

## Proton-Proton Polarization Data: Possible Evidence for Exotic Peripheral Resonances?\*

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(Received 3 January 1972)

We first draw attention to the important fact that proton-proton polarization is very similar to  $\pi^+$ -proton polarization for lab momenta 3 to 17.5 GeV/c. In particular, there is a *double zero* at  $|t| \approx 0.8$  (GeV/c)<sup>2</sup>. This  $pp$  polarization structure is in disagreement with the expectations of simple Regge-pole schemes. We discuss briefly some modifications in these schemes, and suggest the possibility of peripheral resonances even in exotic channels. We also show how the observed  $pp$ ,  $\pi p$ , and  $Kp$  polarizations may be interpreted in terms of an optical model.

### I. INTRODUCTION

During the past two or so years, several impressive schemes for explaining structures in differential cross sections and polarizations in hadron collisions have been developed.<sup>1</sup> One feature common to all these schemes, as they stand at present, is the anticipation of structureless polarization in proton-proton elastic scattering. We first wish to point out in this paper that there is now evidence that this prediction is in disagreement with the experimental situation. This observation is dramatically highlighted by the recent CERN polarization measurements<sup>2</sup> at 10, 14, and 17.5 GeV/c. Indeed, as illustrated in Sec. II,  $pp$  polarizations turn out to be very similar to  $\pi^+p$  polarizations, the interpretation of the latter having been considered an outstanding success in these schemes.

The disagreement with experiment in the  $pp$  case calls for an examination of the current schemes. In Secs. III, IV, and V, we discuss briefly various aspects of the problem from an impact-parameter point of view, and in particular suggest a modification of the idea of exoticity. We note however in Sec. VI that an alternative way of resolving the  $pp$  difficulty is to adopt a solution closer to that of an optical type of model. We then give interpretations (Sec. VII) of all the observed  $pp$ ,  $\pi p$ , and  $Kp$  polarization structures in terms of this latter model. The  $K^+p$  polarization in the region  $0.5 \leq |t| \leq 1.5$  (GeV/c)<sup>2</sup> serves as a further distinctive test between the two approaches.

### II. PROTON-PROTON POLARIZATION DATA; PREDICTIONS OF THE REGGE-POLE SCHEME

Experimental proton-proton polarizations are not structureless as has been anticipated for some time, but in fact turn out to exhibit persistent structure all the way from  $p_{\text{lab}} = 3$  to 17.5 GeV/c.<sup>3,4</sup>

To illustrate this important point, we have drawn in Figs. 1(a) and 1(b) the experimental  $pp$  polarizations at 5.15 and 10 GeV/c, respectively. For comparison, we have included the more familiar  $\pi^+p$  polarizations at these momenta. As can be seen immediately from Fig. 1,  $pp$  and  $\pi^+p$  polarizations not only have similar shapes, but are comparable in magnitude. In particular, just as the  $\pi^+p$  polarization has a double zero around  $|t| \approx 0.6$  (GeV/c)<sup>2</sup>, so has the  $pp$  polarization a double zero around  $|t| \approx 0.8$  (GeV/c)<sup>2</sup>. Moreover, this double zero remains more or less at the same position  $|t| \approx 0.8$  (GeV/c)<sup>2</sup> for  $p_{\text{lab}} = 3$  to 17.5 GeV/c, and is therefore a systematic of proton-proton scattering.

More structure is also indicated at larger  $|t|$ . For example, in the 10-GeV/c data shown in Fig. 1(b), two further zeros possibly occur at  $|t| \approx 2.0$  and 2.7 (GeV/c)<sup>2</sup>.

That  $pp$  polarization has structure for  $|t| \leq 1.5$  (GeV/c)<sup>2</sup> is surprising from the point of view of present Regge schemes,<sup>1</sup> since these schemes have been particularly successful in explaining other polarizations. For example, the double zero at  $|t| \approx 0.6$  (GeV/c)<sup>2</sup> in  $\pi^+p$  elastic polarizations (see Fig. 1 and the Appendix) arises naturally from the Regge pole term  $\text{Ref}_{+-}(\rho)$ , provided that one at the same time takes the other important amplitude, the imaginary Pomernanchuk term, as structureless throughout this  $t$  region [ $|t| \leq 1.5$  (GeV/c)<sup>2</sup>, say]. The same theory<sup>1</sup> however predicts the  $pp$  polarization to be structureless in this region. The  $pp$   $s$  channel is exotic, and therefore contains no rotating Regge piece. Thus the  $pp$  polarization, which is given essentially by

$$\mathcal{P}(pp) \sim -\text{Im}[(\Phi_1 + \Phi_3)\Phi_5^*],$$

where  $\Phi_1, \Phi_3$  are the helicity-nonflip amplitudes and  $\Phi_5$  the single-flip amplitude, will be structureless because it is the product of a smooth imag-

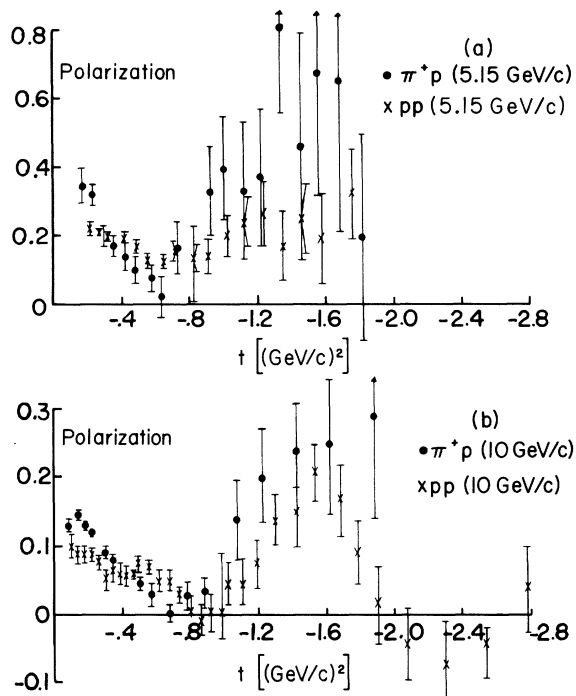


FIG. 1. Proton-proton and  $\pi^+$ -proton polarizations at (a) 5.15 GeV/c, (b) 10 GeV/c (Refs. 2, 3, 4). Note the similarity of structure in the  $pp$  and  $\pi^+p$  cases.

inary Pomeranchuk term and a smooth real  $s^{\alpha(t)}$  term.

We will now examine briefly the cause of this disagreement between the prediction and the experiment.<sup>5</sup>

### III. DIAGNOSIS: LACK OF A PERIPHERAL COMPONENT

The observed double zero in  $pp$  polarization is unlikely to be due to some fortuitous cancellation of amplitudes, even although there are five of them. Rather, it is more likely to be due to some specific characteristic of  $pp$  scattering which is presumably missing in the present Regge approach to the  $pp$  situation.

The Regge-pole prediction of structureless polarization can be understood as arising from the absence of high partial waves in the parametrization used – by high partial waves, we mean those (peripheral) waves at c.m. momentum  $k$  with impact parameter  $b = l/k$  centered around 0.8–1.0 F. By taking the Fourier-Bessel transform, one immediately finds<sup>6</sup> that both the Pomeranchuk piece and the  $s^{\alpha(t)}$  piece are almost pure Gaussian in  $l$  and  $b$ , centered around  $b \approx 0$ . The higher partial waves come in only as the small tails of these Gaussians, thus making it evident why these amplitudes are so smooth and structureless in  $t$ .

It is illuminating to compare, at this impact-parameter level, the above  $pp$  situation with the usual Regge argument for the  $\pi^+p$  case. There, a rotating Regge part  $e^{-i\pi\alpha(t)}s^{\alpha(t)}$  is allowed. It is this piece (as may be understood from its oscillating sine and cosine parts) which contains appreciable high-partial-wave components,<sup>6,7</sup> and these are centered around  $b = R \approx 0.8-1.0$  F for a Regge trajectory slope  $\alpha' \approx 1$ .<sup>6</sup> The contribution of this peripheral band to the  $\rho$  helicity-flip amplitude gives<sup>7</sup>

$$\text{Ref}_{+-}^s(\text{peripheral } \rho) \sim J_1(R\sqrt{-t}),$$

which has a zero in the vicinity of  $|t| \approx 0.6$  (GeV/c)<sup>2</sup>. Whether one gets a single zero or a double zero (the latter being favored in the Regge-pole scheme) in the complete amplitude  $\text{Ref}_{+-}^s(\rho)$  depends on the proportion of the accompanying low- $b$  components in this amplitude. It is important to remember however that the structure in the Regge  $\pi p$  amplitude arises basically from the substantial high-partial-wave components.

It seems natural to conclude therefore that the similarity of the experimental  $\pi^+p$  and  $pp$  polarizations indicates that a significant higher-partial-wave component (from  $b \approx 0.8$  to 1 F) is present also even in the exotic  $pp$  amplitudes.

### IV. POSSIBLE REMEDIES IN THE REGGE SCHEME

Before turning to some consequences of a peripheral piece in  $pp$  amplitudes, let us consider possible remedies within the Regge scheme to account for the double-zero structure. It will be recalled that, in Regge-pole theory, the real part of  $\Phi_5$  is given by

$$-\text{Re}\Phi_5 \sim (\beta_{P'} + \beta_\omega)s^\alpha + (\beta_{P'} - \beta_\omega)s^\alpha \cos\pi\alpha,$$

where  $P'$  and  $\omega$  stand respectively for the even-signature ( $P', A_2, \dots$ ) and odd-signature ( $\omega, \rho, \dots$ ) contributions. With exchange degeneracy (EXD), the residues have no poles or zeros at  $\alpha=0$  and  $\beta_{P'} = \beta_\omega$ , and a structureless  $pp$  polarization follows.<sup>8</sup>

As a simple way out of this difficulty, one might ask whether, by allowing EXD breaking but still keeping a smooth Pomeranchuk contribution and smooth residue functions without zeros or poles, the  $\text{Re}\Phi_5$  can develop a double zero. From the expression above for  $\text{Re}\Phi_5$ , we see that a double zero can indeed be easily obtained at two places: (i) at  $\alpha=0$ ,  $|t| \approx 0.5$  (GeV/c)<sup>2</sup> if  $\beta(t)_{P'} = 0$ , and (ii) at  $\alpha=-1$ ,  $|t| = 1.5$  (GeV/c)<sup>2</sup> if  $\beta(t)_\omega = 0$ . However, both of these solutions are unsatisfactory: The first solution has the consequence that  $pp$ ,  $\bar{p}p$  polarizations would be mirror images (like  $\pi^+p$ ), which is not the case experimentally,<sup>2</sup> while the second so-

lution has the double zero in the wrong place. Thus it would appear that a more complicated type of remedy is required.

Regge cuts are of course another way of obtaining structure in the  $pp$  polarization. Cuts unfortunately reduce the predictive power of Regge schemes, but seem to be necessary in a number of other places.<sup>1</sup> In the present  $pp$  case, it is difficult to see the precise mechanism through which the cuts would produce the systematic  $|t|=0.8$  (GeV/c)<sup>2</sup> structure; some quantitative work seems necessary. However, whatever the mechanism, it would appear from the almost equal magnitudes of the  $pp$  and  $\pi p$  polarizations that one is faced with the unpleasant necessity of introducing modifications into the exotic  $pp$  amplitudes which are as big as the usual nonexotic  $\rho$ -pole contributions in  $\pi p$  elastic scattering.

#### V. A MODIFICATION TO THE PRESENT IDEA OF EXOTICITY

Rather than pursue the effect of cuts, etc., we wish to examine in this paper the consequences of having a peripheral piece in  $pp$  scattering. Since the Regge-pole scheme does not allow such a piece, it would appear that the present ideas of exoticity and strict EXD have to be relaxed. In this regard, it might be useful to introduce two different kinds of resonant effects, just as is often done in nuclear physics:

(i) The compound-nucleus type of resonance where, roughly speaking, the interacting particles lose their identity in the collision; the "nucleus" subsequently decays. (Possibly a quark excitation picture would be appropriate for this type of resonance.)

(ii) The peripheral or quasimolecular type of resonance, corresponding to a rotational level sequence  $l=kR$  between the colliding particles, like a revolving dumbbell. It was Sommerfeld<sup>9</sup> who first showed that the appropriate classical parametrization of this was (what is now called) an  $s$ -channel Regge pole,<sup>6,7</sup> of the form  $N/(l-l_0 - \frac{1}{2}i\Gamma)$ , with  $l_0=kR$ . The amplitude has a simple resonance pole in the  $k$  plane. The value of  $R$  is just the typical range of the forces, equal to about 0.8 to 1 F for strong interactions.

It seems perfectly consistent with all the experimental data to apply the idea of exoticity only to the "compound-nucleus" type of resonances: There are indeed no low-mass  $pp$  (or  $K^+p$ ) resonances. However, one should probably not exclude the possibility of peripheral resonances, even for a process such as  $pp$  scattering. In this way, one can proceed to interpret polarization structures in all reactions.

#### VI. AN ALTERNATIVE APPROACH

There is of course a well-known model which already contains the two most important ingredients, that is, diffraction and a peripheral effect, which seem to be necessary in high-energy scattering. This is the optical model. The diffraction amplitude in this model does have structure, however, in contrast to the usual Pomeranchuk amplitude. One of the reasons often given for taking the Pomeranchuk amplitude structureless<sup>1</sup> is that the experimental  $pp$  differential cross section is smooth. However, the experimental situation for this has changed recently<sup>10,11</sup>: Above 10 GeV/c, a break begins to develop in  $d\sigma/dt$  at  $|t|\approx 1$  (GeV/c)<sup>2</sup>; by 24 GeV/c the break has become very abrupt, thus substantially weakening the argument for a smooth Pomeranchuk contribution. Even below 10 GeV/c, where  $d\sigma/dt$  is indeed smooth, one could conclude that the imaginary Pomeranchuk contribution is likewise smooth only if it were definitely known to be the dominant amplitude throughout [for at least, say,  $|t|\lesssim 1.5$  (GeV/c)<sup>2</sup>]. That it is the dominant amplitude much beyond the very forward direction is highly questionable. It is known from experiment<sup>11</sup> that  $pp$  scattering has an unusually large *real* part (about 30–40% of the imaginary part at  $t=0$ ) at these lower momenta. It is therefore by no means certain whether the imaginary part will be much bigger than the real part beyond, say,  $|t|\approx 0.4$  (GeV/c)<sup>2</sup>. (This latter statement may also be made about  $K^+p$  scattering, which also has a large real part.<sup>12</sup>) Most of all, however, the persistent structure in the  $pp$  polarization for all momenta above 3 GeV/c surely indicates a much richer amplitude structure than might be guessed from the differential cross section alone.

With these remarks in mind, it might be worthwhile at this stage to consider the possibility of the diffraction amplitude having structure (zeros), such as in a straightforward optical model.<sup>13</sup> Actually the difference between the two situations is by no means great: The essential difference is that the Pomeranchuk amplitude has a greater proportion of low partial waves (low  $b$ ) than the corresponding optical-model amplitude.

At present there in fact seems to be no concrete evidence against the optical model possibility. Various attempts<sup>14</sup> have been made in the past to clarify this question of structure by using finite-energy sum rules (FESR) for pion-nucleon scattering; however, no definite conclusion can be drawn from these FESR. Some reasons for this are evident at the outset: For example, (i) FESR becomes less reliable the further one goes from the forward direction; (ii) the  $S$ - and  $P$ -wave contributions are sizable, but these waves are

rather poorly determined and unreliable (this is particularly important since, as mentioned above, the major difference between the Pomeranchuk and optical amplitudes is the proportion of low partial waves); (iii) since the phase shifts are determined from differential cross sections and polarizations only, there is an arbitrary  $t$ -dependent phase  $e^{i\theta(t)}$  between the amplitudes. Likewise, the recent analysis by Halzen and Michael<sup>15</sup> to obtain the  $\pi N$  amplitudes at 6 GeV/c for  $|t| \lesssim 0.6$  (GeV/c)<sup>2</sup> is insufficient to decide the issue. One can therefore conclude that whether the diffraction amplitude has structure or not is still very much an open question.

### VII. INTERPRETATION OF POLARIZATIONS IN AN OPTICAL MODEL

In this section we wish to show how one can give interpretations of elastic polarizations by means of an optical model. The interpretations are fairly straightforward. If the scattering takes place from a sharp absorbing region of radius  $R$ , the diffraction amplitude is of the form  $iJ_1(x)/x$ , with  $x = R\sqrt{-t}$ . One therefore anticipates polarizations of the following two types.

(a) If the (real part of the) helicity-flip amplitude has no *net* strong peripheral piece, then

$$\mathcal{P} \sim J_1(x)/x,$$

with single zeros at the zeros of the Bessel function  $x=3.8, 7.0, \dots$ . How far out we can go in  $x$  depends on how far in  $t$   $\text{Im}(\text{nonflip}) \times \text{Re}(\text{flip})$  continues to give the most important contribution to the polarization.

(b) If the flip amplitude does have a strong peripheral piece, then

$$\mathcal{P} \sim \frac{J_1(x)}{x} J_1(x),$$

with *double* zeros at  $x=3.8, 7.0, \dots$ .

We immediately note that the  $pp$  polarization is of type (b). The double zero at  $|t| \approx 0.8$  (GeV/c)<sup>2</sup> implies a radius  $R \approx 0.8$  F. One therefore expects the next double zero to occur at  $|t| \approx 2.6$  (GeV/c)<sup>2</sup>. Surprisingly enough, the  $pp$  polarization does have two zeros in this region, at  $|t| \approx 2.0$  and 2.7 (GeV/c)<sup>2</sup>, as seen in Fig. 1(b). It is extremely tempting to associate these latter experimental zeros with the double zeros at  $|t| \approx 2.6$  (GeV/c)<sup>2</sup> anticipated from the rough formula; one then has a simple interpretation of the polarization structure not just for small  $|t|$ , but for  $|t|$  probably all the way out to 3 (GeV/c)<sup>2</sup>.

Likewise,  $\pi^+p$  polarizations<sup>2</sup> are of type (b), their approximate mirror symmetry implying that the flip peripheral piece has  $I^t = 1$ , as in the Regge

model. The double zero at  $|t| \approx 0.6$  (GeV/c)<sup>2</sup> corresponds to radius<sup>16</sup>  $R \approx 0.9$  F, and one expects a second double zero around  $|t| \approx 2.3$  (GeV/c)<sup>2</sup>. We note that where measured, both  $\pi^+p$  polarizations do seem to return to zero again around  $|t| \approx 2$  (GeV/c)<sup>2</sup>, as in the  $pp$  case. From this optical model, we expect the  $\pi^+p$  and  $pp$  polarizations to be rather similar for  $|t| \lesssim 3$  (GeV/c)<sup>2</sup>.

Experimentally, the  $K^-p$  polarization<sup>2</sup> has a single zero at  $|t| \approx 1$  (GeV/c)<sup>2</sup> and thus is of type (a), with radius  $R \approx 0.75$  F. This implies that  $K^-p$  either has no peripheral piece to start with, or else has two peripheral pieces (possibly one in each of the two  $s$ -channel isospin amplitudes) which substantially cancel. From the  $K^-p$  polarization alone, one cannot distinguish between these two possibilities. The next single zero is expected at  $|t| \approx 3$  (GeV/c)<sup>2</sup>.

The  $K^+p$  polarization<sup>2</sup> now turns out to play an important role in further deciding between the two models. The Regge scheme again predicts no structure for this exotic process, as in the  $pp$  case. From an optical model, one would expect *either* a single zero or a double zero, corresponding to type (a) or (b), respectively. Unfortunately, the experimental measurements stop around  $|t| \approx 1$  (GeV/c)<sup>2</sup>, so that no conclusions can as yet be drawn. In Fig. 2, we illustrate the situation with the  $K^+p$  polarization<sup>2</sup> at 14 GeV/c. It is impossible to rule out any of the possibilities – no structure, a single zero, or a double zero around  $|t| \approx 1$  (GeV/c)<sup>2</sup>. If any structure at all shows up here, this would be further evidence that the idea of exoticity needs substantial modification.

A comparison of the Regge predictions and the experimental situation, together with optical model interpretations, is summarized in Table I.

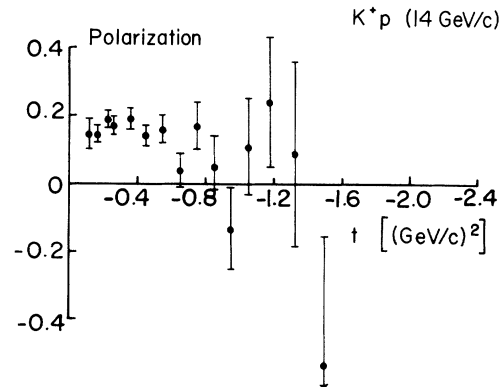


FIG. 2.  $K^+$ -proton polarization at 14 GeV/c (Ref. 2). A crucial test between Regge and optical models: Is there no structure, a single zero, or a double zero at  $|t| \approx 1$  (GeV/c)<sup>2</sup>?

TABLE I. Predictions of Regge scheme for  $\pi^+p$ ,  $pp$ , and  $K^+p$  polarizations for  $|t| \lesssim 3$  (GeV/c)<sup>2</sup>, compared to the experimental data. We also list optical-model interpretations of the observed experimental structures.

Elastic reaction	Regge scheme (smooth Pomeranchuk contribution)	Experiment (observed)	Optical-model interpretation		
			Type	Radius	Structure
$\pi^+p$	Double zeros at $ t =0.5, 2.5$ (GeV/c) <sup>2</sup>	Double zero at $ t =0.6$ (GeV/c) <sup>2</sup>	(b)	0.9 F	Double zeros at $ t =0.6, 2.3$ (GeV/c) <sup>2</sup>
$K^-p$	Single zeros at $ t =1, 2, 3$ (GeV/c) <sup>2</sup>	Single zero at $ t =1$ (GeV/c) <sup>2</sup>	(a)	0.75 F	Single zeros at $ t =1, 3$ (GeV/c) <sup>2</sup>
$pp$	No structure	Double zero at $ t =0.8$ (GeV/c) <sup>2</sup> ; Zeros at $ t =2.0, 2.7$ (GeV/c) <sup>2</sup>	(b)	0.8 F	Double zeros at $ t =0.8, 2.6$ (GeV/c) <sup>2</sup>
$K^+p$	No structure	Uncertain; possible structure at $ t =1$ (GeV/c) <sup>2</sup>	(a) (b)	0.75 F 0.75 F	Single zeros at $ t =1, 3$ (GeV/c) <sup>2</sup> Double zeros at $ t =1, 3$ (GeV/c) <sup>2</sup>

### VIII. CONCLUDING REMARKS

In this paper, we have drawn attention to the fact that  $pp$  polarization has systematic structure, in particular a double zero at  $|t| \approx 0.8$  (GeV/c)<sup>2</sup>. This seems to imply that a peripheral piece is necessary in the  $pp$  amplitudes, coming from a radius  $R \approx 0.8$  F, and indicates that the strict idea of exoticity has to be relaxed. We suggested in particular the possibility of peripheral resonances even in exotic channels.

We also pointed out that one can have simple interpretations of  $pp$ ,  $\pi^+p$ , and  $K^+p$  polarizations from the point of view of an optical model. It is of special importance to find out whether the  $K^+p$  polarization has structure or not in the vicinity of  $|t| \approx 1$  (GeV/c)<sup>2</sup>: In this way, one might be able to distinguish further between the two models.

A systematic fitting of all the available data has of course still to be done. As regards the fitting, we would emphasize that it is important that all the data be used, including scattering at large angles. As mentioned in Sec. VI, the basic difference between present Regge models and the not-so-different optical model is the proportion of the lower partial waves in the various amplitudes. Forward polarization structures are particularly useful in indicating the proportion of peripheral waves, but it is the large-angle differential cross sections which place a severe constraint on the allowable amount of low partial waves. From the discussion here, it would appear that the optical model presents a viable approach to high-energy scattering.<sup>17</sup>

### ACKNOWLEDGMENTS

The authors would like to acknowledge helpful discussions with Professor A. Dar, Professor D. Lichtenberg, Professor K. McVoy, and Professor R. Weiner.

### APPENDIX

In this appendix, we outline the Regge case<sup>1</sup> for the  $\pi^+p$  elastic scattering and indicate briefly the effects of EXD breaking and cuts. The elastic  $\pi^+p$  polarization structure is usually explained by a double zero in  $\text{Re}f_{+-}^t(\rho)$ ; this arises essentially from the following assumptions:

(a) Assume an asymptotic Regge-pole power-law behavior  $s^{\alpha_\rho(t)}$ . It then follows from the Phragmén-Lindelöf theorem<sup>18</sup> (or crossing and analyticity) that the amplitude  $f_{+-}^t(\rho)$  must have the *asymptotic* phase as given in the Regge signature factor

$$f_{+-}^t(\rho) \sim \beta_\rho(t)(1 - e^{-i\pi\alpha_\rho(t)})s^{\alpha_\rho(t)}.$$

So far, no definite statement can be made about zeros, since the pole structure of  $\beta_\rho(t)$  is as yet unspecified.

(b) If EXD is also assumed for Regge residues (supported by the usual argument<sup>1</sup> of duality and absence of exotic resonances),  $\beta_\rho(t)$  cannot have a pole at  $\alpha_\rho(t) = 0$  since  $\beta_{A_2}$  is finite at  $\alpha_{A_2} = \alpha_\rho = 0$ . It is then taken that  $\text{Re}f_{+-}^t(\rho)$  has a double zero at  $\alpha_\rho(t) = 0$ , since

$$\text{Re}f_{+-}^t(\rho) \sim 1 - \cos\pi\alpha_\rho(t) = 2 \sin^2\left[\frac{1}{2}\pi\alpha_\rho(t)\right].$$

(c) Assume a structureless Pomeranchuk

amplitude.

These three assumptions then lead to a plausible explanation of the double-zero and mirror-image character of  $\pi^+p$  polarization<sup>19</sup> (near the forward direction, the  $s$ - and  $t$ -channel amplitudes are approximately equal).

However, the same set of assumptions lead to the prediction of structureless  $pp$  polarization, in disagreement with experiment. Let us examine briefly how EXD breaking and cuts affect the above argument.

With EXD breaking, the strict connection between the asymptotic phase and the number of zeros in  $\text{Ref}_{+-}^t(\rho)$  is relaxed; the pole structure of  $\beta_\rho^t$  is then no longer determined. This brings us back to the preduality Regge scheme with its many different ghost-eliminating possibilities (see, for example, Ref. 20). For example, if

$$\beta_\rho(t) \sim \frac{\gamma(t)}{\Gamma(1 + \alpha) \sin \pi \alpha(t)},$$

then  $\text{Ref}_{+-}^t(\rho)$  will have only a single zero at  $\alpha_\rho(t) = 0$ . The  $\pi^+p$  polarization can still be fitted, of course, by simultaneously dropping assumption (c) and allowing zeros in the diffraction amplitude.

The introduction of cuts further complicates the issue. In particular, it leads to a different phase:

$$f_+^t(\rho \text{ cuts}) \sim \beta_c(t) \left[ \frac{1}{(\ln s)^\lambda} - \frac{e^{-i\pi\alpha c}}{[\ln(-s)]^\lambda} \right] s^{\alpha c(t)}.$$

This may be particularly important if the amplitudes are nowhere near their asymptotic forms at present accelerator energies.

Thus, if one allows EXD breaking and cuts (as seems necessary in  $pp$  scattering, and there is no *a priori* reason why they should not be important in  $\pi^+p$  scattering also), the situation becomes more complicated and the predictive power of the Regge scheme is greatly reduced. It also opens up the possibility of structure in the diffraction amplitude.

\*Work supported in part by the Atomic Energy Commission under Contract No. AT(11-1)-2009B.

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<sup>2</sup>M. Borghini *et al.*,  $\pi^+p$ : Phys. Letters **36B**, 493 (1971);  $K^+p$  and  $\bar{p}p$ : *ibid.* **36B**, 497 (1971);  $pp$ : *ibid.* **36B**, 501 (1971).

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<sup>4</sup>N. E. Booth, G. Conforto, R. J. Esterling, J. Parry, J. Scheid, D. Sherden, and A. Yokasawa, in *High Energy Collisions*, Third International Conference held at the State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

<sup>5</sup>Indications that the usual Regge scheme might have difficulties in baryon-antibaryon scattering have been pointed out in the literature. See for example J. Rosner, Phys. Rev. Letters **21**, 950 (1968).

<sup>6</sup>A. W. Hendry, Phys. Rev. D **5**, 215 (1972).

<sup>7</sup>S. Y. Chu and A. W. Hendry, Phys. Rev. D **4**, 2743 (1971); **4**, 3282 (1971).

<sup>8</sup>A single zero at  $|t| \approx 1.0$  (GeV/c)<sup>2</sup> is simultaneously predicted in  $\bar{p}p$  polarization.

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<sup>10</sup>J. V. Allaby *et al.*, Phys. Letters **34B**, 431 (1971); **28B**, 67 (1968); **27B**, 49 (1968); **25B**, 156 (1967).

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<sup>16</sup>There is no reason why the radius  $R$  should be the same for all elastic processes. Indeed it is unlikely that they are all exactly equal, since the slopes of the forward elastic diffraction peaks for  $pp$ ,  $\pi^+p$ , and  $K^+p$  are all different. For example,  $K^+p$  forward peaks are not so steep as the  $\pi^+p$  peaks, indicating that the effective radius for  $Kp$  scattering is smaller than that for  $\pi p$  scattering; the estimates of the radii from polarization agree with this expectation.

<sup>17</sup>A report with fits to  $pp$  scattering at all angles, based on the general optical-model ideas discussed here, is in preparation [T. Y. Cheng, S. Y. Chu, and A. W. Hendry (unpublished)]. Optical-model fits to  $\pi N$  scattering are already published (Ref. 7).

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