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Two-Particle Correlations in Inclusive Reactions*

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Two-particle inclusive distributions and correlation functions are computed from a model which emphasizes single production of resonance-like hadrons (novas) which cascade decay through pion emission. Strong positive correlations are predicted at small values of the center-of-mass rapidity of the final pions. The magnitude and energy dependence of $\langle n_{\pi^-}\rangle$ and $\langle n_{\pi^-}(n_{\pi^-}-1)\rangle$ are reproduced correctly. Good agreement is obtained with experimental $d\sigma/dy$, $d^2\sigma/dy_1dy_2$, and $C(y_1, y_2)$ from data on $\pi^+p\to\pi^-\pi^-X$ at 18.5 GeV/c.

A wealth of information has become available on single-particle (inclusive) distributions[1] in high-energy hadronic interactions. Many theoretical points of view are consistent with present data,[2] but one may expect that differences between models will be conceded when data are confronted at higher energies. Differences between models may also be found in the analysis of correlations[3] among produced secondaries in data at present machine energies. These correlations may represent obvious interactions among particles in the final state (e.g., resonances), but should also provide insight into the multiparticle production mechanism proper. In this paper, we focus on correlations between two π^- in two-particle inclusive reactions such as $\pi p\to\pi^-\pi^-X$ and $pp\to\pi^-\pi^-X$. Because the $\pi^-\pi^-$ system has exotic quantum numbers, we expect that any correlations will reveal information primarily about the production mechanism.

Correlations will appear among the variables describing the final particles, e.g., the rapidities y, the transverse momenta p_T, and the angle φ between transverse momentum vectors. In some data, a maximum is observed in the two-pion distribution at zero relative rapidity.[4] This may suggest a physical picture in which both secondaries are fragments of the same object (beam or target). It is tempting to pursue this picture in analyzing more extensive data available now from pion-induced reactions.[5] In view of the large number of variables, it is hard to reach conclusions at a general level. Moreover, because it is valuable to have some preconceptions when examining new data, we adopt the quantitative framework offered by the nova model of particle production.[6] In this (fragmentation) model, single excitation of either the beam or the target particle is followed by eventual cascade decay of the resonance-like excited hadron. Although single excitation is far from exhaustive, the model reproduces simply important features of single particle distributions at current accelerator energies. Well-defined predictions are made also for two-particle correlations, which, as we will show, are in reasonable agreement with available data. Our results, displayed in Figs. 1, 2, and 3, represent the first realistic predictions for two-particle correlations at presently available energies. Several experimental analyses are now in progress,[5,7,8] and our detailed model calculations may be of value.

Predicted single-particle distributions should reproduce correctly the observed rapidity and transverse momentum behavior of spectra, and be normalized to $\langle n\rangle\sigma_{\text{inel}}$. Even if the total inelastic cross section σ_{inel} is kept as a normalization constant, as in our case, the fractional number of secondaries of a given charge should be a prediction of the model. When analyzing two-particle spectra, one should reproduce a five-dimensional distribution, $d^5\sigma/dy_1dy_2dp_{T1}dp_{T2}d\varphi$, whose integral is $\langle n(n-1)\rangle\sigma_{\text{inel}}$. The average value $\langle n(n-1)\rangle$ should also be a prediction of the model.

As discussed in detail in Ref. 6, upon summing contributions from excitation of beam and target we write the single-particle rapidity distribution as

$$\frac{d\sigma^i}{dy}=\sum_\alpha c_\alpha\int \rho_\alpha(M)n^i_\alpha(M)A_\alpha(M,y)\,dM\ , \tag{1}$$

where α refers to either target or beam, and i labels the type of observed secondary. The integral extends over the entire mass spectrum. As written, Eq. (1) implies an average over all p_T^2. The function $A(M,y)$ is the normalized decay distribution (in y) of a nova of mass M. To obtain

$A(M, y)$ we begin with the following simple symmetric decay distribution in the nova rest system:

$$\frac{d^2D}{dp_T{}^2dp_L} = \frac{a}{K^2\omega}\exp\left(-\frac{p_T{}^2+p_L{}^2}{K^2}\right) . \qquad (2)$$

Here, $\omega = (p_T{}^2 + p_L{}^2 + m_\pi{}^2)^{1/2}$; a normalizes the distribution to unity, and K (~0.45 GeV/c) is adjusted to reproduce a typical Q value of 330 MeV at each step in the decay chain of the nova.[9] This value of K also leads to good agreement with distributions in p_T. The function $A(M, y)$ is obtained after making a Lorentz transformation on Eq. (2) to the center-of-mass system and performing an integration over $p_T{}^2$.

In Eq. (1), $\rho(M)$ prescribes the weight assigned to the nova of mass M; as discussed in Ref. 6, there is little freedom in the choice of this function. The function $n^i(M)$ is the average number of secondaries of a particular kind (i) expected from the decay of a nova of mass M. To obtain $n^i(M)$, we modify a statistical distribution by strong charge-related effects met in the first few steps of the cecay.[6] As fixed by our average Q value, the total number $n(M)$ of pions of all charges is $2.1(M - M_0)$, where M_0 is the mass of the excited particle (target or projectile). For a positively charged nova of mass M, we find that simple and reasonable approximations are[10]

$$n^-(M) \approx \tfrac{1}{3}[n(M) - 1] , \qquad (3)$$

$$\langle n^-(n^- - 1)\rangle_M \approx \tfrac{1}{9}[n(M) - 1]\,[n(M) - 3] . \qquad (4)$$

The value zero is used when $n(M) < 1$ or 3, respectively.

This is an oversimplified picture, of course,

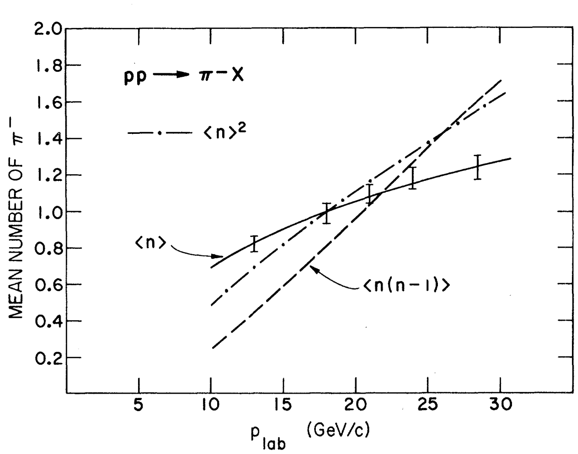

FIG. 1. The calculated values of $\langle n\rangle$, $\langle n\rangle^2$, and $\langle n(n-1)\rangle$ for pp reactions. Here n stands for the number of π^-. The mean is defined with respect to $\sigma_{\rm inel}$ (e.g., $\langle n\rangle = \sum n\sigma_n/\sigma_{\rm inel}$). Data shown are taken from Ref. 11; experimental points have been scaled to correct for the fact that they are defined with respect to $\sigma_{\rm tot}$.

and it is important to verify that our predicted average multiplicity reproduces the energy dependence and actual value[11] of $\langle n^-\rangle$ for $pp \to \pi^- X$. Upon combining Eqs. (3) and (4) with an excitation spectrum,[6] to which details are insensitive,

$$\rho(M) = (M - M_0)^{-2}\exp[-2/(M - M_0)] , \qquad (5)$$

we obtain the results shown in Fig. 1. Given the agreement we achieve with $\langle n^-\rangle$, it would be valuable to check our predictions for $\langle n^-(n^- - 1)\rangle$. It may also be noted in Fig. 1 that our calculated values of $\langle n^-\rangle^2$ and $\langle n^-(n^- - 1)\rangle$ are approximately equal over the energy range 20–30 GeV/c; thus, our results approximate a Poisson distribution in this energy range. However, as $p_{\rm lab}$ increases, we expect that $\langle n^-(n^- - 1)\rangle$ will become decidedly larger than $\langle n^-\rangle^2$. These features are in good agreement with a recent compilation of available data.[12]

Identical computations of $\langle n\rangle$ and $\langle n(n-1)\rangle$ may be done for π^+p induced reactions. At 18.5 GeV/c, where data are available, we find $\langle n^-\rangle = 1.21$ and $\langle n^-(n^- - 1)\rangle = 1.30$; these may be compared with experimental values[13] 1.34 and 1.27, respectively. Although agreement is reached only at the 10% level, it should be stressed that the same model is used to describe meson and baryon excitation and decay.[14] It is instructive that $\langle n(n-1)\rangle/\langle n\rangle$ is somewhat too large. This corresponds in the model to too strong a weight being assigned to the large-M end of the excitation spectrum $\rho(M)$. This end is, of course, poorly known *a priori*. The lesson is that data on correlations probe high-mass excitations, not tested in a sensitive way by single-particle spectra at present energies.[15] New data on the energy dependence of $\langle n(n-1)\rangle/\langle n\rangle^2$ would be particularly valuable and will provide a good test of the fragmentation hypothesis.[16] Keeping Eq. (5) as our best present guess, we are led to two-particle distributions whose normalization will be too large by ~13%. Similar overestimate is therefore expected for $\langle n(n-1)\rangle$ in Fig. 1.

Attaching both observed π^-'s to the same nova, the two-particle inclusive distribution for $\pi^+p \to \pi^-\pi^-X$ is

$$\frac{d^2\sigma}{dy_1dy_2} = \sum_\alpha c_\alpha \int \rho_\alpha(M)\langle n^-(n^- - 1)\rangle_M \times A_\alpha(M, y_1)A_\alpha(M, y_2)dM. \qquad (6)$$

Again, the distribution has been averaged over transverse momenta, but the simple form of Eq. (2) allows easy determination of p_T dependences.

Results obtained from numerical evaluation of Eq. (6) are shown in Fig. 2, along with some recent data.[5,13] The shapes of these distributions, which emerge from our essentially zero-param-

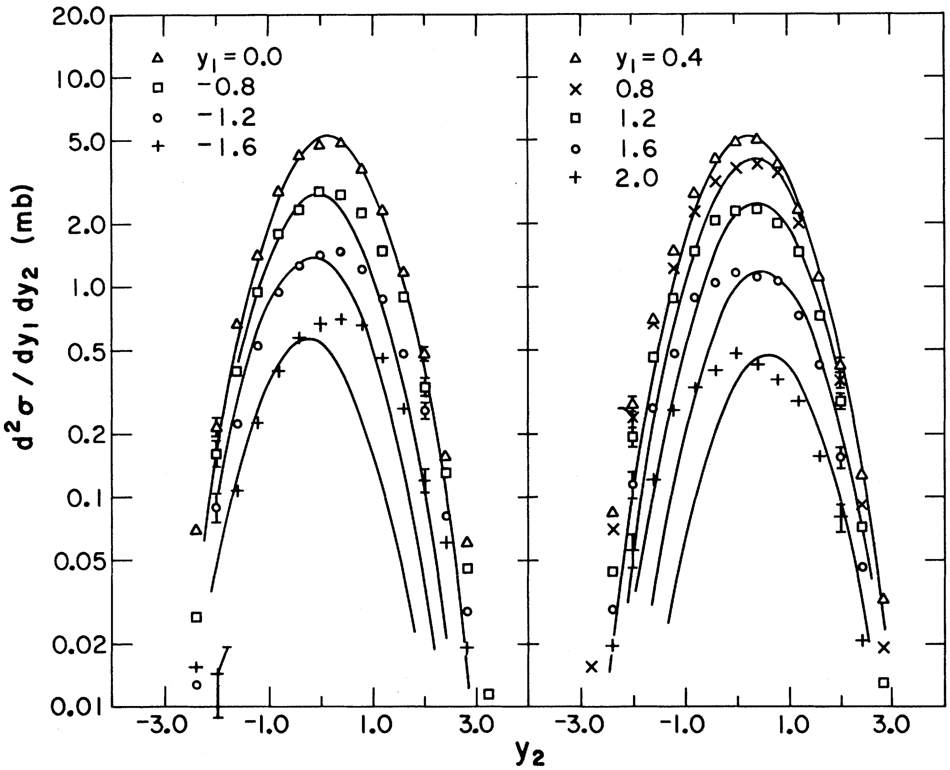

FIG. 2. Calculated distributions $d^2\sigma/dy_1dy_2$ are compared with Notre Dame data on $\pi^+p \to \pi^-\pi^-X$ at 18.5 GeV/c, Refs. 5 and 13; y_1 and y_2 are rapidities of the two π^-'s. Theoretical results are normalized to data at $y_1 = y_2 = 0$ because, as discussed in the text, our prediction for $\langle n(n-1)\rangle$ is $\sim 13\%$ too high. Curves are computed at fixed values of y_1 indicated on the figure, whereas data are averaged over a band $\Delta y = 0.4$ centered at values shown. The relatively stable maximum at $y = 0.4$ in the data corresponds to the maximum at positive rapidity in the single particle distribution.

eter model, are in reasonable agreement with data, especially at small y where the cross section is largest. This shows that the two π^- have a strong tendency to follow each other, a feature stressed by the model.[17] Discrepancies between our calculations and the data appear systematically at larger y, where cross sections have dropped by an order of magnitude. This is not surprising because double excitation (e.g., production of AN^*), is not present in this calculation and is likely to give back-to-back rapid π^- secondaries.[18] At intermediate values of y, experimental distributions are broader than our computations. This effect is connected to the presence of double excitation which is discussed in detail elsewhere.[18] Although our picture does not accommodate all observed features, Fig. 2 shows that it seems to be capable of reproducing the bulk of present data. Our model thus provides dominant effects from which corrections can be contemplated.[18]

Two-particle distributions in pp and Kp reactions are readily calculable and show similar treads. It will be very interesting to compare them with forthcoming data in the 10–30-GeV/c range.[5,7,8]

To summarize effects such as those shown in

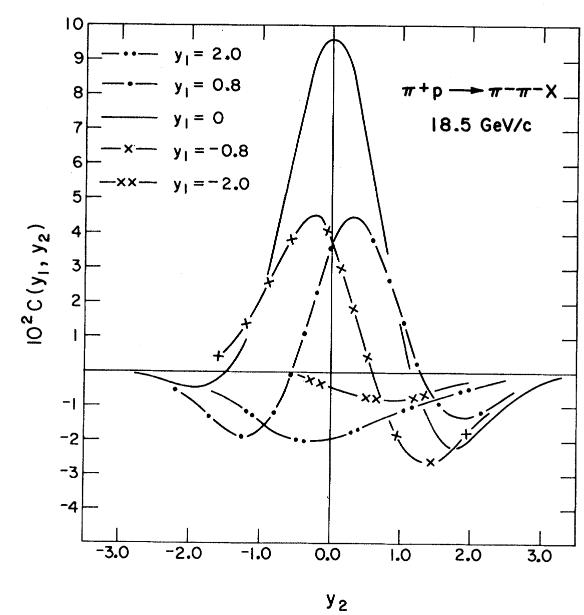

FIG. 3. The correlation function $C(y_1, y_2)$ for $\pi^+p \to \pi^-\pi^-X$ at 18.5 GeV/c is plotted versus y_2 for various values of y_1. See Eq. (7) of the text for the definition of C.

Fig. 2, one can define a correlation function $C(y_1, y_2)$ as follows:

$$C(y_1, y_2) \equiv \frac{1}{\langle n(n-1)\rangle \sigma_{\text{inel}}} \frac{d^2\sigma}{dy_1 dy_2} - \left(\frac{1}{\langle n\rangle \sigma_{\text{inel}}}\right)^2 \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \,. \tag{7}$$

This definition differs slightly from others found in the literature,[2] but seems more appropriate in an energy regime where $\langle n\rangle$ is of order unity. Observe that Eq. (7) agrees with more common definitions if the distribution in n obeys a Poisson law. The integral $\int C(y_1, y_2)\, dy_1 dy_2$ is equal to zero. In Fig. 3, we give values of C computed for several values of rapidity. The pronounced structure is remarkable. The fact that correlations are indeed large may be appreciated from the observation that the ratio

$$C \Bigg/ \frac{1}{\langle n(n-1)\rangle \sigma_{\text{inel}}} \frac{d^2\sigma}{dy_1 dy_2}$$

is 0.4 at $y_1 = y_2 = 0$. Since we keep only dominant single-excitation terms, this may be an overestimate.

As mentioned previously, correlations in y for different values of p_T are calculated easily. An interesting effect is that the two-particle rapidity distributions become broader when p_T is restricted to be small. This effect reflects our sequential decay picture, as expressed by the statistical distribution, Eq. (2). A finer test of the notion of sequential decay is the weakness of any correlation in φ for two π^-, which are rarely emitted in successive steps.[7] Stronger back-to-back φ correlations can be present for $\pi^+\pi^-$. As a final point, we remark that our analysis proceeds in a slightly different way for inclusive processes in which one of the observed final particles has quantum numbers identical to one of the incident particles. Another term must be added to Eqs. (1) and (6), corresponding to the quasielastically scattered primary. The relations given in Eqs. (3) and (4) are also modified. As a result, $d^2\sigma/dy_1dy_2$ at small y is approximately three times as large for $\pi^-p \to \pi^-\pi^-X$ as for $\pi^+p \to \pi^-\pi^-X$ at 18.5 GeV/c. These points will be discussed elsewhere.[18]

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[9]E. Yen and E. L. Berger, Phys. Rev. Letters 24, 695 (1970).

[10]Oversimplifications are involved in attaching fixed pion multiplicity to each excitation mass and in calculating relative weights for each charge configuration. Nevertheless, only small corrections result from including other possibilities, such as double steps in the decay chain.

[11]D. B. Smith, LRL Report No. UCRL-20632, 1971 (unpublished).

[12]A. Białas and K. Zalewski, University of Cracow report, 1971 (unpublished).

[13]J. T. Powers and W. D. Shephard, private communication.

[14]The only extra input is the relative value of the constants c, which are defined through Eq. (1). We take $c_\pi/c_p = 2$, which reproduces the shape of the single-particle spectrum.

[15]In a model-independent way, the character of distributions for small y can be probed by studying single-particle distributions at very high energy or, at fixed energy, by studying higher and higher correlations, which obtain their weight at smaller and smaller y.

[16]J. Benecke *et al.*, Phys. Rev. 188, 2159 (1969); T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970).

[17]This results from the strong yield of heavy novas which produce many slow pions. The rapidity difference is small because both y_1 and y_2 are small.

[18]Detailed discussion of $\pi^{\pm}p \to \pi^-\pi^-X$ and $pp \to \pi^-\pi^-X$, including the contribution of terms in which two novas are formed per event, is given in E. L. Berger, M. Jacob, and R. Slansky, Phys. Rev. D (to be published).