2 are the real functions of s and t from relations (3). [Note that Eqs. (4a) and (4b) are independent of SU(3) and even of factorization for the Regge-pole residues since they involve only line-reversed reactions; the ratios (4c) and (4d) are both dependent on factorization and SU(3) for the pole residues and both are expressed as functions of f, where $f+d=1$. All of the ratios (4) are independent of the t dependence of the Regge-pole residues.]

II. RESULTS

The ratio C_2/C_1 is calculated from Eqs. (4a)–(4d) from differential cross-section data[5] at small t for the various reactions involved.[6] Small interpolations of the data in s or t were sometimes necessary for one or the other reactions in a cross-section ratio in order to properly determine C_2/C_1 at a unique energy and momentum transfer. The results are presented in two graphs because relations (4a) and (4b) depend only on assumptions (i)–(iii) in Sec. I, while relations (4c) and (4d) depend in addition on factorization and SU(3) for the Regge-pole vertices. The errors shown in the figures are statistical and do not take into account possible systematic errors in the data, which are often quoted as 10–20% in our sources.[5]

In Fig. 1(a), three salient features are to be noted: (1) The three determinations of C_2/C_1 [8 and 16 GeV/c from (4a) and 3.9 GeV/c from (4b)] are in agreement as functions of t; (2) from the 8- and 16-GeV/c determinations using Eq. (4a), C_2/C_1 is essentially independent of energy; (3) C_2/C_1 does not vary rapidly with t. The results of Fig. 1(b) are shown for an SU(3) coupling $f=1.2$ (see Sec. IV) where $f+d=1$. This is close to the quark-model SU(6) value $f=1$ which gives almost the same results; the points would be shifted only slightly for $f=1$. The reader can easily determine, from Eqs. (4c) and (4d), how C_2/C_1 varies with f; increasing f moves the 3.95-GeV/c [Eq. (4c)] points upward and the 8-GeV/c [Eq. (4d)] points downward. The salient features of Fig. 1(b) to note are (1) the agreement with Fig. 1(a), especially for smaller t, with a theoretically and experimentally "desirable" choice of f found to be favored, and (2) the flat t dependence, especially of the 8-GeV/c points [very few data are available for the Eq. (4c) determination, and only two points, one at small t and one at large t, can be obtained]. Finally, we remark again that our assumptions break down at some $|t|$ value [above 0.2 (GeV/c)^2] which cannot be determined very well, and we have presented our results up to $|t|\approx 0.5$ (GeV/c)^2. Also, our assumptions should not hold for low energies where complications such as possible lower trajectories, Regge-Regge cuts, etc. may contribute. It is perhaps significant that the different determinations of C_2/C_1 agree most closely for the higher energies and $|t|\lesssim 0.2$ (GeV/c)^2.

In order to relate our determination of C_2/C_1 to the octet/singlet ratio of the Pomeranchukon, we must first obtain the ratio $A_{RP(8)}(s,t)/A_{RP(1)}(s,t)$.

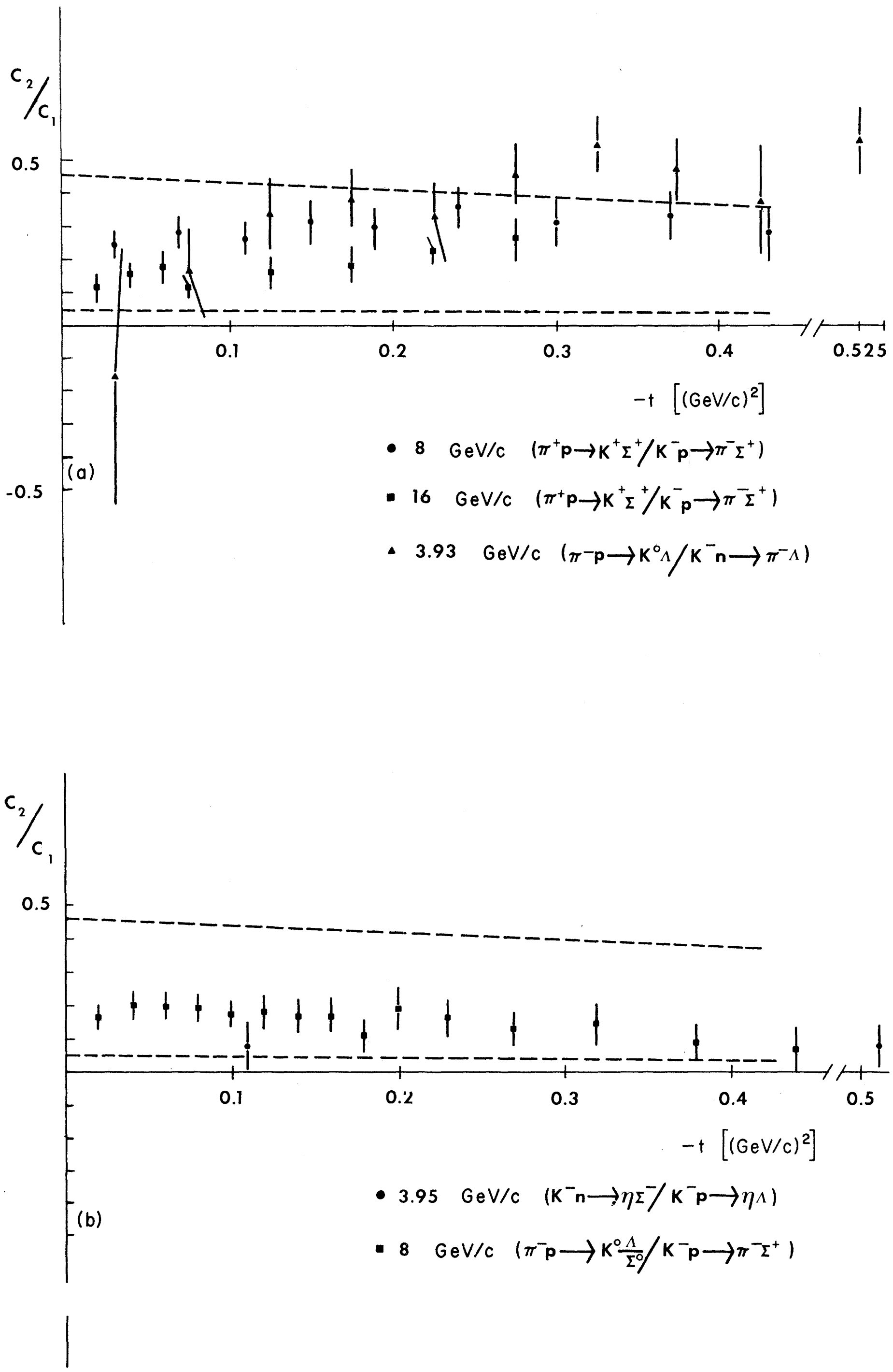

FIG. 1. (a) shows the ratio C_2/C_1 derived from Eqs. (4a) and (4b) using the data sources of Ref. 4. The ratio C_2/C_1 is plotted vs $-t$ in $(\mathrm{GeV}/c)^2$ and at lab momenta shown in the figure. (b) shows, in the same fashion, the ratio C_2/C_1 obtained from Eqs. (4c) and (4d) and an SU(3) coupling of $f=1.2$ where $f+d=1$. This choice of f is explained in the text. The determinations of C_2/C_1 in (a) are independent of factorization and SU(3) for the K^*-K^{**} residues while the determinations shown in (b) depend upon these additional assumptions. Neither determinations depends on the form of the Regge-pole residue as a function of t. The dashed lines shown in both figures are the predictions of the f-dominated Pomeranchukon coupling, from Eq. (9) in the text. The upper dashed line corresponds to 80% Regge-pole absorption for small t in elastic processes ($d_1=0.8$) while the lower dashed line corresponds to 30% ($d_1=0.3$). Setting $d_1\approx 0.6$ in Eq. (9) would give close agreement with the C_2/C_1 values shown. The error bars indicate statistical errors only.

From the definition of C_1, we can write $C_1(s,t) = 1 - d_1(s,t)$ where $d_1 \equiv A_{RP(1)}/A_R$ is the fractional absorption of the Regge-pole A' amplitude by the singlet part of the Pomeranchukon. The ratio we seek is thus

$$\frac{A_{RP(8)}(s,t)}{A_{RP(1)}(s,t)} = \frac{(\gamma_a\gamma_b+\gamma_c\gamma_d)C_2(s,t)}{1-C_1(s,t)} = (\gamma_a\gamma_b+\gamma_c\gamma_d)\frac{C_2(s,t)}{d_1(s,t)} . \qquad (5)$$

The fraction d_1 is nearly constant at small t for absorption which varies smoothly with impact parameter. Estimates range from[7] $d_1 \simeq 0.2$ to[8] $d_1 \simeq 0.8$ with the "weak" and "strong" cut models[9] corresponding to, roughly, $d_1 \simeq 0.3$ and $d_1 \simeq 0.5$, respectively. We shall return below to Eq. (5) in our discussion of a coupling scheme for the Pomeranchukon which gives remarkable agreement with our determination of C_2/C_1.

III. THE f-DOMINATED POMERANCHUKON

It has been shown[10] that the "f-dominated Pomeranchukon" (FDP) coupling scheme[11] emerges via duality from several prominent models, including the multiperipheral model and the dual-resonance model. Carlitz *et al.*[10] show that the scheme accounts well for total cross sections and for "even-signatured" sums of total cross sections at high energies. The FDP couples to the external particles through ordinary Regge trajectories, f and f',[12] with the following factorized form valid near $J=1$, $t=0$, for $ab \to ab$:

$$A_P(J,t) = \sum_{ij} \frac{\beta_{a\bar{a}i}}{J-\alpha_i(t)} B_{ij}(J,t) \frac{\beta_{b\bar{b}j}}{J-\alpha_j(t)} , \qquad (6)$$

with B_{ij} containing the $J \approx 1$ singularity structure and the rapid (approximately exponential) variation in t near $t=0$. The β's are SU(3) coupling coefficients for f, f'. Assuming B_{ij} to be an SU(3) singlet,[10] SU(3)-symmetry breaking (the existence of an octet part) results solely from the splitting of the f and f' trajectories. Projecting out the $\underline{1}$ and $\underline{8}$ parts of Eq. (6), a Mellin transform gives $A_{P(1)}(s,t)$ and $A_{P(8)}(s,t)$. Firstly, since both $A_{P(1)}(J,t)$ and $A_{P(8)}(J,t)$ contain the factor $B_{ij}(J,t)$, both $A_{P(1)}(s,t)$ and $A_{P(8)}(s,t)$ have the same leading energy dependence. Although $A_{P(1)}(s,t)$ and $A_{P(8)}(s,t)$ are both strongly damped in t (from the B_{ij} factor), the ratio $A_{P(8)}/A_{P(1)}$ varies only slowly for $0 \le \alpha_P' \le 1$ GeV^{-2}. This t dependence of $A_{P(8)}/A_{P(1)}$ can be calculated directly from Eq. (6). In fact, if $\alpha_P' = \alpha_f'$ or $\alpha_{f'}'$, then the octet-to-singlet ratio is independent of t. Thus, for large s and small t, $A_{P(8)}(s,t)/A_{P(1)}(s,t) \approx$ constant. This observation, together with the fact that $A_{P(8)}$ and $A_{P(1)}$ are each strongly damped in t (by the same factor) leads in an absorption model to the approximate equality

$$\frac{A_{RP(8)}(s,t)}{A_{RP(1)}(s,t)} = \frac{A_{P(8)}(s,t)}{A_{P(1)}(s,t)} , \qquad (7)$$

where both sides refer to the same elastic process $ab \to ab$; if the left-hand side corresponds to an inelastic process (say, $cd \to ef$), then we must include an additional factor $2\gamma_a\gamma_b(\gamma_c\gamma_d+\gamma_e\gamma_f)^{-1}$ on the left. [In the simplest version of the absorption model, we would have for small t

$$\frac{A_{RP(8)}(s,t)}{A_{RP(1)}(s,t)} = \frac{\iint dt'dt''\Theta(K)K^{-1/2}A_R(s,t')A_{P(8)}(s,t'')}{\iint dt'dt''\Theta(K)K^{-1/2}A_R(s,t')A_{P(1)}(s,t'')} , \qquad (8)$$

which, with $A_{P(8)}(s,t) \propto A_{P(1)}(s,t)$, leads immediately to Eq. (7). The plausibility of Eq. (7), given the (approximate) proportionality of $A_{P(8)}$ and $A_{P(1)}$, leads us to expect that it holds more generally than the oversimplified derivation of Eq. (8). The fact that the data lead to a constant ratio on the left-hand side of Eq. (7) from our analysis of Sec. II supports this expectation.]

Using Eqs. (5) and (7), we can now compare the results of Sec. II with the predictions of the FDP. The relation between $C_2(t)/C_1(t)$ and the SU(3)-breaking parameter $r(t) \equiv [\alpha_P(t) - \alpha_f(t)][\alpha_P(t) - \alpha_{f'}(t)]^{-1}$ defined in Eq. (3.2) of Ref. 7 is given by

$$\frac{C_2}{C_1} = \frac{3d_1[1-r(t)]}{4(1-d_1)[2+r(t)]} . \qquad (9)$$

(We see explicitly in this equation the previously mentioned necessity for specifying the additional parameter d_1, the fractional Regge-pole absorption near $t=0$, in order to relate FDP predictions to our results.) We show in Figs. 1(a) and 1(b) the results of Eq. (9) for the "extreme" cases $d_1 = 0.8$ (upper dashed line) and $d_1 = 0.3$ (lower dashed line). A Pomeranchukon slope of $\alpha_P' = 0.5$ GeV^{-2} was used in $r(t)$. Note the slow t variation as expected. The extremes bracket the values of C_2/C_1 extracted from the data and $d_1 = 0.5$ to 0.7 – corresponding to 50% to 70% Regge-pole absorption in elastic reactions – fits C_2/C_1 very well.

IV. CONCLUDING REMARKS

The agreement of several different determinations of the ratio of the octet absorption to the sum of the Regge-pole and singlet absorption is striking in itself. Turning the argument the other way around, one can say that the presence of an octet component of a predominantly singlet Pomeranchukon leads in an absorption-type model to a consistent understanding of the observed departures from exact exchange degeneracy and SU(3) (or "duality")[13] predictions at high energy in HCEX reaction data (and lack of such discrepancies in the spin-flip-dominated CEX reactions).

We conclude by remarking that an analysis of this type can be applied to *antisymmetric* sums of total cross sections (Ref. 10 considers "symmetric" sums). The presence of octet absorption leads, for vector-tensor exchange, to different d/f ratios than those previously obtained by usual Regge-pole analyses. In particular, the long-standing discrepancy between d/f ratios for A' obtained from Regge-pole fits to total cross sections and the d/f ratios obtained from ratios of HCEX cross sections by such fits[14] may be reduced or eliminated by such an approach. Our rough preliminary findings point to an A' coupling of $f = 1.2 \pm 0.2$ (where $f + d = 1$) as a consistent coupling in both areas. It is interesting that this value is closer to the quark-model SU(6) value $f = 1$ than the previously obtained values of $f \approx 2$ from the pole fits to total cross sections.

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†NDEA title IV predoctoral fellow.

[1]This latter assumption – or observation – was made by V. Barger and R. J. N. Phillips [Phys. Letters 29B, 676 (1969)] in discussing the duality-preserving properties of singlet absorption.

[2]This assumption may be stated another way: If the phases of the absorbing amplitudes differ from that of the pole amplitude, we neglect cross terms in the differential cross sections arising from such phase differences at small t. Assumption (iii) is not at variance with present amplitude analyses. Charge-exchange-amplitude analysis [F. Halzen and C. Michael, Phys. Letters 36B, 367 (1971)] is easily accounted for by lower J-plane singularities than those of *RP* cuts. One of us (P.J.O.) thanks F. Halzen for a discussion of this point. Analysis of hypercharge-exchange amplitudes has been carried out at 4 GeV/c [V. Barger and A. D. Martin, Phys. Letters 39B, 379 (1972)] but is slightly model-dependent. These analyses suggest that at $P_L \lesssim 6$ GeV/c our set of assumptions is less reliable than at higher energies, as expected.

[3]Most simple absorption schemes would yield a C_2 (and C_1) approximately independent of external particles a, b, c, d. In the case of C_2, this would arise from the *RP* (8) amplitude containing a factor $(\gamma_a\gamma_b + \gamma_c\gamma_d)$ which would cancel with the same factor in the denominator. We will in fact assume (iv) independence of C_1 and C_2 on the external particle quantum numbers in writing our relations Eq. (3). As seen in Sec. II, C_2/C_1 seems experimentally to be independent of the external particles for all the cases we consider.

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[6]Since Eqs. (4) are quadratic in C_2/C_1, two roots are obtained. If each quadratic is written so that the product of the cross section ratio [on the left-hand side in Eq. (4)] with $(C_2/C_1)^2$ is positive, then the root with the discriminant taken to be positive is, in all cases, the solution we discuss in this paper. The negative-discriminant root fluctuates over a very wide range.

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[12]Lower trajectories with the same vacuum quantum numbers may also contribute, but they are as yet unknown, and also they are suppressed by the $(\alpha_p - \alpha_i)^{-1}$ factors. We consider only f and f' with ideal mixing as in the applications of Ref. 10.

[13]Octet absorption is not "duality preserving" in the sense of Ref. 1.

[14]We thank A. Ahmadzadeh for a discussion on this point.