PHYSICAL REVIEW D VOLUME 6, NUMBER 7 1 OCTOBER 1972

# Breaking Chiral and Scale Symmetry of a Lagrangian with Massive Gauge Fields*

W. Hudnall

*Physics Department, Syracuse University, Syracuse, New York 13210*

(Received 29 February 1972; revised manuscript received 25 April 1972)

We investigate the effects of adding the $1^{\pm}$ gauge fields to an SU(3) $\sigma$ model of the $0^{\pm}$ fields. Particular attention is given to the requirements of broken scale invariance on the simple model where the "(3, 3*)"-chiral- and scale-symmetry-breaking terms coincide. We also calculate the mass spectrum and the scalar-meson trilinear decay widths.

## I. INTRODUCTION

Phenomenological Lagrangians have been useful in studying SW(3) symmetry and its breaking in the low-energy region. One particular model of 18 $0^{\pm}$ mesons interacting via the most general SW(3)-invariant interaction and the simplest "(3, 3*)"-symmetry-breaking term has given a surprisingly accurate description of the low-mass region.[1]

We extend that Lagrangian by adding $1^{\pm}$ gauge mesons to it. Of course, this extension allows the introduction of various combinations of symmetry-breaking (SB) terms which can result in an extremely complicated Lagrangian.[2] We have added the gauge fields in a simple chiral- *and* scale-invariant manner in order to test a model where the "(3, 3*)"-chiral- and scale-symmetry-breaking terms coincide. We calculate the mass spectrum and scalar-meson trilinear decay widths to check that the model is in reasonable agreement with experiment. We point out that the (broken) scale symmetry of our model yields a very interesting relation between the two isoscalar scalar-meson masses. This relation gives an upper limit for the mass of one isoscalar scalar particle which may actually be violated by the most favored experimental candidate.

## II. THE MODEL

The model of Ref. 1 is described by

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}\,(\partial_\mu M \partial_\mu M^\dagger) - V_0 - V_{\mathrm{SB}}\ , \tag{1}$$

where

$$M_a^b = S_a^b + i\phi_a^b\ , \qquad V_{\mathrm{SB}} = -2(A_1 S_1^1 + A_2 S_2^2 + A_3 S_3^3), \tag{2}$$

and $V_0$ is the most general, nonderivative chiral SW(3)-invariant interaction of the nine scalar ($S$) and nine pseudoscalar ($\phi$) fields. As in Ref. 1, we will define the "equilibrium point" (or "ground state") by

$$\langle M_a^b\rangle_0 = \langle S_a^b\rangle_0 = \alpha_a \delta_a^b\,. \tag{3}$$

We ignore electromagnetic effects, giving the isotopic-spin symmetry limit, where $A_1 = A_2$ and $\alpha_1 = \alpha_2 \equiv \alpha$. For convenience we define a quantity $W = \alpha_3/\alpha$.

Modification of (1) to include the vector and axial-vector gauge fields consists of the usual replacement of the derivative $\partial_\mu$ by the gauge derivative $D_\mu$, the addition of the spin-1 Yang-Mills term and the addition of a term responsible for the gauge field masses. Our Lagrangian then becomes

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}\,(D_\mu M D_\mu M^\dagger) - V_0 - V_{\mathrm{SB}} - \tfrac{1}{2}\,\mathrm{Tr}\,(F^l_{\mu\nu}F^l_{\mu\nu} + F^r_{\mu\nu}F^r_{\mu\nu}) + \mathcal{L}_1 + \mathcal{L}_2\,, \tag{4}$$

where

$$\begin{aligned}
\mathcal{L}_1 &= -\tfrac{1}{2}B\,\mathrm{Tr}\,(l_\mu l_\mu M M^\dagger + r_\mu r_\mu M^\dagger M),\\
\mathcal{L}_2 &= -C\,\mathrm{Tr}\,(l_\mu M r_\mu M^\dagger),\\
V_\mu &= l_\mu + r_\mu\,,\\
A_\mu &= l_\mu - r_\mu\,,\\
F^l_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - ig[l_\mu, l_\nu],\\
F^r_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - ig[r_\mu, r_\nu],\\
D_\mu M &= \partial_\mu M - ig l_\mu M + ig M r_\mu\ .
\end{aligned} \tag{5}$$

Here, $V_\mu$ and $A_\mu$ are the vector and axial-vector fields, respectively, and $g$, $B$, and $C$ are constants which we will determine later.

As in Ref. 1, we expand $V_0$ in a Taylor series in the "normal coordinates,"

$$S = \hat{S} + \langle S\rangle_0\,, \qquad \phi = \hat{\phi}, \tag{6}$$

and, after proper renormalization, the appropriate coefficients of terms of second order in the fields are identified with the $0^{\pm}$ masses. Coefficients of third order (and above) are identified as contributions to the trilinear (and above) couplings.

Note that the terms $\mathcal{L}_1$ and $\mathcal{L}_2$ are both chiral SW(3) and scale-invariant so that only $V_{\mathrm{SB}}$ breaks these two symmetries if $V_0$ is chosen to be scale invariant. $\mathfrak{L}_1$ and $\mathfrak{L}_2$ yield expressions for the gauge field masses when we substitute the normal coordinates (6). The gauge field masses are conventionally included in the Lagrangian via (a scale-breaking) term of the form[3]

$$m_0^2 \,\mathrm{Tr}(V_\mu V_\mu + A_\mu A_\mu). \tag{7}$$

Our $\mathfrak{L}_1$ and $\mathfrak{L}_2$ terms actually yield a more accurate spin-1 meson mass spectrum than does (7).

When we make the substitutions (5) and (6) in (4), we find that the gauge fields must be redefined to eliminate terms of the form $V_\mu \partial_\mu \hat{S}$ and $A_\mu \partial_\mu \hat{\phi}$ and that the scalar and pseudoscalar fields $S$ and $\phi$ must be "renormalized" to give the "physical" fields (denoted by the tilde),

$$\begin{aligned} V^b_{\mu a} &= \tilde{V}^b_{\mu a} + \frac{ig(\alpha_a - \alpha_b)}{2m_{ab}^2 X_{ab}} \partial_\mu \tilde{S}^b_a, \\ A^b_{\mu a} &= \tilde{A}^b_{\mu a} + \frac{g(\alpha_a + \alpha_b)}{2M_{ab}^2 Z_{ab}} \partial_\mu \tilde{\phi}^b_a, \\ \hat{\phi}^b_a &= \frac{1}{Z_{ab}} \tilde{\phi}^b_a, \quad \hat{S}^b_a = \frac{1}{X_{ab}} \tilde{S}^b_a, \end{aligned} \tag{8}$$

where

$$\begin{aligned} Z_{ab}^2 &= 1 - \frac{g^2(\alpha_a + \alpha_b)^2}{4M_{ab}^2}, \\ X_{ab}^2 &= 1 - \frac{g^2(\alpha_a - \alpha_b)^2}{4m_{ab}^2}, \\ m_{ab}^2 &= \frac{g^2 + B}{4}(\alpha_a^2 + \alpha_b^2) - \frac{g^2 - C}{4}(2\alpha_a\alpha_b), \\ M_{ab}^2 &= \frac{g^2 + B}{4}(\alpha_a^2 + \alpha_b^2) + \frac{g^2 - C}{4}(2\alpha_a\alpha_b). \end{aligned} \tag{9}$$

Here $m_{ab}$ and $M_{ab}$, after the usual isoscalar vector mixing formalism is applied to both spin-1 gauge fields, give the vector and axial-vector particle masses, respectively.

We also determine the pion and kaon weak decay constants from the Noether currents as

$$\begin{aligned} F_\pi &= 2\alpha Z_\pi, \\ F_K &= \alpha(1 + W) Z_K. \end{aligned} \tag{10}$$

Then if we determine $g$ [see Eqs. (5) and (8)] from the $\rho$ decay width,[4] we may consider the two possibilities:

$$g^2/4\pi = 3.6, \tag{11a}$$

which corresponds to $\Gamma_\rho \approx 100$ MeV, and

$$g^2/4\pi = 4.5, \tag{11b}$$

which corresponds to $\Gamma_\rho \approx 150$ MeV. We may now determine $Z_\pi$ and $\alpha$ from the two equations

$$F_\pi = 2\alpha Z_\pi = 1.01 m_\pi$$

and (12)

$$Z_\pi^2 = 1 - \frac{g^2\alpha^2}{m_{A_1}^2}$$

to get

$$Z_\pi^2 = \frac{1}{2} \pm \frac{1}{2m_{A_1}} (m_{A_1}^2 - g^2 F_\pi^2)^{1/2}. \tag{13}$$

Of the two solutions to (13) for each value of $g$ chosen, we found that the solution with $Z_\pi^2 > \frac{1}{2}$ gives a mass spectrum more in line with experiment.

The chiral constraints on the scalar and pseudoscalar masses as calculated in Ref. 1 are now altered by the renormalization of the spin-0 mesons. This does not present any serious complication, however, since a renormalization factor is all that is needed in, for example, the pion and kaon mass expressions:

$$\begin{aligned} m_\pi^2 &= \frac{1}{Z_\pi^2}\, 2\frac{A_1 + A_2}{\alpha_1 + \alpha_2}, \\ m_K^2 &= \frac{1}{Z_K^2}\, 2\frac{A_1 + A_3}{\alpha_1 + \alpha_3}. \end{aligned} \tag{14}$$

We have listed input values, mass and $F_K/F_\pi$ predictions for the model in Table I where we have chosen $W = 1.33$ since it gives a reasonable mass fit. We have three comments on the predictions in Table I:

(1) The quantities $m_\omega$ and $m_{\omega_A}$ are predicted to be degenerate with $m$ and $m_{\rho_A} \equiv m_{A_1}$, respectively, since from the model, $m_{\omega_A}^2 = 2(\delta_V^2 + \beta_V^2 W^2) m_{\rho_A}^2$, but $\beta_V^2 \approx 0$, $\delta_V^2 \approx \frac{1}{2}$.

(2) The prediction for $m_{\eta'}$ is higher here than the prediction given in Ref. 1 principally because we have chosen a smaller value of $W$. The prediction for $m_K$ would have been much higher here than in Ref. 1 had it not be moderated by a factor $Z_\pi^2$.

(3) The prediction for $F_K/F_\pi$ is reduced from the value given in Ref. 1 principally because of our choice of a smaller value of $W$.

## III. SCALE INVARIANCE

As in Ref. 1, we have found that the constraints from the chiral invariance of $V_0$ are not sufficient to relate all the scalar masses to one another. We, therefore, investigate the consequences of implementing an approximate scale symmetry on our model in the same manner as in Ref. 1, i.e., in exact analogy to classical physics.

Exact scale invariance of the Lagrangian density (4) would require that $V = V_0 + V_{SB}$ be a homo-

TABLE I. Input values and mass predictions for the choices $g^2/4\pi=4.5$ and $W=1.33$. All masses are given in MeV with experimental values in square brackets (Ref. 5).

| Quantity | $m_\pi$[134.9] | $m_K$[497.7] | $m_\eta$[548.8] | $m_\rho$[765] | $m_{\rho_A}$[1070] | $F_\pi$[136.32] | $\Gamma_\rho$[150] |
|---|---|---|---|---|---|---|---|
| Prediction | ← | | | Input data | | | → |

| Quantity | $m_{K^*}$[892.6] | $m_\omega$[783.9] | $m_\phi$[1019.5] | $m_{K_A}$[1242?] | $m_{\omega_A}$[1286?] | $m_{\phi_A}$[1422?] | $m_{\eta'}$[957.5] |
|---|---|---|---|---|---|---|---|
| Prediction | 908.5 | 765 | 1017.5 | 1252.9 | 1070 | 1423.1 | 1095.4 |

| Quantity | $m_\kappa$[1100?] | $F_K/F_\pi$ [1.28?] | $Z_\pi^2$ | $Z_K^2$ | $\theta_V=\theta_A$ | $\theta_p$ |
|---|---|---|---|---|---|---|
| Prediction | 1036.9 | 1.168 | 0.643 | 0.647 | −35.26° | −3.69° |

geneous function of order four in the fields $\phi$, $S$, $V_\mu$ and $A_\mu$. The particular choice of $V_{SB}$ given in (2) obviously violates this criterion. However, since this choice of $V_{SB}$ seems reasonable,[1] we require only $V_0$ to be scale-invariant. This allows us to test one simple model where $V_{SB}$ is a "(3, 3*)"-chiral- and scale-symmetry-breaking term. Recall that we are considering $V_0$ to be a function of $S$ and $\phi$ only. Then using Euler's theorem for homogeneous functions of order 4, we require

$$\mathrm{Tr}\left(\phi\frac{\partial V_0}{\partial\phi}+S\frac{\partial V_0}{\partial S}\right)=4V_0. \qquad (15)$$

We may obtain the consequences of the scale invariance of $V_0$ on the masses and couplings by the appropriate differentiation of (15) with respect to $\phi$ and $S$.

We obtain a relation between the $\sigma$ and $\sigma'$ masses and the $\sigma$-$\sigma'$ mixing angle $\theta_s$ by differentiating (15) with respect to the scalar field, evaluating the result at the equilibrium point and using an extremum condition on $V$ (see Ref. 1) to get

$$\sum_{a=1}^{3}\alpha_a\left\langle\frac{\partial^2 V_0}{\partial S_a^a\partial S_b^b}\right\rangle_0=-3\left\langle\frac{\partial V_{SB}}{\partial S_b^b}\right\rangle_0 \quad (b=1,2,3). \qquad (16)$$

We now use expression (2) for $V_{SB}$ and the scalar analog of the pseudoscalar $\eta$-$\eta'$ mixing formalism and eliminate $\theta_s$ from the resulting expressions to arrive at Eq. (6.5) of Ref. 1,

$$(W^2+2)(m_\sigma m_{\sigma'})^2=-\frac{36}{\alpha^2}[2(A_1)^2+(A_3)^2]+\frac{6}{\alpha}(2A_1+WA_3)(m_\sigma^2+m_{\sigma'}^2). \qquad (17)$$

We determine the quantities $A_1/\alpha$ and $A_3/\alpha$ from (14) (which introduces renormalization constants into the relation) and use the values $W=1.33$ and $Z_\pi^2=0.643$ to get

$$m_{\sigma'}^2=\frac{(882.97)^2-m_\sigma^2}{1-m_\sigma^2/(633.71)^2}. \qquad (18)$$

A graph of Eq. (18) is provided in Fig. 1 where we see that at least one of the $\sigma$-$\sigma'$ masses is greater than 883 MeV and the other is below 633 MeV. We may also solve Eqs. (16) for the scalar mixing angle. A few representative values are given in the following chart:

| $m_\sigma$ (MeV) | $m_{\sigma'}$ (MeV) | $\theta_s$ |
|---|---|---|
| 0 | 882.9 | 127.8° |
| 425 | 1043.4 | 116.6° |
| 633.71 | ∞ | 82.1° |

It becomes clear now that the primary result of our requirement of scale invariance is the mass relation (18). Comparing the chart above with the results of Ref. 1, we see that the effect of the gauge fields on (17) is to lower the values of the asymptotes of (17) for each value of $W$ chosen. We list a few asymptotes for different choices of $Z_\pi^2$ at $W=1.33$ below:

| $Z_\pi^2$ | Asymptotes (MeV) |
|---|---|
| 1 (~Ref. 1) | 788 |
| 0.758 | 687 |
| 0.643 (present model) | 634 |
| 0.5 | 557 |

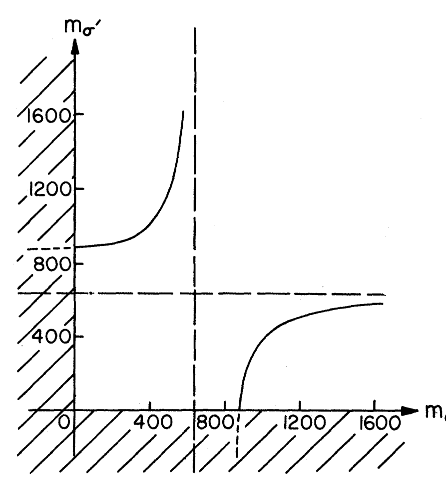


FIG. 1. Graph of the scale relation [Eq. (18)] between the $\sigma$ and $\sigma'$ masses in MeV.

TABLE II. Some scalar-meson on-shell coupling constants. The last term in each coupling constant is the contribution to that particular coupling from $\mathfrak{L}_1$ and $\mathfrak{L}_2$. $\theta_s$ is the $\sigma$-$\sigma'$ mixing angle, $\theta_p$ is the $\eta$-$\eta'$ mixing angle.

$$g_{\sigma\pi\pi}=\frac{\delta'}{\alpha}\left[m_\pi{}^2(1-2Z_\pi{}^2)+Z_\pi{}^2m_\sigma{}^2\right]+\frac{\delta'}{\alpha}(m_\sigma{}^2-2m_\pi{}^2)(1-Z_\pi{}^2)=\frac{\delta'}{\alpha}(m_\sigma{}^2-m_\pi{}^2)$$

$$g_{\sigma'\pi\pi}=\frac{\beta'}{\alpha}\left[m_\pi{}^2(1-2Z_\pi{}^2)+Z_\pi{}^2m_{\sigma'}{}^2\right]+\frac{\beta'}{\alpha}(m_{\sigma'}{}^2-2m_\pi{}^2)(1-Z_\pi{}^2)=\frac{\beta'}{\alpha}(m_{\sigma'}{}^2-m_\pi{}^2)$$

$$g_{\epsilon\pi\eta}=\frac{\delta}{\alpha}\left[(m_\epsilon{}^2-m_\eta{}^2)+(1-Z_\pi{}^2)(m_\eta{}^2+m_\pi{}^2-m_\epsilon{}^2)\right]+\frac{\delta}{\alpha}(1-Z_\pi{}^2)(m_\epsilon{}^2-m_\eta{}^2-m_\pi{}^2)=\frac{\delta}{\alpha}(m_\epsilon{}^2-m_\eta{}^2)$$

$$g_{\epsilon\pi\eta'}=\frac{\beta}{\alpha}\left[(m_\epsilon{}^2-m_{\eta'}{}^2)+(1-Z_\pi{}^2)(m_{\eta'}{}^2+m_\pi{}^2-m_\epsilon{}^2)\right]+\frac{\beta}{\alpha}(1-Z_\pi{}^2)(m_\epsilon{}^2-m_{\eta'}{}^2-m_\pi{}^2)=\frac{\beta}{\alpha}(m_\epsilon{}^2-m_{\eta'}{}^2)$$

$$g_{\epsilon K\bar{K}}=\frac{1}{\alpha(1+W)}\left[Z_K^2m_\epsilon{}^2+m_K^2(1-2Z_K^2)\right]+\frac{(m_\epsilon{}^2-2m_K^2)(1-Z_K^2)}{4m_{KA}{}^2Z_K^2\alpha}\left[m_{A_1}{}^2Z_\pi{}^2(1+W)+m_\rho{}^2(1-W)\right]$$

$$g_{\sigma K\bar{K}}=\frac{\delta'-\sqrt{2}\beta'}{\alpha(1+W)}\left[Z_K^2m_\sigma{}^2+m_K^2(1-2Z_K^2)\right]+\frac{(m_\sigma{}^2-2m_K^2)(1-Z_K^2)}{4m_{KA}{}^2Z_K^2\alpha}\left[(\delta'-\sqrt{2}\beta')m_{A_1}{}^2Z_\pi{}^2(1+W)+(\delta'+\sqrt{2}\beta')m_\rho{}^2(1-W)\right]$$

$$g_{\sigma' K\bar{K}}=\frac{\beta'+\sqrt{2}\delta'}{\alpha(1+W)}\left[Z_K^2m_{\sigma'}{}^2+m_K^2(1-2Z_K^2)\right]+\frac{(m_{\sigma'}{}^2-2m_K^2)(1-Z_K^2)}{4m_{KA}{}^2Z_K^2\alpha}\left[(\beta'+\sqrt{2}\delta')m_{A_1}{}^2Z_\pi{}^2(1+W)+(\beta'-\sqrt{2}\delta')m_\rho{}^2(1-W)\right]$$

$$\delta=\frac{\cos\theta_p-\sqrt{2}\sin\theta_p}{\sqrt{6}},\qquad \beta=\frac{\sin\theta_p+\sqrt{2}\cos\theta_p}{\sqrt{6}}$$

$$\delta'=\frac{\cos\theta_s-\sqrt{2}\sin\theta_s}{\sqrt{6}},\qquad \beta'=\frac{\sin\theta_s+\sqrt{2}\cos\theta_s}{\sqrt{6}}$$

$$\alpha=F_\pi/2Z_\pi$$

Since the most likely experimental[5] candidate for the $\sigma$ is around 700 MeV (but very broad), it is seen that it violates the upper bound of 634 MeV. Considering the $Z_\pi{}^2$ dependence of our upper bound it is perhaps too early to rule out the simple and esthetic theory in which the same symmetry-breaking object is responsible for chiral- and scale-symmetry violation. However, if a definitive set of low-energy $\pi$-$\pi$ $s$-wave phase shifts is determined and if no low-energy $\sigma$-type resonance is seen it may be possible to do this.

## IV. SCALAR-MESON TRILINEAR COUPLINGS

To calculate the $\tilde{S}\phi\phi$ coupling constants, we must account for contributions from $V_0$, from the gauge-invariant kinetic term, and from $\mathfrak{L}_1$ and $\mathfrak{L}_2$ in Eq. (4). We list the more tractable on-shell coupling constants for those vertices related by the chiral symmetry of $V_0$ in Table II. We point out that the last term in each coupling constant is the contribution to that particular coupling from $\mathfrak{L}_1$ and $\mathfrak{L}_2$. The first term in each case is identical in form to the respective couplings derived from the model where the gauge field masses are added via (7) rather than $\mathfrak{L}_1$ and $\mathfrak{L}_2$.[3] It is interesting that, for the couplings involving no strange mesons, the couplings in Table II are identical to those derived in Ref. 1 before the constant $\alpha$ is eliminated via the pion decay constant (10).

The numerical results for the at rest decay widths of the scalar mesons are presented in Table III. Data from Table I and Sec. III have been used wherever possible. Although not listed in Table III, the broadening of the widths due to $\mathfrak{L}_1$ and $\mathfrak{L}_2$ is typically 50–60% if no strange particles are involved and 20–30% if strange particles are in-

TABLE III. Scalar-meson decay (at rest) widths using the trilinear couplings given in Table II. Where possible, the values of masses in the various formulas were taken from Table I. Values for $m_\sigma$, $m_{\sigma'}$, and $\theta_s$ were taken from Sec. III, the value $\theta_p=-3.69°$ was predicted from the model, and values for $m_\epsilon$ were chosen freely. All masses and widths are given in MeV.

| | | | | | |
|---|---|---|---|---|---|
| $m_\sigma$ | 425 | 600 | 630 | 633.71 | 882.97 |
| $\Gamma(\sigma\to\pi\pi)$ | 77.9 | 190.16 | 192.1 | 192.3 | 3.6 |
| $m_{\sigma'}$ | 1043.4 | | | | |
| $\Gamma(\sigma'\to\pi\pi)$ | 102.2 | | | | |
| $\Gamma(\sigma'\to K\bar{K})$ | 522.5 | | | | |
| $m_\sigma$ | 1000 | 1100 | | | |
| $\Gamma(\sigma\to K\bar{K})$ | 159 | 773.7 | | | |
| $m_\epsilon$ | 962 | 996 | 1016 | 1240 | 1320 |
| $\Gamma(\epsilon\to K\bar{K})$ | not possible | 65.6 | 456.24 | ··· | ··· |
| $\Gamma(\epsilon\to\pi\eta)$ | 140 | 172.5 | 193.4 | 527.7 | ··· |
| $\Gamma(\epsilon\to\pi\eta')$ | not possible | ⟶ | ⟶ | 6 | 46.1 |
| $m_\kappa$ | 1036.9 | | | | |
| $\Gamma(\kappa\to K\pi)$ | 567.8 | | | | |
| $\Gamma(\kappa\to K\eta)$ | not possible | | | | |

volved. The width calculations are all first-order $\tilde{S}\tilde{\phi}\tilde{\phi}$ vertex estimations and no attempt is made to include contributions from higher-order graphs.

We note that many of the widths in the Table III are smaller than those given in Ref. 1. To see how this comes about, we examine, for example, $g_{\sigma\pi\pi}$:

$$g_{\sigma\pi\pi} = \frac{\delta'}{\alpha}(m_\sigma^2 - m_\pi^2)$$

$$= \frac{2\delta Z_\pi}{F_\pi}(m_\sigma^2 - m_\pi^2).$$

Then the reduction of the $\sigma \to \pi\pi$ width in this model is due to the factor $\delta' Z_\pi$. In the models of Ref. 3, however, the width reduction is greater, due to a factor $\delta' Z_\pi^3$ in $g_{\sigma\pi\pi}$, where $Z_\pi^2 \simeq \frac{1}{2}$ (see the first paragraph in Sec. IV). Although some of the widths quoted in Table III are close to some of the experimental evaluations in the literature[5] they are still slightly too broad. The models of the type described in Ref. 3, while giving much smaller widths as noted above, do not produce as reasonable a mass spectrum.

ACKNOWLEDGMENTS

The author would like to thank Y. Ueda and particularly J. Schechter for many helpful discussions and encouragement. Thanks are also extended to M. Singer for pointing out errors in a previous version of this article.

*Work supported by the U. S. Atomic Energy Commission.

[1]J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971). Further references are given here and in Ref. 2.

[2]S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[3]Recent treatments are: A. Bhargava and T. Dass, Phys. Rev. D 1, 649 (1970); T. Suzuki, T. Uchida, and C. Hattori, Progr. Theoret. Phys. (Kyoto) 46, 220 (1971).

[4]We calculate the width for the decay $\rho^+(k) \to \pi^+(p) + \pi^0(q)$ from the model as

$$\Gamma_\rho = \sum_{s_i} \frac{|M|^2 |\vec{k}|}{8\pi m_\rho^2},$$

where $|\vec{k}|^2 = \frac{1}{4}(m_\rho^2 - 4m_\pi^2)$,

$$\sum_{s_i} |M|^2 = \left(\frac{g}{\sqrt{2}} + \frac{Cg\alpha^2}{\sqrt{2}m_{A_1}^2 Z_\pi^2} - \frac{m_\rho^2}{2}\frac{g^3\alpha^2}{\sqrt{2}m_{A_1}^4 Z_\pi^2}\right)^2 \times (m_\rho^2 - 4m_\pi^2),$$

and $C\alpha^2 = m_\rho^2 - m_{A_1}^2 Z_\pi^2$ from (9). We note that letting $C=0$ gives us $Z_\pi^2 \approx \frac{1}{2}$ and $g^2/4\pi = 4.9$ which corresponds to $\Gamma_\rho \approx 218$ MeV. In any case, we determine $B\alpha^2 = m_\rho^2 + m_{A_1}^2 Z_\pi^2$ from (9).

[5]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

PHYSICAL REVIEW D VOLUME 6, NUMBER 7 1 OCTOBER 1972

# Test for Scalar-Boson-Mediated Weak-Interaction Theories in High-Energy Neutrino-Scattering Processes

S. Krishna
*Department of Physics, Indian Institute of Technology, Kanpur, India*

(Received 13 March 1972)

Pais and Treiman have shown that in neutrino-scattering processes lepton-pair locality implies explicit dependence of the cross section on two-lepton-system variables. We suggest here that violation of their theorem occurring to the first order in the weak-interaction coupling constant in the scalar-boson-mediated theories may serve as an unambiguous test at energies much smaller than those required for production and subsequent tests on the weak bosons. In a typical model we find substantial violations of the lepton-energy-dependence theorem. On the contrary, violation of the angle-dependence theorem is small.

## I. INTRODUCTION

The structure of the weak-interaction Lagrangian has recently become a topic of renewed interest with the expectation of new possibilities implied by the proposed neutrino-scattering experiments at high energies. The familiar current-current effective Lagrangian has well-known troubles at high energies. A first step towards a complete Lagrangian would be to look for deviations from the implications of the local current-current Lagrangian. Important results in this regard are the