PHYSICAL REVIEW D VOLUME 6, NUMBER 7 1 OCTOBER 1972

# Derivation of Neutral-Pion Momentum Spectra from Photon Momentum Spectra*

R. G. Glasser†
*Argonne National Laboratory, Argonne, Illinois 60439*
(Received 27 April 1972)

The spectrum of neutral-pion momentum (projected on an arbitrary axis) can be measured by measuring the corresponding component of the individual photon spectrum. The only assumption required is that the photons are predominantly produced from the pion decay.

## I. INTRODUCTION

In many high-energy experiments there is a moderately high detection efficiency for photons. In these experiments the number of neutral pions which can be observed from the observation of both photons in its normal decay is much less than the number of individual photons. If the predominant source of photons is from neutral-pion decay, then the spectrum of projected pion momentum along arbitrary axes in any convenient frame of reference can be derived from the (derivative of the) corresponding photon spectrum.

## II. LORENTZ TRANSFORMATION

Consider the two-body decay: $A \to B + C$. In the reference frame of particle $A$, particle $B$ has the 4-momentum $(\vec{P}, E)$. The magnitude of $E$ is constant from energy-momentum conservation, so if $A$ is unoriented, the vector $\vec{P}$ has an isotropic distribution (and is of constant magnitude).

In any other frame $S$, in which particle $A$ has the 4-velocity $(\vec{\eta}, \gamma)$, the 4-momentum of $B$ is given by the Lorentz transformation[1]:

$$\vec{P}_S = \vec{P} + \left(\frac{\vec{\eta}\cdot\vec{P}}{\gamma+1} + E\right)\vec{\eta}\ , \tag{1a}$$

$$E_S = \gamma E + \vec{\eta}\cdot\vec{P}\ . \tag{1b}$$

From (1b) follows the well-known result[2] that the energy distribution of $B$ in frame $S$ (for fixed $\vec{\eta}$) is rectangular, since $E_S$ is proportional to the cosine of the angle between $\vec{\eta}$ and $\vec{P}$:

$$\rho(E_S\,|\,\vec{\eta})dE_S = \frac{dE_S}{2|\vec{\eta}||\vec{P}|}\ ,\qquad \gamma E - |\vec{\eta}||\vec{P}| \le E_S \le \gamma E + |\vec{\eta}||\vec{P}|\ . \tag{2}$$

A similar result can be obtained for the projection of the momentum on an arbitrary direction. (See Fig. 1) Let $\hat{n}$ be a unit vector defining the direction. Then, if $q$ denotes the projection of $\vec{P}_S$ along $\hat{n}$:

$$q = \hat{n}\cdot\vec{P}_S = \hat{n}\cdot\vec{P} + \left(\frac{\vec{\eta}\cdot\vec{P}}{\gamma+1} + E\right)\vec{\eta}\cdot\hat{n}$$

which may be written:

$$q = \vec{m}\cdot\vec{P} + \hat{n}\cdot\vec{\eta}E\ , \tag{3}$$

where

$$\vec{m} = \hat{n} + \frac{\vec{\eta}\cdot\hat{n}}{\gamma+1}\vec{\eta}\ . \tag{4}$$

The magnitude of $\vec{m}$ is

$$|\vec{m}| = [1 + (\hat{n}\cdot\vec{\eta})^2]^{1/2}\ . \tag{5}$$

Since for fixed $\vec{\eta}$ the value of $q$ is seen from (3) to depend only on the cosine of the angle between $\vec{P}$ and $\vec{m}$ in the frame of $A$, the distribution of $q$ is rectangular:

$$\rho(q|\vec{\eta})dq = \frac{dq}{2|\vec{m}||\vec{P}|}\ ,\qquad \hat{n}\cdot\vec{\eta}E - |\vec{m}||\vec{P}| \le q \le \hat{n}\cdot\vec{\eta}E + |\vec{m}||\vec{P}|\ . \tag{6}$$

It should be noted that in (6) the distribution of $q$ (which is the component of $\vec{P}_S$ along $\hat{n}$) depends on $\vec{\eta}$ only through its projection on $n$. It should also be noted that the components of $\vec{P}_S$ along different directions are not statistically independent, so that although we could apply this theorem to any three independent directions, we do not get the vector distribution of $\vec{P}_S$ by multiplying the three distributions. (See also the remarks at the end of Sec. III.)

## III. PARENT-DAUGHTER RELATION FOR $\pi^0$ DECAY

The theorem of the previous section was given for an arbitrary two-body decay. The relation to be derived in this section is valid only when one of the decay products is of zero mass and is most interesting for the case of the two-photon decay of the neutral pion, so the algebra will be simplified by considering only this case. The energy and

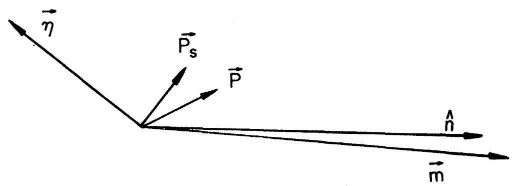


FIG. 1. Shows the relationship between some of the vectors involved in the two-body decay. $\vec{\eta}$ is the spatial part of the 4-velocity of the parent particle in frame $S$. $\vec{P}$ is the momentum of the daughter particle in the rest frame of the parent. $\vec{P}_S$ is the corresponding momentum of the daughter in frame $S$. $\vec{m}$ is a vector in the plane of $\vec{\eta}$ and the arbitrary unit vector $\hat{n}$, such that $\hat{n}\cdot\vec{P}_S$ depends only on $\vec{m}\cdot\vec{P}$.

magnitude of momentum of the photon in the pion rest frame are then given by $E=|\vec{P}|=M/2$ (where $M$ is the neutral pion mass).

As shown by (6), $q$, the projection of the photon momentum on $\hat{n}$, depends on the pion momentum only through $v$, its projection on $\hat{n}$. So we can specialize (6) to the present case:

$$\rho(q|v)=\frac{1}{(M^2+v^2)^{1/2}}\ ,\qquad \frac{v-(M^2+v^2)^{1/2}}{2}\leqslant q\leqslant\frac{v+(M^2+v^2)^{1/2}}{2}\ . \tag{6$'$}$$

Now, if the pion momentum component is distributed according to some distribution $g(v)$, then the photon momentum component is distributed according to

$$f(q)=\int_{q-M^2/4q}^{\infty}\frac{g(v)dv}{(M^2+v^2)^{1/2}}\ ,\quad q>0$$
$$=\int_{-\infty}^{q-M^2/4q}\frac{g(v)dv}{(M^2+v^2)^{1/2}}\ ,\quad q<0\ . \tag{7}$$

Change the variable in (7) to $r=q-M^2/4q$ obtaining

$$f\left(\frac{r+\sqrt{r^2+M^2}}{2}\right)=\int_r^{\infty}\frac{g(v)dv}{(M^2+v^2)^{1/2}}\ ,\qquad f\left(\frac{r-\sqrt{r^2+M^2}}{2}\right)=\int_{-\infty}^{r}\frac{g(v)dv}{(M^2+v^2)^{1/2}} \tag{7$'$}$$

and differentiate (7′) with respect to $r$:

$$g(r)=-\left(\frac{r\pm\sqrt{M^2+r^2}}{2}\right)f'\left(\frac{r\pm\sqrt{M^2+r^2}}{2}\right)\ . \tag{8}$$

This gives the desired relation between the photon spectrum $f(q)$ and the parent pion spectrum $g(v)$. For photon momenta large compared to the pion mass we get the approximate relation:

$$g(r)\approx -rf'(r) \tag{9}$$

which is similar to the result derived by Sternheimer.[3]

There are two equations in each of (6)–(9) since in principle the entire pion spectrum $g(v)$ can be obtained from the values of $f(q)$ for only positive (or negative) values of $q$. If $g(v)$ is symmetric under reflections in the origin, then $f(q)$ is also and the spectrum can be determined separately from the data with $q\geqslant M/2$.

There is also a similar relation for the energy spectra. From (2) the distribution of photon energy, $k$, for fixed pion energy $E_\pi$ is

$$\rho(k|E_\pi)=\frac{1}{(E_\pi{}^2-M^2)^{1/2}}\ ,\qquad \frac{E_\pi-P_\pi}{2}\leqslant k\leqslant\frac{E_\pi+P_\pi}{2} \tag{10}$$

and this leads by the same algebraic derivation as for the momentum components to

$$h(s)=-\frac{s+(s^2-M^2)^{1/2}}{2}\,d'\left(\frac{s+(s^2-M^2)^{1/2}}{2}\right). \tag{11}$$

In (11) $h(s)$ is the pion energy spectrum and $d(k)$ is the photon energy spectrum.

We can directly apply (11) to deduce pion energy spectra from $\gamma$ energy spectra, and (8) to deduce longitudinal-momentum spectra from $\gamma$ longitudinal-momentum spectra. For the transverse momentum there is a slight problem. Equation (8) applies directly to any one component of transverse momentum but there are two (algebraically) independent directions transverse to any primary direction and the distributions are not (statistically) independent. So we cannot get the total transverse-momentum distribution this way. The individual components of transverse momentum are, however, actually observed in many experiments, and (8) can be applied to this data and the result converted to the total momentum transverse to the primary direction.

Throughout this section the spectral functions have been considered as probability distributions of their variable. One can also interpret them as cross sections. The only change necessary is to insert on the right-hand side of Eqs. (6), (7′), and (10) a factor of 2 and on the right-hand side of Eqs. (8), (9), and (11) a factor of $\frac{1}{2}$ to account for the two photons produced by each neutral pion.

## IV. ACKNOWLEDGMENTS

I would like to thank Gordon Charlton, whose persistent questions about how the photon detection efficiency of the Argonne 12-foot bubble chamber could be used effectively led to these results.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†On leave from University of Maryland, College Park, Maryland.

[1]R. Hagedorn, *Relativistic Kinematics* (Benjamin, New York, 1964), p. 9.

[2]Bruno Rossi, *High Energy Particles* (Prentice Hall, Englewood Cliffs, N. J., 1952) p. 199.

[3]R. M. Sternheimer, Phys. Rev. 99, 277 (1955).

PHYSICAL REVIEW D VOLUME 6, NUMBER 7 1 OCTOBER 1972

# Octet Pomeranchukon Component and Absorption in Hypercharge-Exchange Reactions*

P. J. O'Donovan and T. E. Prickett†
*Physics Department, Arizona State University, Tempe, Arizona 85281*
(Received 7 June 1972)

General features of absorption are used to derive simple relations for hypercharge-exchange differential cross sections at small $t$, with the Pomeranchukon permitted to have an octet as well as a singlet part. The data for these reactions are then used to extract, as a function of $s$ and $t$, the ratio of the octet absorption term to the sum of the Regge-pole and singlet absorption terms. Four independent determinations yield a unique ratio which is essentially independent of $s$ and exhibits a very slow $t$ variation. It is noted that these features, as well as the sign and magnitude of the ratio, are in excellent agreement with $f$-dominated Pomeranchukon predictions.

## I. INTRODUCTION

It is well known from the high-energy inequality of $\pi p$ and $Kp$ total cross sections that the Pomeranchukon cannot be a pure unitary singlet. An octet component of the Pomeranchukon is an obvious candidate for the major nonsinglet part and, as we discuss below, several models for the Pomeranchukon exhibit such a component.

The presence of an octet component has consequences for Regge-pole absorption. We use the term "absorption" in the present paper to denote any of several prescriptions for reducing the $s$-channel lower partial waves of a Regge-pole amplitude. Only the most general features of absorption will be required to derive the consequences of the presence of "octet absorption" in addition to "singlet absorption." Specifically, we assume that at high energies (i) *the Pomeranchukon factorizes and has both singlet and octet components in the t channel,* (ii) *the* $K^*$ *and* $K^{**}$ *Regge poles are exchange-degenerate,* and (iii) *the phase of the absorbing amplitude is, for small t, approximately opposite to that of the Regge-pole amplitude.*[1,2] These assumptions are clearly of an approximate nature, and we correspondingly consider general features of differential cross sections in limited kinematic regions, and not more sensitive features such as polarizations and crossing properties.

Omitting spin indices for the moment, we can thus write an amplitude for an inelastic reaction $ab \to cd$ as

$$A(s,t)=A_R(s,t)+A_{RP(1)}(s,t)+A_{RP(8)}(s,t) \qquad (1)$$

at high energies. The three terms on the right-hand side represent the Regge-pole term, the singlet absorption term, and the octet absorption term, respectively. Equivalently, Eq. (1) can be written as

$$A(s,t)=A_R(s,t)[C_1(s,t)+(\gamma_a\gamma_b+\gamma_c\gamma_d)C_2(s,t)], \qquad (2)$$

where

$$C_1(s,t)\equiv 1+[A_{RP(1)}(s,t)/A_R(s,t)],$$

$$C_2(s,t)\equiv A_{RP(8)}/(\gamma_a\gamma_b+\gamma_c\gamma_d)A_R(s,t),$$

and $\gamma_a$ is the Clebsch-Gordan coefficient for $d$-type coupling of particle $a$ to $P(8)$.[3] [$P(8)$ transforms as the isosinglet member of an octet.] From assumption (iii), $C_1(s,t)$ and $C_2(s,t)$ are real functions of $s$ and $t$ (the over-all phase is the pole phase contained in $A_R$). Our simplifying assumptions limit the range of validity of Eq. (2) to large $s$ and small $t$; the limits are hard to estimate but $P_{\rm lab}\gtrsim 8$ GeV/$c$ and $|t|\lesssim 0.2$ (GeV/$c$)$^2$ are safe limits and perhaps, as the data suggest,