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Resonance Contributions, Radiative Widths, and Stray Baryonic States in $K^+\Lambda$ Photoproduction*

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We have analyzed the reaction $\gamma p \to K^+\Lambda$ below 2.2-GeV c.m. energy using an isobar model. Energy-dependent multipoles are parametrized by a Breit-Wigner resonance form with an unknown phase included on the coupling. Evidence is found for the existence of a $D_{13}(1670)$ resonant state with a width of about 100 MeV. In addition to the $D_{13}(1670)$, other important states below 1.9 GeV are the $S_{11}(1700)$, $P_{11}(1750)$, and $P_{13}(1860)$. Approximate values are obtained for the radiative widths of these states.

I. INTRODUCTION

Over the past decade there have been several phenomenological studies of $K^+\Lambda$ photoproduction. A good review of the early work is given by Thom,[1] whose analysis was adequate to explain the experimental situation below approximately 2-GeV c.m. energy in 1966. Since then, there have been new developments both from the experimental and theoretical side. It is now desirable to perform an energy-dependent multipole analysis within the spirit of duality and to make use of known results from studies of strong interactions and pion photoproduction. The recent analysis of Schorsch, Tietge, and Weilnböck[2] did this to a certain extent. Their analysis is within the broad framework of duality; however, the masses and widths of their resonance states tend to deviate considerably from the "book" values,[3] and it is therefore difficult to calculate radiative decay widths of the isospin-$\frac{1}{2}$ nucleon resonance states from their results. In addition, important new data are now available for the process $\gamma p \to K^+\Lambda$. The new data[4] combined with the earlier data[5] indicate a very interesting structure. There seems to be a rather pronounced dip in the cross section around 1750-MeV c.m. energy. This dip is more pronounced in the backward direction and tends to fade out somewhat, though not completely, for scattering in the forward direction. It would be interesting to know whether or not this structure can be explained in terms of well-established nucleon resonances.[6]

Thus, there are several reasons why it is particularly appropriate to do a phenomenological analysis at this time. (1) We have new data which indicate considerable structure in the differential cross section. (2) Recent analyses[7,8] of the process $\pi^- p \to K^0\Lambda$ yield information on the $K\Lambda$ partial widths $\Gamma_{K\Lambda}$ of certain isospin-$\frac{1}{2}$ nucleon resonance states. This information coupled with our recently gained knowledge about the radiative widths of some of these states[9-11] allows us to determine within reasonable limits the contributions made to the process $\gamma p \to K^+\Lambda$ from some of the nucleon resonances. (3) A very interesting conjecture made by Donnachie[12] concerning the possibility of stray baryonic states which do not couple significantly to the πN channel can be tested in part by a study of $K^+\Lambda$ photoproduction. (4) New and confirming information can be obtained regarding the radiative decay widths of the isospin-$\frac{1}{2}$ nucleon states which are important in the $\gamma p \to K^+\Lambda$ process. (5) Although we have enough differential-cross-section and polarization data for an energy-dependent multipole analysis we still do not have enough for a full analysis using the preferred method developed by Cutkosky[13]; however, the type of study here may be of value when it is possible to use these techniques.

After developing the formalism we shall examine several solutions which result from different initial input. First, we consider a solution in which known information regarding masses, widths, and decay rates determined from studies of πN and γN states is utilized. Second, we look for improvements in this solution by allowing for stray baryonic states. Third, we look for improvement by allowing for slightly different masses, widths, and decay rates in the known nucleon resonance states. Finally, we discuss our new results, which relate to stray states and radiative widths, and their implications for symmetry assignments.

II. FORMALISM

We use a model which includes resonance terms in the direct channel, a resonance coupling phase,[2,7] and nonresonant terms in both S and P waves. The parametrization is similar to the approach used in Ref. 2 and the same as that used in Ref. 11.

For the resonant part of a magnetic multipole amplitude we assume

$$M_{l\pm}(\text{res}) = \frac{-ie^{i\phi}(\Gamma^M_{\gamma p}\Gamma_{K\Lambda})^{1/2}}{2[qkj(j+1)]^{1/2}(W_r - W - \frac{1}{2}i\Gamma)} \tag{1}$$

and for an electric multipole amplitude $E_{l\pm}(\text{res})$ we have a similar expression $M \to E$ except for j which is equal to the final orbital angular momentum l for $M_{l\pm}$ and is given by $j = l \pm 1$ for $E_{l\pm}$. Here k and q are the c.m. momenta of the photon and kaon, respectively, W is the total c.m. energy, W_r is the mass of the resonant state, and ϕ is the phase angle. The product of the partial widths is given by

$$(\Gamma^M_{\gamma p}\Gamma_{K\Lambda})^{1/2} = [2kRv_n(kR)]^{1/2}[2qRv_l(qR)]^{1/2}\gamma^M, \tag{2}$$

and similarly for $M \to E$. R is an interaction radius chosen to be 1 F, the v's are barrier factors,[14] and γ^E and γ^M are products of reduced widths, assumed to be constants, which depend upon the strength of the interaction.[15] We choose $n = l$ except for E_{l-} when $n = l - 2$. (See in this connection, Table I.) The energy dependence of the total width is approximated by the relation

$$\Gamma = \frac{pv_l(Rp)}{p_r v_l(Rp_r)}\Gamma_r, \tag{3}$$

so Γ reduces to the width Γ_r at the resonant energy W_r. The index r means evaluated at the resonant energy, and the momentum p is some characteristic c.m. momentum for the process.

We have tried both the incident photon momentum and the equivalent c.m. pion momentum for p. The latter of these was used in the quoted results which appear in this paper. We have also tried altering the energy dependence of the resonances in the regions of their tails.[8] As one might expect, the current data are neither sufficiently accurate nor complete to enable us to make a significant distinction among these different momentum and energy dependences. In addition, the basic results seem to be reasonably invariant under such modifications.

For the background we have used

$$E^b_{0+} = \frac{(a+ib)R^2k}{[2(1+Rq)(1+R^3k^3)]^{1/2}}, \tag{4}$$

$$M^b_{1-} = \frac{(a'+ib')R^3qk}{[2(1+R^3q^3)(1+R^3k^3)]^{1/2}}, \tag{5}$$

where a, b, a', and b' are adjustable real parameters, which is the method of Orito[16] and Schorsch *et al.*[2]

Calculation of the differential cross section and polarization is standard. Relevant formulas are collected by Thom[1] and need not be reproduced here.

In making comparisons with data it is the γ^E, γ^M, and ϕ which may vary unless they are already known. Once the parameters are determined, one may calculate the product of the partial widths from Eq. (2). If the partial width $\Gamma_{K\Lambda}$ is known from other studies, then the radiative width Γ_γ may be calculated from

$$\Gamma_\gamma = \Gamma^E_{\gamma p} + \Gamma^M_{\gamma p}, \tag{6}$$

or in terms of the $A_{1/2}$ and $A_{3/2}$ helicity amplitudes[17]

$$\Gamma_\gamma = \frac{k^2}{\pi}\frac{M_N}{W_r}\frac{2}{2J+1}(|A_{1/2}|^2 + |A_{3/2}|^2), \tag{7}$$

where M_N is the nucleon mass and $J = l \pm \frac{1}{2}$ is the total angular momentum. A further discussion of the interrelationship among the parameters and the amplitudes is given in the Appendix.

One could, of course, include other terms in all

TABLE I. Amplitudes and barrier factors.

State	Parity	Amplitude	Multipole transition	Barrier factor
S_{11}	−	E_{0+}	$E1$	v_0
P_{11}	+	M_{1-}	$M1$	v_1
P_{13}	+	E_{1+}	$E2$	v_1
		M_{1+}	$M1$	v_1
D_{13}	−	E_{2-}	$E1$	v_0
		M_{2-}	$M2$	v_2
D_{15}	−	E_{2+}	$E3$	v_2
		M_{2+}	$M2$	v_2
F_{15}	+	E_{3-}	$E2$	v_1
		M_{3-}	$M3$	v_3
F_{17}	+	E_{3+}	$E4$	v_3
		M_{3+}	$M3$	v_3
G_{17}	−	E_{4-}	$E3$	v_2
		M_{4-}	$M4$	v_4

three channels – s, t, and u. However, the reason for omitting poles in the t channel is to keep from doing any double counting.[18,19] We have done calculations with additional s- and u-channel poles and find that the situation is not improved. The background terms we have included can, at present, adequately account for possible pole contributions. Hence, we have a rather pure direct-channel model with only resonance and background terms included.

There are certainly objections which can be made in regard to the parametrization. For example, one could introduce a different interaction radius for each resonance state, or allow for different energy dependence in the partial widths with additional parameters, or additional terms could be added in the background, and we have already mentioned other possible momentum and energy dependences for the total width. While it is important to realize that improvements in the parametrization can be found, the level of the data simply does not justify modifications which introduce many additional parameters, and the energy dependence of the resonance shapes does not seem to be as crucial as whether or not a given state is included or omitted. We therefore feel justified in using as simple an approach as possible. For more ideas in regard to the principles of parametrization one can consult the informative discussion by Moorhouse and Rankin.[20]

It would be helpful if we knew exactly how much the results depend on the particular parametrization. The best we can do at this stage is give an estimate which is based upon calculations with other momentum and energy dependences, and by examining the χ^2 structure in the neighborhood of a minimum. First, we believe the general results regarding the important resonance contributions are correct; and second, we estimate that the radiative widths determined for the most important states are not off by more than about 40%. The phases can undergo changes of about 25° without causing great trouble in the resulting fit. Of course, the relative phases between two resonant states are the physically interesting quantities rather than the magnitude of the given phase for a single state.

III. RESULTS FROM STUDIES OF RELATED PROCESSES

We can utilize the results obtained in studies of $\pi^- p \to K^0\Lambda$ to obtain approximate ranges on the partial widths $\Gamma_{K\Lambda}$ for various nucleon states.[7,8] This information is summarized in Table II. In addition, information is available on the radiative widths of certain states from photoproduction work.[9-11] These results are also summarized in Table II. It is now possible to utilize these results to obtain a probable range for the product $\Gamma_{\gamma p}\Gamma_{K\Lambda}$ of some of the partial widths.

TABLE II. Approximate ranges for partial widths.

State(mass)width (MeV)	$\Gamma_{K\Lambda}$ (MeV)	Γ_γ (MeV)
S_{11}(1700)245	6–10	0.2–1.5
P_{11}(1750)300	15–55	0.1–1.5
P_{13}(1860)300	3–7	
D_{13}(2040)255	5–40	
D_{15}(1670)140	0–0.4	0–0.03
F_{15}(1688)125	0–0.02	0.3–0.4
F_{17}(1990)240	0.2–1.0	
G_{17}(2190)250		0–2

IV. COMPARISON WITH DATA

Our first input for comparison with data from threshold to approximately 2.2-GeV c.m. energy consists of only known nucleon resonance states (plus background) with parameters which are in general agreement with the results of Table II. It appears that the P_{11}(1470), S_{11}(1550), and D_{13}(1520) states which are well below threshold can be safely omitted. In addition, we omit the F_{15}(1688) since the product $\Gamma_{\gamma p}\Gamma_{K\Lambda}$ is small. The D_{15}(1670), which is not photoproduced,[10,21] is also left out. There are reasons, based upon the quark model,[21] for omitting the second S_{11} and D_{13} states; however, in view of possible mixing[22,23] one can not safely rule them out. Furthermore, we do not wish to be ruled by the quark model in making a comparison with data. We therefore have as input the states S_{11}(1700), P_{11}(1750), P_{13}(1860), F_{17}(1990), D_{13}(2040), and G_{17}(2190). Our best results for this combination which we call solution A are shown in Table III. It is clear that this solution is not adequate and modifications are required. There are several possibilities. We can allow for possible stray states which do not couple significantly to the πN channel, and we can look for solutions which have reasonable changes in the mass and width parameters of the known nucleon resonance states. A combination of stray states plus mass and width changes is also a possibility.

As our first modification (solution B) we allow for a possible stray F_{15} baryonic state conjectured by Donnachie.[12] We do not find great improvement and the optimum position for the mass of this F_{15} state tends to be somewhat higher than that suggested in Ref. 12. Next we try a stray D_{13} state (solution C) and then both stray F_{15} and D_{13} states (solution D). Significant improvement is thus seen when the D_{13}(1670) is included. It is interesting to

TABLE III. Values of parameters for various solutions.

State, W_r (MeV), Γ_r (MeV) (for A, B, C, D)	Parameters, γ (MeV), ϕ (deg)	Solutions A	B	C	D	E	State, W_r (MeV), Γ_r (MeV) (for E)
S_{11}, 1700, 245	γ^E	0.88	0.91	0.39	0.38	0.75	S_{11}, 1696, 200
	ϕ	158	161	−173	−175	147	
P_{11}, 1750, 300	γ^M	0.11	0.05	0.83	0.85	0.11	P_{11}, 1700, 400
	ϕ	−95	−105	−153	−153	146	
P_{13}, 1860, 300	γ^E	0.17	0.19	0.24	0.23	0.16	P_{13}, 1881, 295
	γ^M	0.65	0.63	0.50	0.45	0.68	
	ϕ	−94	−91	−156	−158	−96	
D_{13}, 1670, 90	γ^E			0.69	0.71		
	γ^M			0.10	0.08		
	ϕ			82	79		
D_{13}, 2044, 256	γ^E	0.14	0.16	0.02	0.02	0.02	D_{13}, 2050, 300
	γ^M	0.41	0.46	0.36	0.34	0.48	
	ϕ	43	35	−40	−50	36	
F_{15}, 1903, 50(B)	γ^E		0.06		0.08		
	γ^M		−0.00		−0.03		
1930, 112(D)	ϕ		−173		112		
F_{17}, 1190, 240	γ^E	−0.03	−0.05	0.02	−0.11	0.10	F_{17}, 1980, 230
	γ^M	0.42	0.32	0.29	0.25	0.34	
	ϕ	4	−6	−58	−38	−9	
G_{17}, 2190, 250	γ^E	0.64	0.61	−0.11	−0.01	−0.59	G_{17}, 2180, 290
	γ^M	−0.29	−0.23	0.54	0.53	0.49	
	ϕ	−107	−103	−38	−45	33	
Background	$a\times10^3$	−0.25	−0.16	4.78	4.50	−0.68	
	$b\times10^3$	2.82	2.20	12.84	12.78	1.63	
	$a'\times10^3$	4.19	3.65	9.66	9.81	3.61	
	$b'\times10^3$	−0.49	−0.73	−1.25	−1.42	−2.66	
χ^2/N		3.4	3.2	1.9	1.7	3.0	

note that the transition is almost totally electric dipole. Finally, we allow for small changes in the resonance parameters W_r and Γ_r without stray states (solution E). Very little improvement is found by this approach. We have also found solutions similar to B, C, and D with small changes in the masses and widths of the known resonance states. The over-all χ^2 is better, but since N (= data points minus variable parameters) decreases the resulting χ^2/N is not improved and we do not show these results. The parameters for solutions A through E are tabulated in Table III, and graphical comparisons with the data are shown in Fig. 1 through Fig. 4 for solutions A, C, and D. Each solution in Table III is unchanged if all phase angles undergo the same transformation $\phi \rightarrow \phi+\psi$, where ψ is an arbitrary angle, provided the background parameters are also rotated by the same angle ψ. That is, the γ's, masses, widths, and χ^2 are invariant under such a rotation. It is there-

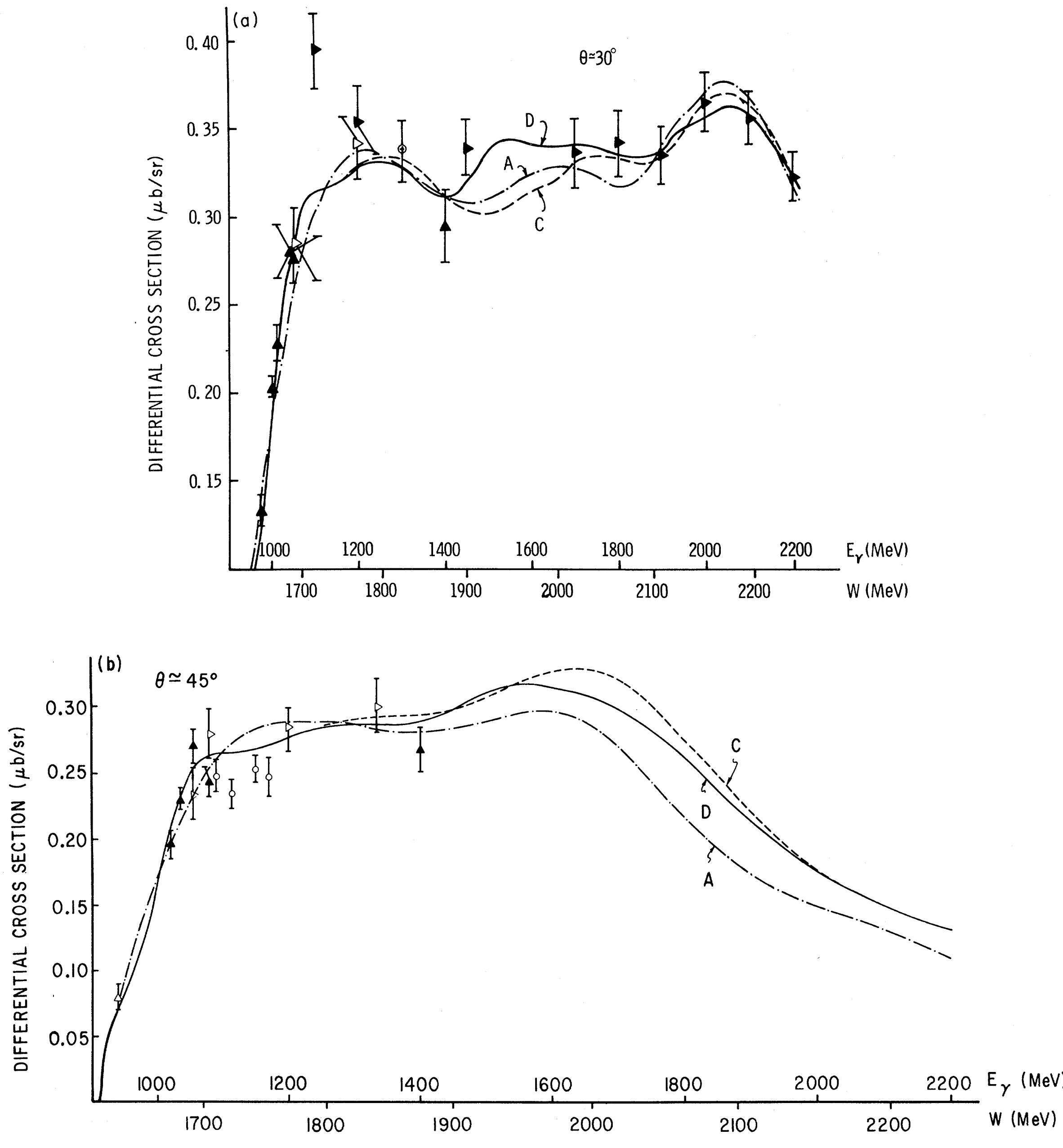

FIG. 1. *(Continued on the following page)*

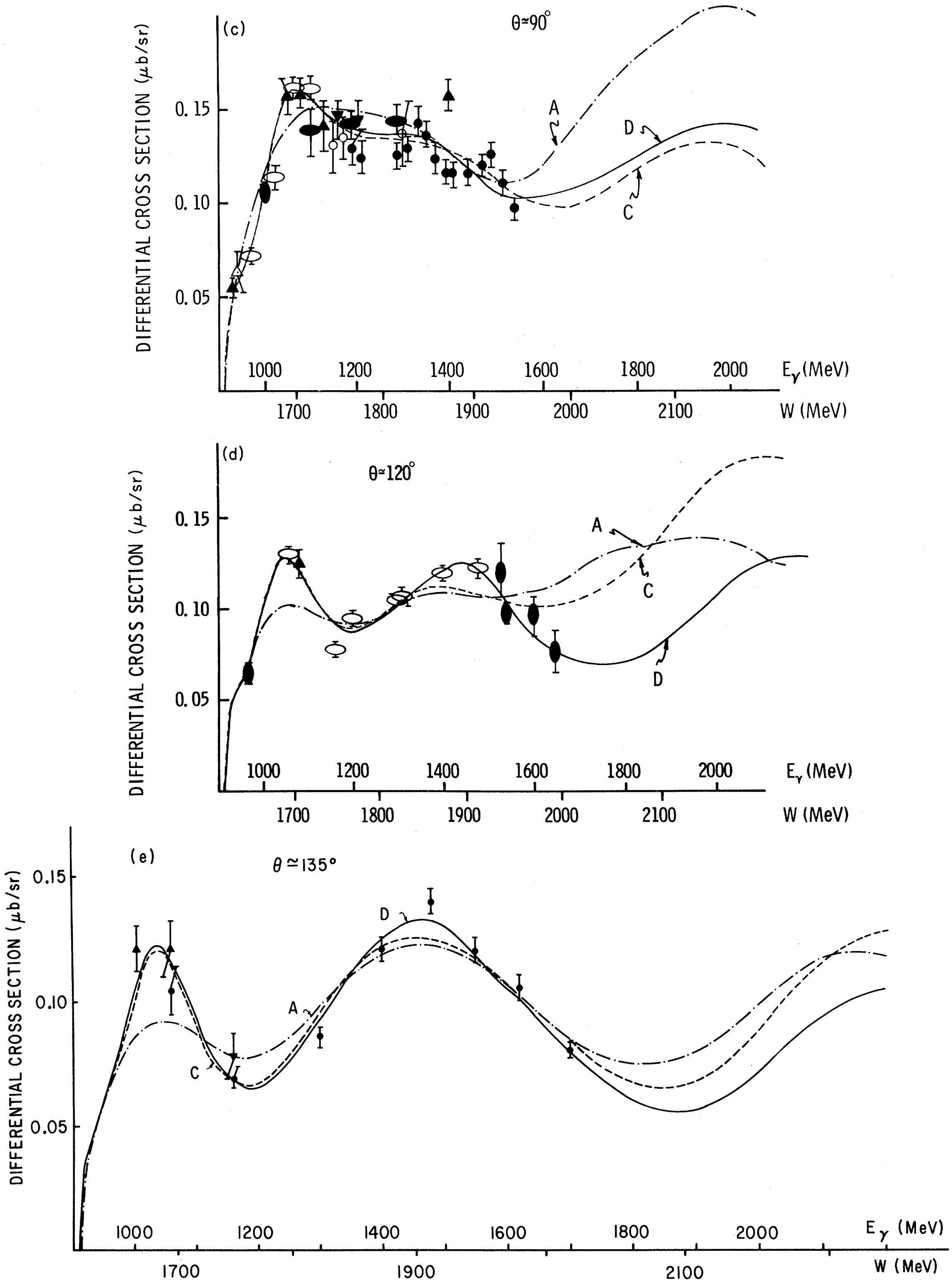

FIG. 1. Differential cross section as a function of energy at fixed c.m. scattering angle θ for solutions A, C, and D. The identification of data follows: ▲ R. L. Anderson *et al.*, ▷ C.W. Peck, ⬬ D. E. Groom and J. H. Marshall, ⊕ A. Bleckman *et al.*, △ F. M. Renard and Y. Renard, ○ T. Fujii *et al.*, ● H. Göing *et al.*, ■ M. Grilli *et al.*, ▼ A. J. Sadoff *et al.*, ▶ P. Feller *et al.*, ⬭ D. Décamp *et al.*, ⬮ Th. Fourneron.

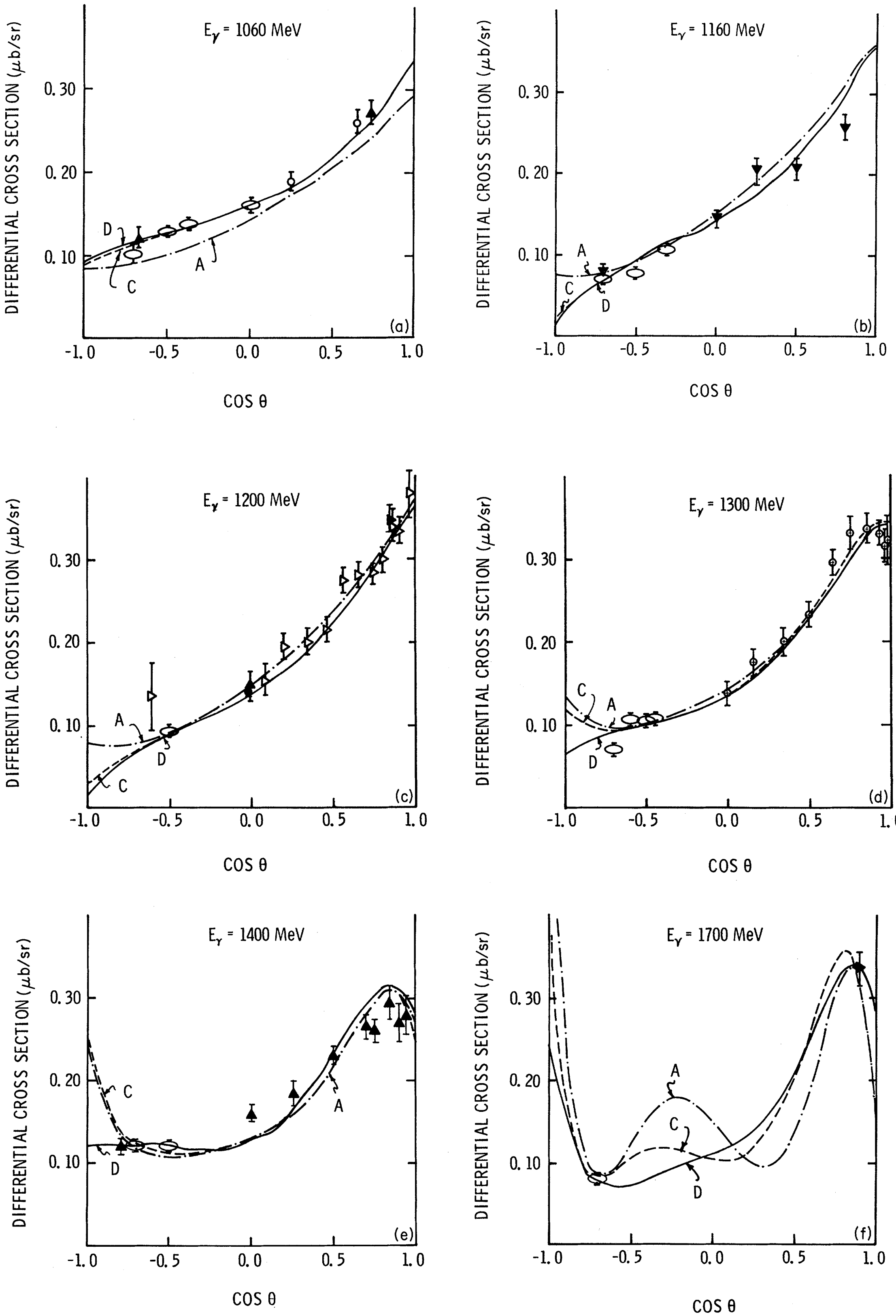

FIG. 2. Differential cross section as a function of θ at fixed energy.

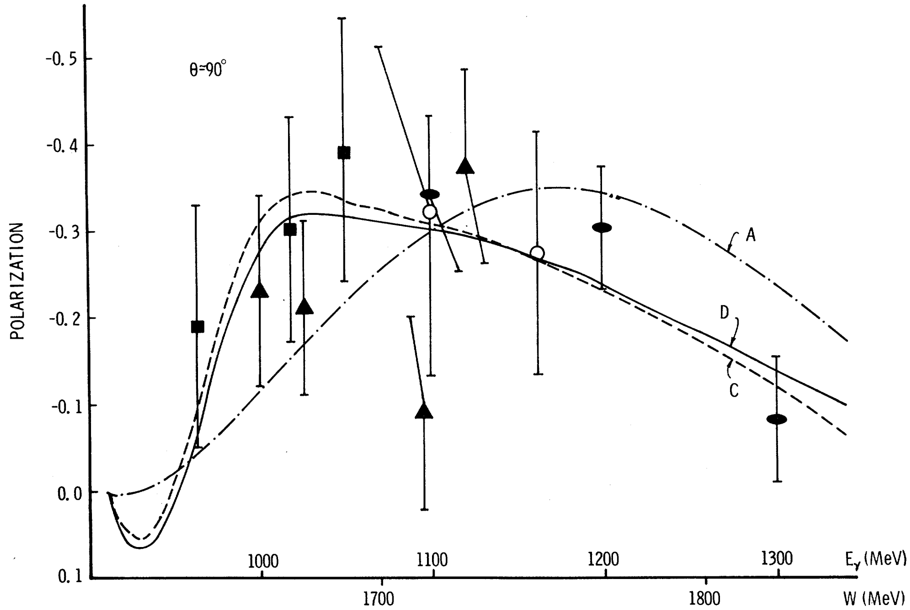

FIG. 3. Polarization of the Λ hyperon in direction $\hat{k} \times \hat{q}$ as a function of energy at fixed angle. The identification of the polarization data is the same as for Fig. 1 except for the solid triangle ▲ which now corresponds to H. Thom *et al.*

fore possible to choose one state as the standard and determine all phases relative to this standard.

In view of the need for the $D_{13}(1670)$ in fitting the current data, perhaps some comments are in order regarding the negative results we obtained in trying to replace the $D_{13}(1670)$ by other nearby states. It is clear that the $D_{15}(1670)$ cannot be used in lieu of the $D_{13}(1670)$. When such a replacement is attempted the best value of χ^2/N is greater than 3.

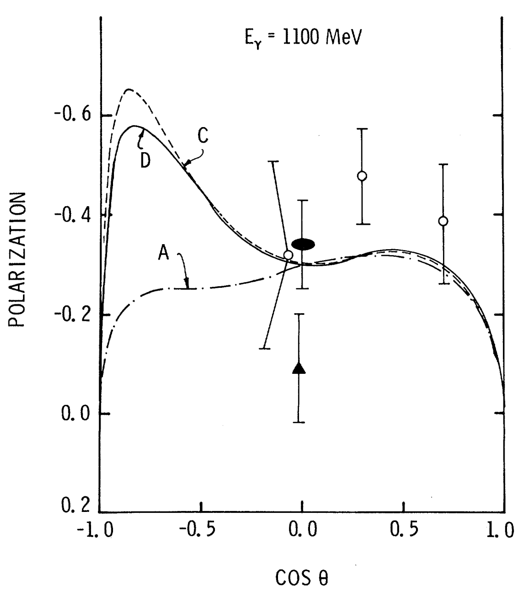

FIG. 4. Polarization of the Λ hyperon in direction $\hat{k} \times \hat{q}$ as a function of angle at fixed energy.

If the mass of the $S_{11}(1700)$ state is allowed to be around 1670 MeV the best value of χ^2/N is greater than 2.2, and the combination $S_{11}(1670)$ and $D_{15}(1670)$ is no better than the $S_{11}(1670)$ alone as a possible replacement. In all of these cases rather liberal ranges were allowed for the masses and widths of states in the 1670-MeV region. In addition to these cases, many random starting points for type-E solutions have been tried and none of these have been successful.

Hence we conclude that, although the most likely replacement for the $D_{13}(1670)$ is an $S_{11}(1670)$ with a width of around 100 MeV, the solution with the $D_{13}(1670)$ is significantly better. In addition, if one is to use the S_{11} with such a low mass and narrow width then these parameters must be explained since they are rather different from the values obtained in the compilations.[3] Consequently, we have some confidence in the need for the $D_{13}(1670)$ state which contributes almost entirely through an electric dipole transition.

We are not unaware of the perils of multiple minima in a multidimensional parameter space. However, the space has been well searched and within the framework of the parametrization used the need for the $D_{13}(1670)$ appears inescapable.

V. DISCUSSION OF RESULTS

Perhaps the most important result is the need for the $D_{13}(1670)$ resonance in obtaining a fit to the data. This state can be identified with the elusive $D_{13}(1700)$ reported earlier.[3,24] This D_{13} state fits very nicely in Dalitz's quark model[25,26] classification $^4\{8\}_{3/2}[\underline{70}, 1^-]_1$ provided we allow for mixing[22,23] between the $^4\{8\}$ and the $^2\{8\}$ states which occur within the $[\underline{70}, 1^-]_1$.

Since we have an acceptable solution (D) in which the masses and widths of the nucleon resonance states are close to those reported in other studies,[3] it is meaningful to calculate radiative widths for some of these states. We show these results in Table IV. Here, we have made some assumptions (see Table II) about the partial widths $\Gamma_{K\Lambda}$ in order to be able to calculate the radiative widths Γ_γ. Error estimates for these values of Γ_γ are around 40%; however, in view of the uncertainties (statistical, model dependence, and values for $\Gamma_{K\Lambda}$) this estimate may be optimistic. We have not calculated the radiative widths for some of the higher-mass states for two reasons. First, the values of $\Gamma_{K\Lambda}$ are either not known or not as well known; and second, the determination of the product $\Gamma_\gamma\Gamma_{K\Lambda}$ above 1900-MeV c.m. energy is much more likely to undergo changes in the future as more data become available.

Although we have extended the analysis out to 2200-MeV c.m. energy, the results at the higher energies are based upon very few data points as can be seen by looking at the figures. Thus, while the products of the partial widths are reasonably well determined (to about 15%) in the lower-energy region (below 1900 MeV), we must await further experimental results at the higher energies before placing too much confidence in the results above 1900 MeV. In particular, note that the data around $\theta_{c.m.} = 135°$ are fitted very nicely; however, we predict a third peak [Fig. 1(e)] for which there are no data for comparison.

The major difficulty at low energy seems to be our inability to fit the $E_\gamma = 1.1$ GeV point from Feller *et al.*[4] There is also difficulty with the $\theta_{c.m.} = 90°$, $E_\gamma = 1.4$ GeV point of Anderson *et al.*[5] We also have some difficulty, though not as serious, with the data of Fujii *et al.*[5] around $\theta_{c.m.} = 45°$. It is possible that these problems stem from normalization of data rather than inadequate solutions. For this reason, we have doubled the error bars on the 1.1-GeV and 1.4-GeV points in computing the final values of χ^2/N reported in Table III. These two points are the only ones we have altered from the reported values, and all minimizing was done with no alteration in these two points.

It is unclear just how one should compare our resonance couplings, or equivalently the $A_{1/2}$ and $A_{3/2}$ amplitudes, which have a phase (see the Appendix) and the ones determined by other methods where the phase was zero. This observation was also made by Moorhouse and Rankin[20] in their study of pion photoproduction. Perhaps the safest approach at present is to concentrate on radiative widths Γ_γ since the phase is not important there, as can be seen from Eq. (7). We thus compare our results with the radiative widths as calculated from the harmonic-oscillator quark model.[9,22,26-28]

If the $P_{13}(1860)$ state is placed in a $^2\{8\}_{3/2}[\underline{56}, 2^+]_2$ multiplet (see Ref. 28 for notation) then its radiative width is expected to be about 0.5 MeV and we find 1.0 MeV with an estimated error of around 40%. There is currently some controversy[23,27] about the proper classification for the $P_{11}(1750)$. Our value of $\Gamma_\gamma = 0.24$ MeV tends to favor either a $^2\{8\}_{1/2}[\underline{70}, 0^+]_2$ or a $^2\{8\}_{1/2}[\underline{56}, 0^+]_2$ assignment. In order to single out one of these, the sign of the $A_{1/2}$ amplitude must be determined with no ambiguity.[27]

The $S_{11}(1700)$ is usually placed in a $^4\{8\}_{1/2}[\underline{70}, 1^-]_1$ multiplet.[25] Both our result of $\Gamma_\gamma(S_{11}(1700)) = 0.27$ MeV and the value of $\Gamma_\gamma(S_{11}(1550)) = 0.42$ MeV from Ref. 11 can be understood in terms of mixing between the $^4\{8\}_{1/2}$ and the $^2\{8\}_{1/2}$ members of the $[\underline{70}, 1^-]_1$.[22,23,28] These values are in excellent agreement with Lipes's relativistic quark model[29] if a mixing angle of $\theta_s \simeq 37°$ (in the notation of Ref. 22) is assumed. In addition, if we assign the $D_{13}(1670)$ found in this analysis to a $^4\{8\}_{3/2}[\underline{70}, 1^-]_1$ multiplet then the same type of mixing must occur between this D_{13} and the $D_{13}(1520)$ which is believed to be properly assigned to a $^2\{8\}_{3/2}[\underline{70}, 1^-]_1$. The third and last nucleonic member of the $^4\{8\}_J[\underline{70},1^-]_1$ multiplet is the $D_{15}(1670)$. No mixing can occur for this state and its photoproduction is thus forbidden.[21] We find it rather interesting that in our analysis we must include precisely those members of the $^4\{8\}_J[\underline{70}, 1^-]_1$ multiplet for which photoproduction through mixing is allowed.

The remaining states used in this analysis are really not sufficiently well determined to warrant making statements about their radiative widths and possible multiplet assignments. Clearly, more experimental work, and analysis thereof, must be

TABLE IV. Approximate values for radiative widths from solution D.

State	$\Gamma_\gamma\Gamma_{K\Lambda}$ (MeV^2) (from D)	$\Gamma_{K\Lambda}$ (MeV) (from Table II)	Γ_γ (MeV) (calculated)
$S_{11}(1700)$	2.14	8	0.27
$P_{11}(1750)$	9.68	40	0.24
$P_{13}(1860)$	6.07	6	1.0
$D_{13}(1670)$	0.48		
$D_{13}(2040)$	3.21		
$F_{15}(1930)$	0.06		
$F_{17}(1990)$	0.65		
$G_{17}(2190)$	16.23		

done before a complete picture can be formed in regard to multiplet assignments.

VI. CONCLUSIONS

We have found rather good evidence for the existence of a $D_{13}(1670)$ nucleon resonance by studying the reaction $\gamma p \to K^+\Lambda$. The stray F_{15} state, though helpful, does not seem to be essential in view of the current level of the data. In addition, we find important contributions from the $S_{11}(1700)$, $P_{11}(1750)$, and $P_{13}(1860)$ states, with incompletely determined contributions from the $F_{17}(1990)$, $D_{13}(2040)$, and $G_{17}(2190)$ at higher energy. The radiative widths for the lower-mass states have been calculated and compared with model determinations. It is important to remember; however, that these values are only roughly determined at present and, as emphasized by Moorhouse and Rankin,[20] anyone wishing to use numbers from multipole analyses of photoproduction should do so keeping the problems in mind regarding the current level of the data as well as methods of parametrization.

ACKNOWLEDGMENTS

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APPENDIX

The relations among the $A_{1/2}$ and $A_{3/2}$ helicity amplitudes defined by Walker[9] and Copley *et al.*,[17] and the parameters used in this analysis are given by

$$A_{1/2} = \xi e^{i\phi}\{[v_n(Rk_r)]^{1/2}\alpha^{\pm}\gamma^{E} \pm [v_l(Rk_r)]^{1/2}\beta^{\pm}\gamma^{M}\} \quad \text{for } J = l \pm \tfrac{1}{2}, \tag{A1}$$

$$A_{3/2} = \xi e^{i\phi}\{[v_l(Rk_r)]^{1/2}\alpha^{\pm}\gamma^{M} \mp [v_n(Rk_r)]^{1/2}\beta^{\pm}\gamma^{E}\} \quad \text{for } J = l \pm \tfrac{1}{2}. \tag{A2}$$

Here, $n = l$ for $J = l + \frac{1}{2}$, $n = l - 2$ for $J = l - \frac{1}{2}$, and

$$\alpha^{\pm} = (l + \tfrac{1}{2} \pm \tfrac{3}{2})^{1/2}, \quad \beta^{\pm} = (l + \tfrac{1}{2} \mp \tfrac{1}{2})^{1/2}.$$

In (A1) and (A2) we have written

$$\xi = -R[2\pi W_r q_r v_l(Rq_r)]^{1/2}/(k_r M_N \Gamma_{K\Lambda})^{1/2}. \tag{A3}$$

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Fluctuations, Dips, and Duality*

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We examine data for backward elastic scattering of pions by protons. The width and frequency of the dips seen, as a function of energy, are strongly suggestive of Ericson fluctuations. The implications of this observation are discussed.

INTRODUCTION

According to the usual dual picture, elastic scattering is to be considered as the result of Pomeranchukon exchange plus Regge exchange which is dual to s-channel resonances. At high enough energies the resonances in $\pi^- p$ scattering overlap, and their contribution, plus the diffractive contribution, which itself is presumably smooth, makes for a differential cross section which becomes increasingly smooth as energy increases. It has been pointed out recently by Frautschi[1] that there may be regions of energy and scattering angle where fluctuation phenomena of a type well known in other branches of physics and especially in nuclear physics[2,3] exist. These fluctuation phenomena in nuclear physics arise from the superposition of a large number of Breit-Wigner amplitudes with some average spacing and width, but with the value of each fluctuating from the mean in a random way.

The statistical theory of nuclear reactions predicts the mean values of cross sections in terms of average level widths and spacings. The statistical models of Hagedorn[4] and Frautschi[5] predict average level densities for hadronic matter, and one should be able to find fluctuation phenomena in particle physics if these models have validity. A statistical model implies fluctuations. Some years ago these fluctuations were looked for in p-p large-angle scattering and not found. However, p-p scattering involves an exotic s channel, and hence this is not surprising.[1]

PION-NUCLEON SCATTERING

Pion-nucleon scattering with both signs of pion charge have nonexotic s channels and hence should exhibit fluctuation phenomena. If one looks in the few-GeV energy region, at backward angles where diffractive scattering is of minimal importance, one has a good chance of seeing fluctuation phenomena. Figure 1 shows $\pi^- p$ elastic-scattering data[6,7] at 180° in the c.m. energy range 1.5–2.6 GeV. Also shown are resonances listed in the Particle Data Group tables[8] which have been determined mainly by phase-shift analysis. The existing phase-shift fits fail to reproduce new data[6,7] at large angles near 180°. We have fitted these large-angle data with a superposition of Breit-Wigner amplitudes varying widths, angular momentum, and resonance energies. Our fit to the 180° data is shown in