volved. The width calculations are all first-order $\tilde{S}\tilde{\phi}\tilde{\phi}$ vertex estimations and no attempt is made to include contributions from higher-order graphs.

We note that many of the widths in the Table III are smaller than those given in Ref. 1. To see how this comes about, we examine, for example, $g_{\sigma\pi\pi}$:

$$g_{\sigma\pi\pi}=\frac{\delta'}{\alpha}(m_\sigma{}^2-m_\pi{}^2)$$

$$=\frac{2\delta Z_\pi}{F_\pi}(m_\sigma{}^2-m_\pi{}^2).$$

Then the reduction of the $\sigma\to\pi\pi$ width in this model is due to the factor $\delta' Z_\pi$. In the models of Ref. 3, however, the width reduction is greater, due to a factor $\delta' Z_\pi{}^3$ in $g_{\sigma\pi\pi}$, where $Z_\pi{}^2\simeq\frac{1}{2}$ (see the first paragraph in Sec. IV). Although some of the widths quoted in Table III are close to some of the experimental evaluations in the literature[5] they are still slightly too broad. The models of the type described in Ref. 3, while giving much smaller widths as noted above, do not produce as reasonable a mass spectrum.

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[4]We calculate the width for the decay $\rho^+(k)\to\pi^+(p)+\pi^0(q)$ from the model as

$$\Gamma_\rho=\sum_{s_i}\frac{|M|^2\,|\vec{k}|}{8\pi m_\rho{}^2},$$

where $|\vec{k}|^2=\frac{1}{4}(m_\rho{}^2-4m_\pi{}^2)$,

$$\sum_{s_i}|M|^2=\left(\frac{g}{\sqrt{2}}+\frac{Cg\alpha^2}{\sqrt{2}m_{A_1}{}^2Z_\pi{}^2}-\frac{m_\rho{}^2}{2}\,\frac{g^3\alpha^2}{\sqrt{2}m_{A_1}{}^4Z_\pi{}^2}\right)^2\times(m_\rho{}^2-4m_\pi{}^2),$$

and $C\alpha^2=m_\rho{}^2-m_{A_1}{}^2Z_\pi{}^2$ from (9). We note that letting $C=0$ gives us $Z_\pi{}^2\approx\frac{1}{2}$ and $g^2/4\pi=4.9$ which corresponds to $\Gamma_\rho\approx 218$ MeV. In any case, we determine $B\alpha^2=m_\rho{}^2+m_{A_1}{}^2Z_\pi{}^2$ from (9).

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Test for Scalar-Boson-Mediated Weak-Interaction Theories in High-Energy Neutrino-Scattering Processes

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Pais and Treiman have shown that in neutrino-scattering processes lepton-pair locality implies explicit dependence of the cross section on two-lepton-system variables. We suggest here that violation of their theorem occurring to the first order in the weak-interaction coupling constant in the scalar-boson-mediated theories may serve as an unambiguous test at energies much smaller than those required for production and subsequent tests on the weak bosons. In a typical model we find substantial violations of the lepton-energy-dependence theorem. On the contrary, violation of the angle-dependence theorem is small.

I. INTRODUCTION

The structure of the weak-interaction Lagrangian has recently become a topic of renewed interest with the expectation of new possibilities implied by the proposed neutrino-scattering experiments at high energies. The familiar current-current effective Lagrangian has well-known troubles at high energies. A first step towards a complete Lagrangian would be to look for deviations from the implications of the local current-current Lagrangian. Important results in this regard are the lepton-pair-locality theorems shown to constitute the full content of lepton-pair locality in neutrino processes by Pais and Treiman.[1] Their results, explicit expressions for lepton energy and angle dependence for cross sections, are expected to hold to the lowest order in the weak-interaction coupling constant $G(\sim 10^{-5} m_p^{-2})$ in a local current-current theory and also in vector-boson-mediated models of weak interactions. Higher-order weak interactions would be expected to be comparatively large in these theories at energies around 300 GeV. But at energies of 30–40 GeV, higher-order processes will not be very important according to our present ideas about weak interactions. Consequently, violations of lepton-pair locality are expected to be small at these energies.

On the other hand, in models of weak interactions in which scalar bosons mediate, the lowest-order term in G is the result of a fourth-order interaction involving the exchange of two scalar bosons.[2] In these models, therefore, at energies comparable to the masses of the scalar bosons, substantial violations of the lepton-pair-locality theorem are expected. Tests for violations of lepton-pair-locality theorems may therefore serve as tests for the scalar-boson-mediated weak-interaction theories at energies much smaller than would be required for production and subsequent tests on the intermediate boson.

We have estimated such effects in a specific model – a simplified version of the one proposed by Gupta and Patil.[3] This particular model has been chosen because it is possibly the best among the currently proposed models, being free of exotic hadrons and a host of other new particles which the earlier theories needed to reproduce the results of the $V-A$ theory at small energies.

II. THE LEPTON-PAIR-LOCALITY THEOREMS

Consider the process

$$\bar{\nu}_l(q_1) + p(p_1) \to l^+(q_2) + \alpha(p),$$

where $\alpha(p)$ is a hadronic system with total momentum p. We work throughout in the laboratory frame ($\vec{p}_1 = 0$) and take the mass of the lepton equal to 0.

We define $L = q_2 - q_1$ and $N = q_2 + q_1$. Let us choose the z direction along $\vec{L}$, the y axis along $\vec{L}\times\vec{k}$, where k is some vector characteristic of the hadronic complex, and the x axis along $(\vec{L}\times\vec{k})\times\vec{L}$. Further, let θ be the angle between $\vec{k}$ and the z axis, and ϕ the angle between the x axis and the projection of $\vec{N}$ on the xy plane. If there are n particles in the system α, we will need in all $3n-1$ variables to describe the scattering process. Four of them are chosen to be

(1) E, the laboratory energy of the incident neutrino,

(2) the angle ϕ as defined above,

(3) $t = (q_2 - q_1)^2$, the square of the momentum transfer from the leptons, and

(4) $s = p^2$, the squared invariant mass of the final-state hadronic complex.

The theorems proved by Pais and Treiman are, in our notation, as follows:

The energy theorem. The dependence of the cross section integrated over the rest of the phase space including the ϕ domain is given as a function of the neutrino energy by

$$\frac{d^2\sigma}{ds\,dt} = \frac{G^2}{32\pi m_n{}^2 E^2}(AE^2 + BE + C)\,, \tag{1}$$

where A, B, and C are functions of s and t only.[4]

The angle theorem. The cross section integrated over all the final-state variables except ϕ is given by

$$\frac{d\sigma}{d\phi} = a\cos 2\phi + b\sin 2\phi + c\cos\phi + d\sin\phi + e\,. \tag{2}$$

The two theorems follow from the fact that the hadronic part of the matrix element for the process is independent of the vector N and hence of the two lepton-system variables, N itself having two constraints: $N^2 = -t$ and $N\cdot L = 0$.

III. THE MODEL

The model proposed by Gupta and Patil has the following Lagrangian:

$$\mathcal{L}_W = g_+ W^+[(S_2^1 - P_2^1)\cos\theta_C + (S_3^1 - P_3^1)\sin\theta_C] + gW^- \sum_{l=e,\mu} L_l^-(1-\gamma_5)\nu_l$$
$$+ g'\left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} + \frac{X^0}{\sqrt{3}}\right)\sum_{l=e,\mu} L_l^-(1-\gamma_5)l^- + \text{H.c.} + g_\pi \sum_{\alpha,\beta=1}^{3} \pi_\beta^\alpha P_\alpha^\beta\ .$$

Here $W^\pm$ are heavy scalar intermediate bosons, L_e^- and L_μ^- are charged heavy leptons, S_β^α and P_β^α ($\alpha, \beta = 1, 2, 3$) are scalar and pseudoscalar hadronic currents, and θ_C is the Cabibbo angle. The P_β^α are also the sources of the nine pseudoscalar mesons π_β^α containing the octet π^0, η^0, etc. The third term couples the hadrons π^0, η^0, and X^0, which act as the "neutral intermediate bosons," directly to the leptons.

The fourth term is the SU(3)-symmetric strong interaction of the pseudoscalar mesons with the hadrons. Further, $g_\pi = \sqrt{2}\, g_{NN\pi}$ is the pion-nucleon coupling constant.

$$g' = g, \quad g_+ g_\pi = g^2 ,$$

and

$$\left(\frac{g^2}{4\pi}\right)^2 \frac{1}{2M^2} = \frac{G}{\sqrt{2}} .$$

The charged heavy leptons and the heavy scalar intermediate bosons are assumed, for the sake of simplicity, to have the same mass M.

In calculating violation of the Pais and Treiman theorem, we work with a simplified version of the model in which the hadronic scalar and pseudoscalar currents S_2^1, P_2^1, and P_1^1 are constructed out of the nucleon fields only.

IV. VIOLATION OF THE ENERGY-DEPENDENCE THEOREM

We consider here the elastic process

$$\bar{\nu}_\mu(q_1) + p(p_1) \rightarrow \mu^+(q_2) + n(p_2).$$

This process in our simplified version of the Gupta-Patil model occurs through the box diagram shown in Fig. 1.

The amplitude is

$$T = \frac{2g^4 \cos\theta_C}{(2\pi)^4} \bar{v}(q_1)\gamma^\lambda(1-\gamma_5)v(q_2) \int \frac{d^4q\, q_\lambda \bar{u}_n(p_2)[\gamma\cdot(q+p_1-L) - m_n](1-\gamma_5)u_n(p_2)}{[q^2-\mu^2][(q-q_2)^2 - M^2][(q-L)^2 - M^2][(q+p_1-L)^2 - m_n{}^2]} , \qquad (3)$$

where μ is the mass of the pseudoscalar meson and m_n is the mass of the nucleon. First the q integration is performed. We are interested in extracting terms involving N. Expanding the result in powers of $1/M^2$, we find that N occurs as the coefficient of the $1/M^4$ term. The contribution of terms involving higher powers of N will be very small, since they involve higher powers of $1/M^2$. Retaining only terms of interest to order $1/M^4$, we obtain

$$T = \frac{G\cos\theta_C}{\sqrt{2}} l^\lambda J_\mu \left[g_{\mu\lambda}\left(1 + \frac{N\cdot p_1}{6M^2}\right) + \frac{N_\mu p_{1\lambda}}{6M^2}\right] , \qquad (4)$$

where

$$l_\lambda = \bar{v}(q_1)\gamma_\lambda(1-\gamma_5)v(q_2)$$

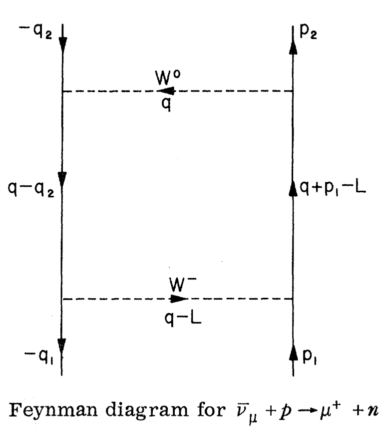

FIG. 1. Feynman diagram for $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$.

and

$$J_\lambda = \bar{u}_n(p_2)\gamma_\lambda(1-\gamma_5)u_n(p_1) .$$

The cross section given by this matrix element is, again retaining only terms of the lowest order in $N\cdot p_1/M^2$,

$$\left(\frac{d\sigma}{dt}\right)_{\text{st}} = \left(1 + \frac{2N\cdot p_1}{3M^2}\right)\left(\frac{d\sigma}{dt}\right)_{\text{cc}} ,$$

where $(d\sigma/dt)_{\text{st}}$ is the differential cross section calculated in the scalar theory and $(d\sigma/dt)_{\text{cc}}$ the differential cross section in the current-current theory.

The percentage violation of the theorem, x, defined by

$$x = \left\{ \left[\left(\frac{d\sigma}{dt}\right)_{\text{st}} - \left(\frac{d\sigma}{dt}\right)_{\text{cc}}\right] \Big/ \left(\frac{d\sigma}{dt}\right)_{\text{cc}} \right\} \times 100$$

is shown for various energies and different masses of the scalar boson in Table I. We notice that substantial violations of the locality theorem occur at energies much smaller than would be required for the production of the weak bosons. We would like to point out again that such large violations occur only in theories in which the lowest-order weak process occurs through a fourth-order interaction. Further, radiative corrections which would also induce similar effects are expected to be of the order of 1% or less.

Inelastic processes involving additional hadrons in the final state do not, in principle, involve sub-

TABLE I. Violation of the theorem of Pais and Treiman in scalar-boson theories of weak interactions. t = constant << M^2.

Mass of the scalar boson (GeV)	Laboratory energy of the neutrino (GeV)	x (%)
10	10	12.5
	20	25
	30	37.5
	50	62.5
15	15	8.3
	30	16.6
	45	25
	60	33.2
20	20	6.3
	50	15.7
	75	23.4
	100	31.5

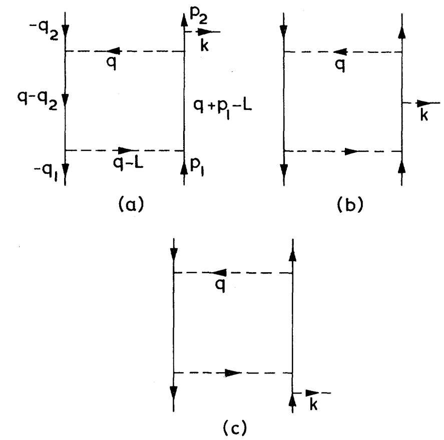

FIG. 2. Diagrams contributing to the reaction $\bar{\nu}_\mu + p \to n + \mu^+ + \eta^0$.

stantial modifications. The actual numbers involved, of course, will be different in specific reactions.

V. VIOLATION OF THE ANGLE THEOREM

To evaluate violations of theorem (2), we considered a simple illustrative example:

$$\bar{\nu}_\mu + p \to n + \mu^+ + \eta^0.$$

Diagrams contributing to this process are shown in Fig. 2. Diagram 2(a) does not contribute to the violation of the angle theorem, since in this case the only hadronic momentum occurring inside the q-integration box is p_1. Additional terms from this diagram therefore contain only $N \cdot p_1$, which is independent of the angle ϕ. In the case of diagram 2(b), we find that terms involving N occur only as coefficients of $1/M^6$ in the amplitude. The contribution of these terms to the violation of the angle theorem is therefore negligible. The diagram 2(c) does contribute to the violation of the angle theorem through terms like $N \cdot k$. But in this case, at the energies and masses we have considered (cf. Table I), we find violation of the theorem to be of the order of 5% or less. Therefore, measurable violations of the angle theorem do not seem to occur in scalar-boson-mediated theories.

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