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Fluctuations, Dips, and Duality*

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We examine data for backward elastic scattering of pions by protons. The width and frequency of the dips seen, as a function of energy, are strongly suggestive of Ericson fluctuations. The implications of this observation are discussed.

INTRODUCTION

According to the usual dual picture, elastic scattering is to be considered as the result of Pomeranchukon exchange plus Regge exchange which is dual to s-channel resonances. At high enough energies the resonances in $\pi^- p$ scattering overlap, and their contribution, plus the diffractive contribution, which itself is presumably smooth, makes for a differential cross section which becomes increasingly smooth as energy increases. It has been pointed out recently by Frautschi[1] that there may be regions of energy and scattering angle where fluctuation phenomena of a type well known in other branches of physics and especially in nuclear physics[2,3] exist. These fluctuation phenomena in nuclear physics arise from the superposition of a large number of Breit-Wigner amplitudes with some average spacing and width, but with the value of each fluctuating from the mean in a random way.

The statistical theory of nuclear reactions predicts the mean values of cross sections in terms of average level widths and spacings. The statistical models of Hagedorn[4] and Frautschi[5] predict average level densities for hadronic matter, and one should be able to find fluctuation phenomena in particle physics if these models have validity. A statistical model implies fluctuations. Some years ago these fluctuations were looked for in p-p large-angle scattering and not found. However, p-p scattering involves an exotic s channel, and hence this is not surprising.[1]

PION-NUCLEON SCATTERING

Pion-nucleon scattering with both signs of pion charge have nonexotic s channels and hence should exhibit fluctuation phenomena. If one looks in the few-GeV energy region, at backward angles where diffractive scattering is of minimal importance, one has a good chance of seeing fluctuation phenomena. Figure 1 shows $\pi^- p$ elastic-scattering data[6,7] at 180° in the c.m. energy range 1.5–2.6 GeV. Also shown are resonances listed in the Particle Data Group tables[8] which have been determined mainly by phase-shift analysis. The existing phase-shift fits fail to reproduce new data[6,7] at large angles near 180°. We have fitted these large-angle data with a superposition of Breit-Wigner amplitudes varying widths, angular momentum, and resonance energies. Our fit to the 180° data is shown in

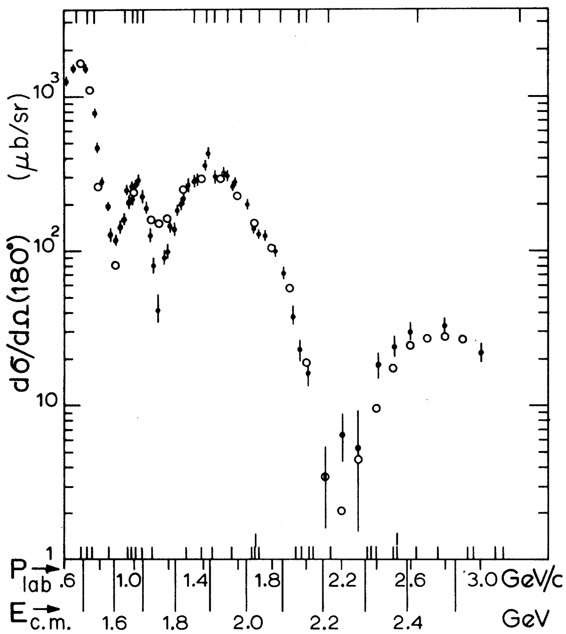

FIG. 1. Measured $\pi^- p$ elastic differential scattering cross section (solid points) (Refs. 6, 7) as a function of c.m. energy, and multilevel Breit-Wigner fit (circles). The energy levels indicated at the top of the diagram are from the Particle Data Group tables (Ref. 8). The levels indicated at the bottom of the diagram are from the fit of this paper.

Fig. 1. We do not describe here the details of the fit parameters. There are, in all, 36 resonances contributing, with an average width of 260 MeV. The sensitivity of the fits to the widths of the new postulated levels is however not great. The average width of nucleon resonances in the Particle Data Group tables, we note, is 190 MeV.

It is important to note that in the dip region near 2.2 GeV there are several resonances contributing. This was necessary to achieve a good fit and hence it is suggestive that this dip and neighboring peaks are fluctuation phenomena rather than the effect of the presence or absence of individual strong resonances.

FLUCTUATION TESTS

We now apply two criteria to test the measurements of Fig. 1 for fluctuation characteristics:

(1) The dips should have an energy spread of the order of the average width Γ of an individual Breit-Wigner level.[2] The dip near 2.2 GeV has a width of the order of 200 MeV, which is about right. The next dip lower down is slightly narrower, as might be expected.

(2) The spacing of dips or peaks should be[3] $\simeq 2\Gamma$. Taking the c.m. energy range 1.7–2.5 GeV one finds three peaks yielding $\Gamma \simeq 200$ MeV.

We have then a strong indication that one is indeed seeing fluctuation phenomena. It would be useful to measure accurately 180° scattering at higher energies. This would improve the statistics and perhaps give an indication of how level widths vary with energy if the fluctuations are still strong enough to be seen.

We comment now on the well-known dip that exists near $t = -2.8$ (GeV/c)^2 in both $\pi^- p$ and $\pi^+ p$ scattering in the energy range of a few GeV. This has been described[9] as being due to an interference between the forward diffractive scattering peak and a rather flat background. In the light of what has been said above we would like to make this description more precise as follows. In the extreme forward direction diffraction is predominant (Pomeranchukon exchange). As we go to larger angles, multi-Pomeranchukon exchange (multiple scattering) becomes dominant and at the same time the differential cross section falls rapidly. At large momentum transfers the contribution of resonances begins to compete strongly with the diffractive scattering, and the relatively flat contribution from the resonances interferes destructively with the rapidly falling diffraction amplitude due to Pomeranchukon exchange.

It would be of interest to apply appropriate tests to scattering data away from the backward direction. It has been suggested by Frautschi[1] that some large-momentum-transfer dips may in fact be fluctuation effects.

We remark finally that we consider the above study to be of considerable interest, if alone to indicate that the level spacings and widths only follow some simple pattern in an average way, the fluctuations being a manifestation of the complexity of hadronic interactions.

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Empirical Systematics of πN Amplitudes*

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The structure of πN s-channel helicity amplitudes as a function of momentum transfer and energy is examined in FESR (finite-energy sum rule) Regge fits, direct amplitude analysis of data, and phase-shift analyses. Information on the separated amplitudes from these various sources is found to agree. The value of helicity correlation measurements in testing the zero structure of helicity amplitudes is demonstrated. The approximation of s-channel helicity conservation for the $I_t = 0$ amplitude above 5 GeV/c is justified from polarization data. The partial-wave and impact-parameter decompositions of the FESR Regge results are presented.

I. INTRGDUCTION

Two-body phenomenology has reached the stage where knowledge of separated amplitudes is essential for further progress. Sufficient experimental information now exists at 6 GeV/c to completely decompose πN scattering amplitudes into isospin and helicity components.[1] However it will be some time before sufficient data on spin correlation parameters exist to isolate the energy dependence of πN amplitudes directly from data. Fortunately finite-energy sum rules (FESR) provide an alternative method for extracting the amplitudes. An FESR analysis generally relies on an effective-pole parametrization for the high-energy amplitudes. Whether or not all the effective poles have physical significance is irrelevant so far as the amplitude determination is concerned. A quantitative FESR Regge fit[2] to the πN differential cross section and polarization data correctly predicted later measurements[3] of spin correlation parameters R and A and is completely consistent with direct amplitude analysis at 6 GeV/c.[1] Since this approach also matches on to phase-shift analysis, through FESR, we have confidence in its representation of the energy dependence of the amplitudes as well as their t dependence.

The FESR Regge fits have been parametrized in terms of t-channel helicity amplitudes, since fixed-t dispersion relations are simpler in terms of t-channel amplitudes. However the important physical systematics due to absorption are more apparent in an s-channel picture.[4] Furthermore, it has been established that the Pomeranchukon-exchange contribution approaches s-channel helicity conservation with increasing energy.[5] Consequently it is worthwhile to cast the result of FESR analyses in terms of s-channel helicity amplitudes and then to look for the empirical peripheral systematics of absorptive effects.

The outline of the paper is as follows. In Sec. II we review the evidence for s-channel helicity conservation of the Pomeranchukon. In Sec. III we observe that measurements of helicity correlation parameters give a direct reflection of the dip structure in separated πN amplitudes. In Sec. IV we examine the s and t dependence of the s-channel helicity amplitudes for πN scattering as obtained from FESR Regge fits. The empirical systematics of the πN amplitudes are compared with Regge-pole and -cut models. Finally in Sec. VI we check the peripherality of the $I_t = 0, 1$ exchanges from partial-wave projections of the high-energy amplitudes. Details of the formalism and the notation are relegated to the Appendix.

II. s-CHANNEL HELICITY CONSERVATION OF $I_t = 0$ EXCHANGE

The theoretical discussion of the πN observables at high energy is greatly simplified by the hypothe-