σ Term in Pion-Nucleon Scattering

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When an error in earlier work is corrected, it is shown that the requirements of t-channel unitarity *do not* affect the validity of the extrapolation procedure of Cheng and Dashen. This conclusion also holds in an extension of our earlier analysis to the case where the σ field transforms as a member of the $(N/2, N/2)$ representation of chiral $SU(2)\times SU(2)$.

It was reported in an earlier communication[1] that the method of Cheng and Dashen[2] provided a calculation of approximately three times the σ term, rather than the σ term itself. In fact, that conclusion is incorrect. When an error in normalization in Ref. 1 is corrected,[3] we find that the method of Cheng and Dashen does indeed calculate the σ term from on-shell pion-nucleon scattering data. The proper conclusion to be drawn from our study is that the requirements of t-channel unitarity do not affect the validity of the extrapolation procedure of Cheng and Dashen.

Because of an unfortunate oversight, the choice of normalization of the σ propagation in Ref. 1 is not consistent with the normalization used in our work on the unitarization of the scattering amplitude.[4] In Ref. 4 we defined $\Delta_0(s)$ to be the two-point function for the scalar field σ^{ab}, projected to isospin 0, while in Eq. (22) of Ref. 1 we defined $\Delta_0(s)$ to be the propagator of the field.

$$\sigma^{ab}(x) = \delta^{ab}\sigma(x)\,. \tag{1}$$

The two definitions are related by

$$[\Delta_0(s)]_{\text{Ref. 4}} = \tfrac{1}{3}[\Delta_0(s)]_{\text{Ref. 1, Eq.(23)}}\,. \tag{2}$$

The analysis of Secs. III B and IV of Ref. 1 used the normalization of Ref. 4, rather than that of Eq. (23), Ref. 1. If one uses a consistent normalization throughout, the disagreement with Ref. 2 disappears.

We take this opportunity to extend our analysis to include a σ term which is a member of the $(N/2, N/2)$ representation of chiral $SU(2)\times SU(2)$. (In Ref. 1 we only considered the case $N=1$.) For consistency we shift all normalizations to that of Ref. 4. The σ field is defined by the equal-time commutator

$$[A_0^a(x), \partial^\mu A_\mu^b(0)]\delta(x^0) = \sigma^{ab}(x)\delta^4(x)\,, \tag{3}$$

and the propagator[4]

$$\begin{aligned}\Sigma(s)^{ab,cd} &= \int d^4x\, e^{-iq\cdot x}\langle 0|T\{\sigma^{ab}(x)\sigma^{cd}(0)\}|0\rangle \\ &= N^2(N+2)^2\Delta_0(s)[P_0]^{ab,cd} + [\tfrac{2}{5}(N-1)(N+3)]^2\Delta_2(s)[P_2]^{ab,cd}\,.\end{aligned} \tag{4}$$

The Ward identities for πN scattering again lead to Eq. (15) of Ref. 1, where now

$$F_N(t)\delta^{ab} = i\langle p_2|\sigma^{ab}(0)|p_1\rangle\,. \tag{5}$$

Separating the one-particle reducible part of the σ field from the πN amplitude leads to the on-shell representation

$$T(\nu, \nu_B; m_\pi^2, m_\pi^2)^{(+)} = \text{nucleon pole} - \frac{m_\pi^2}{N(N+2)}[1-F_0(t)]\Delta_0(t)^{-1}F_N(t) + F_\pi^{-2}q^\mu k^\nu \tilde{T}^{(+)}_{\mu\nu}|_{q^2=k^2=m_\pi^2}\,. \tag{6}$$

The application of unitarity to the $\pi\pi$ scattering amplitude gives[4]

$$\begin{aligned}[F_0(t)-1]\Delta_0(t)^{-1} &= R_0(t) \\ &= -m_\pi^{-2}F_\pi^{-2}[N(N+2) - 2(2-t/m_\pi^2)]\,,\end{aligned} \tag{7}$$

so that for sufficiently small ν and ν_B

$$M(\nu, \nu_B; m_\pi^2, m_\pi^2)^{(+)} \simeq \frac{g_{\pi N}^2}{m}\left(\frac{\nu_B^2}{\nu_B^2-\nu^2}\right) - \frac{1}{N(N+2)}[N(N+2) - 2(2-t/m_\pi^2)]C + \alpha\nu_B + \beta\nu^2\,, \tag{8}$$

where we have assumed that the background term is a slowly varying function, and have defined

$F_\pi^{-2}F_N(t) \simeq C$.

For $\nu = \nu_B = 0$, one finds

$$M(0, 0; m_\pi^2, m_\pi^2)^{(+)} = -C, \tag{9}$$

in agreement with Ref. 2. Thus the inclusion of significant t-channel unitarity corrections does not affect the soundness of the Cheng-Dashen extrapolation method.[5]

It is instructive to correlate the t dependence of the on-shell $\pi\pi\sigma$ form factor with the q^2 and k^2 dependence of the off-shell vertex $F_0(q, k)$. This is obtained from the Ward identity[4]

$$q^\mu k^\nu F_{\mu\nu}(q, k) = \left(1 - \frac{q^2}{m_\pi^2} - \frac{k^2}{m_\pi^2}\right) N(N+2)\Delta_0(t) + m_\pi^2 F_\pi^2[1 - F_0(q, k)], \tag{10}$$

where we have assumed $(m_\pi^2 - q^2)\Delta_\pi(q) \simeq 1$. Now assume that $F_{\mu\nu}(q, k)\Delta_0(t)^{-1}$ is well represented by the most general quadratic function of the momenta consistent with (10) and the mass-shell constraint. Then

$$F_{\mu\nu}(q, k) = F_\pi^{-2}\Delta_0(t)\{g_{\mu\nu}[A + (q^2 + k^2)B + q\cdot kD] - (q_\mu q_\nu + k_\mu k_\nu)[2m_\pi^{-4} + \tfrac{1}{2}m_\pi^{-2}A + B] - k_\mu q_\nu D\} \tag{11}$$

and

$$q^\mu k^\nu F_{\mu\nu}(q, k) = F_\pi^{-2}\Delta_0(t)(t - q^2 - k^2)\left(\frac{q^2 + k^2}{m_\pi^4} - \frac{1}{4}(2 - q^2 - k^2)A\right), \tag{12}$$

where A, B, and D are arbitrary constants. It is easily seen that $q^\mu k^\nu F_{\mu\nu}(q, k) \equiv 0$ along the plane $\nu_B = (t - q^2 - k^2)1/4m = 0$. [There do exist rapidly varying terms of $O(q^4)$, but they vanish along this plane.] In the tree approximation $F_{\mu\nu} \sim g_{\mu\nu}$; however, unitarity requires that $F_{\mu\nu}$ have a more complicated structure. Nevertheless, the Cheng-Dashen extrapolation method remains valid.

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[1]H. J. Schnitzer, Phys. Rev. D 5, 1482 (1972).

[2]T. P. Cheng and R. Dashen, Phys. Rev. Letters 26, 594 (1971). See also L. S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D 4, 2801 (1971).

[3]This was first pointed out by Professor P. Auvil (private communication). See also P. Auvil, J. Brehm, and S. Prasad (unpublished).

[4]H. J. Schnitzer, Phys. Rev. Letters 24, 1384 (1970); Phys. Rev. D 2, 1621 (1970).

[5]See also J. Brehm and S. Prasad, Univ. of Massachusetts (Amherst) report (unpublished).