[2]Y. Georgelin, J. Stern, and J. Jersák, Nucl. Phys. B27, 793 (1971); D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 2105 (1971). More speculative applications are reviewed in R. Brandt and G. Preparata, in Springer Tracts in Modern Physics, edited by G. Höhler (Springer, New York, to be published), Vol. 60.

[3]J. D. Bjorken, Phys. Rev. 179, 1577 (1969).

[4]W. Kummer, Nucl. Phys. B42, 141 (1972).

[5]W. Kummer and G. Segrè, Nucl. Phys. 64, 585 (1965); W. Kummer, Acta Phys. Austr. 32, 387 (1970); W. Kainz, Ph. D. thesis, Technical University of Vienna, 1971 (unpublished).

[6]R. Jackiw and H. Schnitzer, Phys. Rev. D 5, 2008 (1972).

[7]We believe that the light-cone expansions for other operator densities are on a less firmer footing theoretically. Also the only support has come so far from one single experiment, deep-inelastic electron-positron scattering, where current densities alone are important.

[8]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, 1971 Coral Gables Conference on Fundamental Interactions at High Energies, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1.

[9]D. J. Gross and S. Treiman, Phys. Rev. D 4, 1059 (1971).

[10]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[11]K. Wilson, Phys. Rev. 179, 1499 (1969); R. A. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

[12]To achieve simplicity of notation the indices of internal symmetry are omitted as well as the nonconnected contributions.

[13]J. W. Meyer and H. Suura, Phys. Rev. 160, 1366 (1967).

[14]A close inspection of the projection technique of Ref. 9 shows that really some terms are being added which are as singular as the original ones.

[15]W. Kummer, A. P. Balachandran, and H. Pietschmann, Ann. Phys. (N.Y.) 29, 161 (1964).

[16]A very compact and general derivation of all sum rules for the matrix element $\langle p|[j^\mu j^\nu]|p\rangle$ from E.T.C. and N.P.C. can be found in D. A. Dicus, R. Jackiw, and V. Teplitz, Phys. Rev. D 4, 1733 (1971).

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Duality Constraints on Inclusive Reactions. II. The Role of the Harari-Freund Conjecture

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We present a scheme for the generalization of the two-component theory of Harari and Freund to multiparticle amplitudes and, in particular, to inclusive reactions. Our scheme is based on duality and borrows the terminology of dual perturbation theory but we do not restrict our generalizations to any specific model. We obtain a criterion for the absence of secondary (nonscaling) Regge contributions in inclusive reactions. We show how inclusive reactions may approach their limiting values from below, in contrast to total cross sections which always have positive secondary contributions. The scheme suggests a novel interpretation of diffraction dissociation which we discuss in a separate paper.

I. INTRODUCTION

Phenomenological duality[1] has had striking success in accounting for certain features of hadronic interactions, especially for two-body processes and, via the optical theorem, for total cross sections. The predictions of duality (supplemented by the absence of exotic resonances) include (1) exchange degeneracy among Regge trajectories and Reggeon residues[2]; (2) certain quark-model results such as the "magic" ϕ-ω mixing angle[3]; (3) the *fall* of the total cross section, $\sigma(a+b\to\text{anything})$, to its asymptotic value when the channel (ab) is a nonexotic channel, i.e., the positivity of secondary Regge-pole contributions; (4) the approximate energy independence of the total cross section, $\sigma(a+b\to\text{anything})$, at surprisingly low energies when (ab) is an exotic channel, i.e., the absence of secondary Regge-pole contributions when (ab) is exotic.

A cornerstone of this approach is the two-component theory of Harari[4] and Freund[5] which assumes, at least as a first approximation, that the *imaginary part* of a two-body scattering amplitude may be written as the *sum* of two terms,

$$\mathrm{Im}\,A(s,t)=P(s,t)+R(s,t)\,. \tag{1}$$

Here, R denotes the contributions of direct-channel resonances which build (via finite-energy sum rules) the Reggeons[6] in the crossed channel. The first term, P, denotes the contribution of nonresonant background in the direct channel which builds up the crossed-channel Pomeranchukon contribution. A modified scheme has recently been proposed which includes multiple-scattering corrections,[7] so that[8]

$$\mathrm{Im}\,A=P+R+R\otimes P+R\otimes R+R\otimes R\otimes P\,. \tag{2}$$

The success of the above description of two-body scattering encourages attempts to generalize this scheme to multiparticle amplitudes. This problem is especially interesting experimentally since the n-particle inclusive cross section is related to a discontinuity of the $(n+2)$-particle forward scattering amplitude[9] in much the same way as the total cross section is related to the two-body forward amplitude by the optical theorem.

By assuming Regge theory for the three-to-three forward elastic amplitude $(ab\bar{c}\to ab\bar{c})$ this leads to Mueller's expression[9] for the single-particle inclusive cross section (in the region of a fragmenting into c)

$$\frac{d\sigma}{dp_T^{c2}dx}=f(x,p_T^c)+g(x,p_T^c)\left(\frac{s}{s_0}\right)^{\alpha_{b\bar{b}}-1}, \tag{3}$$

where $x=2p_L^c/\sqrt{s}$ and p_T^c, p_L^c are the transverse and longitudinal components of the momentum of c in the center-of-mass frame. The function f describes the limiting distribution (we have assumed the Pomeranchukon to be a factorizable pole with intercept unity). The nonscaling term on the right-hand side of Eq. (3) is dominated by a Regge pole in $(b\bar{b})$ with intercept $\alpha_{b\bar{b}}$ (usually taken to be ~0.5 corresponding to the ρ,ω,f^0,A_2). We refer to such nonscaling contributions as the secondary contributions. It should be noted that the range of validity of Mueller's hypothesis [Eq. (3)] is questionable. This is neatly illustrated in dual models as we discuss in Sec. VI. For all the results of this paper, however, we shall assume that, indeed, the terms with Regge behavior dominate the cross section. Generalizations of the two-component theory have been suggested in the context of the dual-resonance model[10-12] supplemented by specific assumptions about the importance of certain terms. In a previous communication[13] (referred to as I from here on) we have suggested a generalization of the Harari-Freund (HF from here on) conjecture which, while it is in the same spirit as Refs. 10, 11, and 12, has very significant differences. In I we were led to several new and powerful phenomenological predictions on the basis of this generalization.

In this paper, we will elaborate on some of the details of the discussion given previously[14] and will obtain some further consequences of our scheme.

Section II will be devoted to a detailed discussion of our model for multiparticle meson amplitudes, leading to a set of rules for implementing the Harari-Freund conjecture in inclusive reactions. Our scheme is a sort of pictorial "skeleton" of a complete theory – we do not have any burdensome formulas, our arguments are based on a sequence of duality diagrams.

The rest of the paper is devoted to the application of this scheme to (mostly single-particle) inclusive production spectra. In Sec. III, we discuss the conditions for the absence of secondary Regge-pole contributions to inclusive cross sections (i.e., conditions for early scaling). Such conditions have recently been the cause of some controversy and we include a critical discussion of other suggested criteria. A simple criterion for amplitudes involving baryons only exists to the extent that planar duality is reasonable for baryon reactions.

In Sec. IV, we will show that there is no necessity in our scheme for the secondary (nonscaling) Regge-pole contributions in inclusive reactions to be positive. This result should be contrasted with the case of total cross sections which reach their asymptotic limit from above.

The treatment of pionization, given in Sec. V, is a simple extension of the earlier discussion. We also discuss the general n-particle inclusive reaction in our scheme.

Finally, in Sec. VI, we compare our scheme with the dual perturbation theory. We also comment on the validity of Mueller's ansatz in dual models.

The treatment of diffraction dissociation has been left to a separate paper[15] (referred to as III). We are led to a novel description of diffraction dissocation in which the Pomeranchukon-Pomeranchukon-Reggeon vertex vanishes.

It is worth observing that the rules for our scheme presented at the end of Sec. II are themselves very simple although their derivation may appear complicated to the reader unfamiliar with duality diagrams. With the aid of these rules, the constraints of duality in multiparticle processes and the essential role of the Pomeranchukon may be easily visualized as is demonstrated in the later sections.

II. THE "SKELETON" MODEL OF MULTIPARTICLE AMPLITUDES

Our discussion in I began with a classification of the duality diagrams for a two-body scattering amplitude. Examples of such diagrams are given in Fig. 1, where we illustrate four equivalent ways of representing the basic duality diagram that contributes to R. The first two emphasize the equivalence between resonances in the s channel ($ab \to cd$) with Reggeons in the t channel ($a\bar{c} \to \bar{b}d$). The third figure is the corresponding quark duality diagram[16] and the fourth figure is an abbreviated form[17] of the third which is in common usage and we find the simplest and most useful for our purposes. (The diagram in which the quarks circulate in the opposite direction should be added; however, we will never need to consider this separately.) In the most naive dual theory this diagram may be associated with a (st) beta function of the Veneziano model,[18] but our analysis does not depend on this association. (See Sec. VI for further discussion of this point.)

Two basic properties of all such duality diagrams are illustrated by Fig. 1.

(1) Quantum numbers: The SU(3) properties are determined by the quark content.

(2) Topology: The shape of the diagram indicates which channels have discontinuities and illustrates precisely the nature of the discontinuity (resonance, Regge pole, Pomeranchukon, Reggeon-Pomeranchukon cut, etc.).

Each diagram is meant to represent some function (of a complete theory) having the analytic structure suggested by the topology of the diagram. For example, it is assumed that the amplitude corresponding to Fig. 1 has resonances in the s channel and in the t channel, but not in the u channel ($a\bar{d} \to c\bar{b}$). It has Regge asymptotic behavior in s at fixed t, $(-s)^{\alpha(t)}$ and in t at fixed s, $(-t)^{\alpha(s)}$. In addition to the diagrams of Fig. 1, one must add the two corresponding to cyclically inequivalent permutations of the external particles. One has resonances in the u channel and in the t channel and behaves, for large u at fixed t, as $(-u)^{\alpha(t)}$. When added to Fig. 1, this gives the full, signatured Regge-pole contribution in the t channel to the amplitude. The diagram having resonances in s and u is assumed to vanish faster than any power as $s \to \infty$ for fixed t. It does, however, contribute to the imaginary part of the amplitude in the s channel and may be important near threshold (see Sec. VI for more discussion of such contributions).

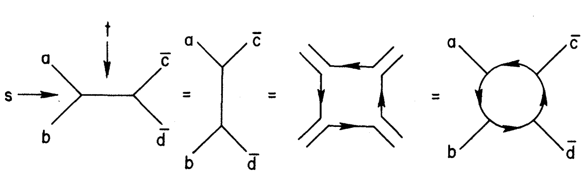

FIG. 1. Equivalent ways of representing a duality diagram.

Starting from these three basic diagrams, many other types of duality diagrams are required for consistency with unitarity, some of which are shown in Fig. 2.[19] We assume that the full scattering amplitude is represented by the sum of all distinct duality diagrams, a feature shared by the dual perturbation theory.[20] We shall not assume that the more complicated "higher-order" diagrams are small compared with the simpler ones. In this respect, we are not assuming that a perturbative theory is necessarily realistic. We shall later (Sec. III) point out that it is likely that a large class of these diagrams must be large for phenomenological reasons. We shall classify all diagrams according to their quantum number and topology content and require that each diagram is consistent with the HF conjecture.

Consider the duality diagram of Fig. 2(a). This diagram illustrates why the additivity assumption of the HF hypothesis [Eq. (2)] is by no means trivial. The particles in the initial state (ab) appear to be able to resonate but the particles in the final state (cd) cannot. The diagram represents a matrix element which mixes a resonance with nonresonant background. Consistency with the HF hypothesis requires that it vanish.[21] In general, then, *whenever a single external meson is connected to a quark loop, the corresponding duality diagram is assumed to vanish.* It is at this point that we diverge most crucially from other treatments.

Figure 2(b) is the diagram that has classically been identified as the primordial Pomeranchuk-

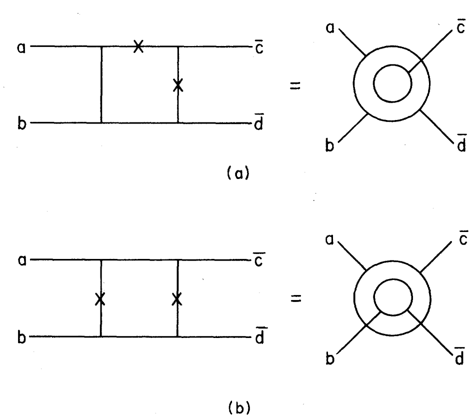

FIG. 2. (a) This diagram violates exchange degeneracy and the Harari-Freund conjecture. (b) The Pomeranchukon diagram. Secondary Reggeons must be absent if HF is satisfied.

on,[22-24] P. It has vacuum quantum numbers in the $a\bar{c}$ channel and in dual perturbation theory[23,24] has a j-plane singularity whose position is independent of the external quantum numbers (although in the naive dual perturbation theory it has the wrong intercept and gives a unitarity-violating cut). We also identify this diagram as P. The diagram appears to have resonance poles in the t channel, but any pole is connected to a quark loop to which no other particles are connected. By the discussion of the previous paragraph, such a contribution vanishes so that the residues of the t-channel poles in Fig. 2(a) must vanish. As we emphasized in I, the absence of Reggeons in the t channel is necessary for the maintenance of the HF hypothesis.[25]

Other diagrams, such as those shown in Fig. 3, give Reggeon-Reggeon cuts. Note that, according to the rule developed above, Fig. 3(a) has no resonances in the direct channel.

Figure 4 illustrates two diagrams obtained from Fig. 1 and Fig. 2(b) by adding holes. Holes may contribute to renormalization corrections as in Fig. 4(a). However, they may also lead to Reggeon-Reggeon cuts as in Fig. 4(b). Although the properties of these cuts are not well determined, they do have lower intercepts and may be ignored at high energy. Indeed the whole discussion of constant total cross sections in exotic reactions assumes cut effects are negligible.[26] For the remainder of the paper, we shall therefore ignore them.

In addition, there are further types of diagrams which are constructed by adding handles[20] to any of the diagrams so far considered. Two such diagrams are shown in Fig. 5, both of which have Reggeons and Reggeon-Pomeranchukon cuts in the t channel as well as other presumably less important contributions. Figure 5(a) contributes both to R and $R\otimes P$ terms in Eq. (2), and so such diagrams will be quite important in general. The addition of a handle may be thought of primarily as an absorption correction. The topology of diagrams with handles is a bit more deceptive than that of the diagrams previously considered. For example, Fig. 5(b) has a discontinuity in the s channel even though particles a and b are not adjacent on the quark loop boundary. It thus has a Reggeon contribution in the t channel regardless of whether the s channel is exotic or not, apparently in violation of the HF conjecture. That this discontinuity is in fact small may be seen as follows: The diagram has the form $R\otimes P$ where R is purely real in the s channel and P is taken to be purely imaginary and strongly peaked near $t=0$; hence the discontinuity vanishes. This fortuitous vanishing of the discontinuity of Fig. 5(b) guarantees the HF hypoth-

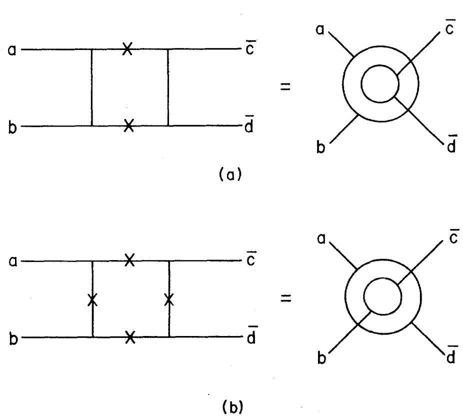

FIG. 3. Diagrams giving rise to Regge-Regge cuts.

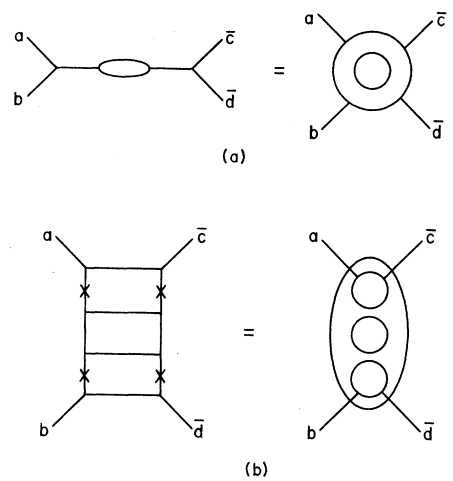

FIG. 4. (a) A renormalization correction. (b) Example of a diagram which has renormalization loop that builds Regge cuts.

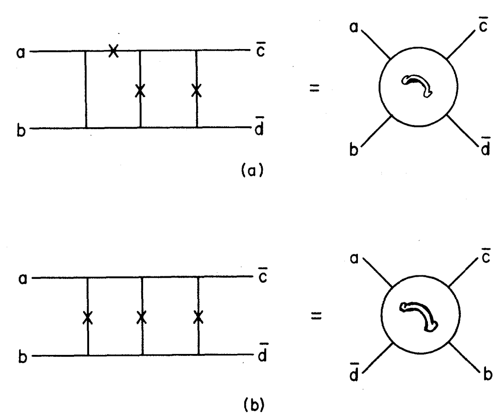

FIG. 5. (a) This diagram has a discontinuity in all channels. (b) We argue that the s_{ab} discontinuity of this diagram is very small.

esis for the imaginary part of the amplitude.[27] We therefore assume in general that the *addition of handles to a diagram does not affect which channels have discontinuities and provides absorptive corrections to resonances or Reggeons in those channels.*

This discussion suggests how the first assumption of Harari's model[7] summarized by Eq. (2) arises in a crossing-symmetric, unitary theory. In the imaginary part of the amplitude Reggeons plus Reggeon-Pomeranchukon cuts in the t channel are dual to s-channel resonances plus their absorption (leaving peripheral resonances prominent, see Fig. 6). Our scheme suggests that this picture is only correct to the extent that the Pomeranchukon is purely imaginary.[28]

For the sake of clarity, we summarize rules for constructing our skeleton model:

(1) Every scattering amplitude can be represented as the sum of all distinct orientable duality diagrams.

(2) Diagrams having only one external particle attached to a single quark loop are assumed to vanish. Similarly, no resonance (or Reggeon) is exchanged between two states unless quarks are exchanged. This is the implementation of the HF conjecture.

(3) The leading contribution to the exchange mechanism between separate quark loops is the Pomeranchukon. The real part of the Pomeranchukon is negligible compared to the imaginary part for small momentum transfers.

(4) Diagrams with handles are assumed to contribute only to the real part of the amplitude in channels formed by nonadjacent particles. Handles contribute primarily to absorptive corrections.

(5) When the quantum numbers of a pair of particles connected to each other by a quark in a particular duality diagram are exotic then the contribution of that diagram vanishes identically. This rule is implicit in any scheme based on duality diagrams.

These rules have been obtained for meson amplitude. In Sec. IV we will discuss the duality diagrams which include baryons.

Our rules are immediately applicable to an arbitrary multiparticle scattering process and provide a generalization of the HF conjecture. In the remainder of this paper, we shall apply this scheme to several multiparticle processes of experimental and theoretical interest.

The simplest diagram of any class of diagrams with a particular quantum number and topology content will be referred to as a *primitive diagram.* In other words the addition of holes and handles (according to rule 4) to a primitive diagram does not alter which channels have singularities. It is clearly sufficient to study these primitive diagrams only if we wish to discuss the high-energy behavior of the amplitude. We have shown that Figs. 1 and 2(b) (plus diagrams formed by noncyclic permutations of external particles) are the primitive diagrams for the four-point function, i.e., Fig. 5(b) need not be considered separately due to rule 4. It remains an assumption that this rule also applies to multiparticle amplitudes with handles.

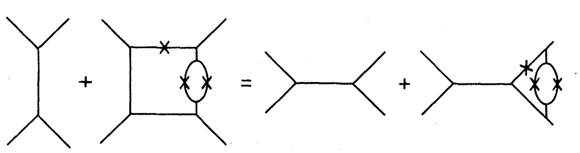

FIG. 6. An illustration of absorbed Reggeons dual to peripheral resonances.

III. EXOTICITY CONDITIONS FOR EARLY SCALING IN INCLUSIVE REACTIONS

We have derived, in I, a condition for the approximate energy independence of single-particle inclusive reactions, $a+b\to c+x$. We will now elaborate on that discussion.

In the limit corresponding to the fragmentation of a into c,[29] $(a\!:\!c|b)$, the leading contributions

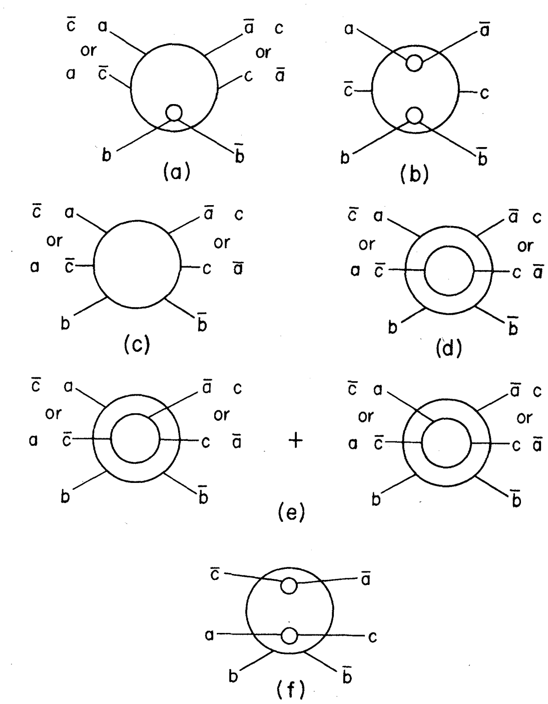

FIG. 7. Contributions to the six-point function in the region of a fragmenting to c excluding diagrams contributing to the diffraction dissociation of a into c.

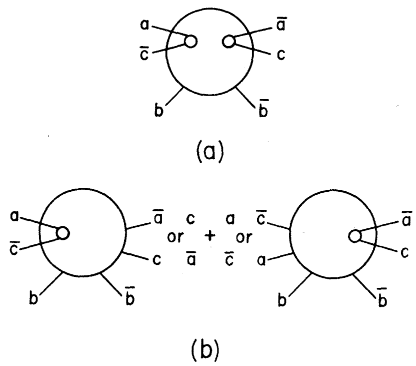

FIG. 8. Contributions to the diffraction dissociation of a into c.

come from those diagrams for the process $a+b+\bar{c} \to a+b+\bar{c}$ which have (i) a nonzero forward discontinuity in $ab\bar{c}$ since this discontinuity gives the inclusive cross section[9] and (ii) either the Pomeranchukon or Reggeons in the $(b\bar{b})$ channel.

In Figs. 7 and 8, we display all such primitive diagrams. Figures 7(a), 7(b), and 8(a) all have the Pomeranchukon in the $(b\bar{b})$ channel and contribute to the limiting fragmentation. Figure 7(b) has the Pomeranchukon in $(a\bar{a})$ also and is the only diagram contributing to the limiting distribution in the pionization limit (see Sec. V). Figure 8 is relevant for diffractive dissociation and will be discussed in more detail in III. All other diagrams displayed have Reggeons in the $(b\bar{b})$ channel and are the dominant contributions to the energy dependence in the fragmentation limit. It is easy to check that there is only one criterion that will eliminate all diagrams which have Reggeons in $(b\bar{b})$. The criterion is

$$(ab) \text{ and } (b\bar{c}) \text{ must be exotic}. \qquad \text{(A)}$$

We do not need to specify that $(ab\bar{c})$ be exotic since when (ab) and $(b\bar{c})$ are exotic only Figs. 7(f) and 8(a) survive and these do not have any energy dependence anyway. There are also baryon reactions for which (ab) and $(b\bar{c})$ exotic does not imply $(ab\bar{c})$ exotic (e.g., $p+\Omega^- \to \Xi^- + X$). We do not expect our simple duality scheme to work for such processes since the corresponding two-body reactions, baryon-antibaryon scattering, cannot be described in a simple dual model without exotic resonances.

In any case, consideration of reactions involving baryons only gives criterion A if planar duality is applicable to baryons. The assumption of planar duality is common to most approaches (e.g., Refs. 30 and 31).

From our presentation, which involved examining all the relevant diagrams for the six-point function, the existence of any simple criterion may seem fortuitous. However, we shall see in Sec. V how a general criterion can be obtained for an n-point function without having to draw any diagrams explicitly, just by using special ordering properties that duality imposes on quark lines.

We stress that our criterion A eliminates all secondaries in the *whole* fragmentation region. In certain limited regions of phase space, we may expect other, weaker, criteria to be good to some approximation. Thus, for instance, in the triple-Regge region (x near unity, where x is the Feynman scaling parameter) Fig. 7(c) may be expected to be the leading nonscaling diagram [if $(a\bar{c})$ is not exotic] since this has the triple-Regge behavior. To the extent that Fig. 7(c) (the tree diagram) does dominate in this region, the condition $(ab\bar{c})$ exotic is sufficient to ensure early scaling. This is the condition proposed by Chan *et al.*[30,31] who, however, suggest a larger region of validity. It is true that the range of validity of the triple-Regge formula is not well established but it cannot be appropriate when $(s_{a\bar{c}})$ is large. The region in which $s_{a\bar{c}}$ is large *is* however included in the fragmentation region of a into c which is defined as a limit at large s_{ab} in which s_{bc}/s_{ab} and $s_{a\bar{c}}$ are held

TABLE I. A demonstration of the great importance of the Pomeranchukon (nearly constant) contributions to total cross sections at rather low kinetic energies.

Total cross section	Type	$\sigma(\infty)$ (mb)	Dependence on center-of-mass kinetic energy, T (MeV)
K^+p	Exotic	18	Constant for $T \geq 600$
K^-p	Nonexotic	18	Falls below 36 for $T \geq 500$
π^+p	Nonexotic	23–24	Falls below 46 for $T \geq 800$
π^-p	Nonexotic	23–24	Falls below 46 for $T \geq 700$
pp	Exotic	39–40	Constant for $T \geq 800$
$\bar{p}p$	Nonexotic	39–40	Falls below 78 for $T \geq 800$

fixed. We have seen that in general there is only one criterion appropriate to the whole fragmentation region. Chan and Hoyer[31] suggest that "higher-order loop corrections" are small compared with tree diagrams in order to justify their criterion for a large region of phase space. However, the Pomeranchukon is a loop diagram and it is known to have couplings of the same order of magnitude as normal Regge trajectories.[32] We emphasize this point, which seems to cause a lot of confusion, by listing some total cross-section data[33] in Table I. This table shows that the Pomeranchukon contribution is larger than the secondary contributions at remarkably small energies – typically of the order of 700-MeV center-of-mass kinetic energy. It is therefore exceedingly dangerous to ignore "loop contributions" at the energies under discussion in inclusive reactions.

The criterion suggested by Ellis, Finkelstein, Frampton, and Jacob[34] which says that reactions scale when $(ab\bar{c})$ and (ab) are exotic amounts to neglecting certain of our diagrams since they arbitrarily neglect the $b\bar{c}$ channel. They assert that since $s_{b\bar{c}}$ is negative in the physical region, the singularities in $s_{b\bar{c}}$ are negligible. However, in a dual model the poles at positive values of $s_{b\bar{c}}$ imply a smooth energy-dependent secondary contribution at negative values of $s_{b\bar{c}}$ which will be just as large as the average energy dependence arising from s_{ab} (which is positive). It is easy to see from our diagrams that the channel $(b\bar{c})$ is just as important as (ab) in determining secondary contributions. The present data are not adequate to separate contributions with sufficient accuracy to determine which criterion is correct. We feel that a discussion of the HF conjecture must be a prerequisite for any exoticity criterion since it is the crucial element in the discussion of total cross sections; it is the lack of such a discussion in Refs. 30, 31, and 34 that makes their conclusions unsatisfactory. We should point out that in order to determine which reactions scale early it is necessary to have a precise study of a reaction over a range of relatively low lab momenta (3–10 GeV/c, say) and values of x not too close to unity. This is not so useful, of course, for observing limiting distributions and establishing properties of the Pomeranchukon which has been subject to most attention up till now. In I, we listed some reactions which distinguish our criterion, the most interesting case being $K^+p \to \pi^- + X$. This should scale at low energies in both fragmentation regions according to previous rules, but only in the proton fragmentation region according to ours.

To conclude this section, we would like to discuss a very recent proposal by Tye and Veneziano.[12] In their scheme the inclusive cross sections are built from the square of production amplitudes. A crucial assumption is that as far as possible one should only include tree diagrams in the production amplitudes. This leads to the same primitive diagrams for total cross sections as we consider. However, at the level of the six-point function one-loop Pomeranchukon diagrams must be added in the production amplitudes to allow for diffraction dissociation (the eighth component in Ref. 12). In addition, many interference terms are neglected for simplicity in Ref. 12 while our rule 2 only eliminates a subset of them. A subtler difference is that our primitive diagrams are supposed to include handles (corresponding to absorption) and therefore we do not predict that even the diagonal terms are positive, while their most important prediction is based on positivity of each of the seven components in their scheme. On the other hand as they do not assume rule 2 they do not predict the vanishing of secondaries when the s channel is exotic (they call this the strong HF hypothesis as opposed to the weak form that they obtain) and therefore they do not have exchange degeneracy. This allows a diagonal diagram in which there is a Pomeranchukon and a secondary Reggeon in the same channel. This secondary Reggeon may give a negative contribution since the Pomeranchukon is positive and dominates (since it has a higher intercept) ensuring positivity for the diagram as a whole. Such negative secondaries are known to be necessary[12] and are discussed in detail in Sec. IV.

IV. THE SIGN OF SECONDARY CONTRIBUTIONS

Before turning to diagrams having Reggeons in $(b\bar{b})$ we would like to remind the reader of the third consequence of the HF hypothesis for two-body reactions which was mentioned in the opening paragraph of the paper, viz., the fall of total cross

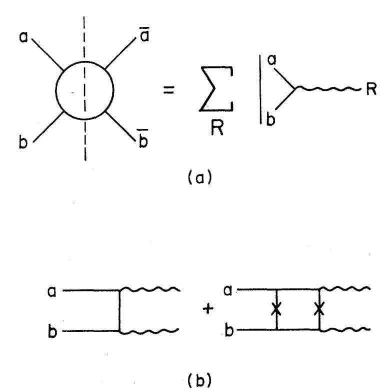

FIG. 9. (a) Illustration of the positivity of resonance contributions. (b) The square of this contribution contains $P \otimes R$ term that may be negative.

sections to their asymptotic values. This result depends on the fact that the discontinuity of the resonance component of the forward elastic amplitude must be positive as shown in Fig. 9(a), and that interference diagrams, such as Fig. 2(a), vanish. It is not obvious, *a priori,* that absorption corrections of the form $R\otimes P$ cannot make the over-all secondary component negative. For instance, in the square of Fig. 9(b), the $R\otimes P$ term may be negative although the net contribution of the square (which involves Pomeranchukon terms like P and $P\otimes P$) is, of course, positive. It is a phenomenological fact that these absorption corrections are not strong enough to cause the total cross sections to rise to their asymptotic limits.

Let us now examine single-particle inclusive reactions. In order to gain some insight into the kinds of processes that build up each contribution, we will look at the missing-mass discontinuity explicitly. The contributions that build the Pomeranchukon in $(b\bar{b})$ when squared and summed over X, Y, and Z are illustrated in Figs. 10(a)–10(c) and so each of these individually contributes a positive component to the limiting distribution.

Since it was a useful approximation in the two-body case, we might ask whether, in the absence of absorption corrections, all single-particle inclusive cross sections necessarily fall to their limiting values in our scheme. If this were so, it would be catastrophic since it can be shown rigorously, by use of energy-conservation sum rules,[35] that some inclusive reactions *must rise* to their limiting values. To examine this, we return to a consideration of those primitive diagrams for the fragmentation $(a{:}c|b)$ having Reggeons in the $(b\bar{b})$ channel. Figure 7(c) and all of Fig. 8 arise from the square of the production mechanisms shown in Fig. 11(a). The crucial point is that although the square of the first two diagrams in Fig. 11(a) and their interference with the third diagram have Reggeons in $(b\bar{b})$, the square of the third gives Fig. 8(a), having only a Pomeranchukon in $(b\bar{b})$. Since this Pomeranchukon term is positive in $(b\bar{b})$ there is no reason why the interference terms, Figs. 8(b), could not be negative. If so, they will dominate over the other contributions with Reggeons in $(b\bar{b})$ for a sufficiently large ratio of s_{ab} to $s_{ab\bar{c}}$ (see the discussion of diffraction dissociation in III), leading to an approach to limiting fragmentation from *below*. Similarly, there are further interference terms [Figs. 7(d) and 7(e)] which arise from squaring Fig. 11(b) and occur in processes in which a and c have different quantum numbers. Once again, the square of the third diagram gives the Pomeranchukon in $(b\bar{b})$ and is positive so that it dominates over the other terms in the square of Fig. 11(b), which have Reggeons in $(b\bar{b})$. This means that the interference terms [Fig. 7(e)] which need not be positive may dominate over the other terms with Reggeons in $(b\bar{b})$ and the possibility again arises of the fragmentation limit being reached from *below*.

In summary, the primitive diagrams suggest that the inclusive reaction might rise to its asymptotic value in two circumstances.

(1) If a or b can diffractively dissociate into c, or

(2) If the quantum numbers of a, b, and c do not force the interference terms of Fig. 7(e) to vanish.

These results only apply to reactions involving baryons if we only need to include planar diagrams. It is clear that in the case of nonplanar diagrams there are so many possible interference terms that a much weaker set of conditions would allow negative secondary terms.

The above considerations have completely neglected the possible effects of absorption. As we have noted, absorption does not change the sign of secondaries in total cross sections but there is no *a priori* reason why it could not. Thus, for the six-point function where the possibilities for absorption are much more complicated, the negative

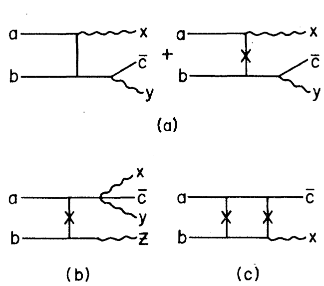

FIG. 10. Terms contributing to the Pomeranchukon in the inclusive reaction $(a{:}\,c|b)$. There are no interference terms between 10(a), 10(b), and 10(c) by rule 2.

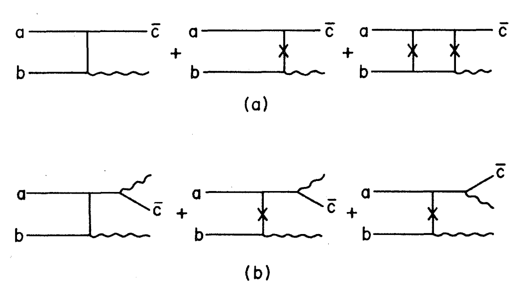

FIG. 11. (a) When squared, this sum gives rise to Fig. 7(c) and Fig. 8. (b) Diagrams contributing to the interference terms 7(e) and 7(f).

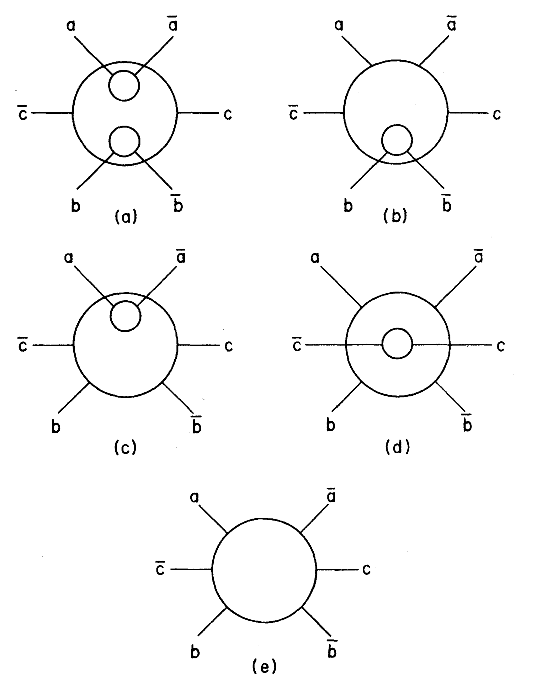

FIG. 12. Diagrams that contribute to the pionization region $(a|c|b)$.

nonscaling secondaries may arise from this mechanism.

V. PIONIZATION AND THE GENERAL N-POINT FUNCTION

In the pionization limit, $(a|c|b)$, the dominant contributions have either a Pomeranchukon or a Reggeon in both the $(a\bar{a})$ and $(b\bar{b})$ channels. All other diagrams are assumed to vanish exponentially; this is equivalent to assuming the Mueller hypothesis.[9] The surviving diagrams are shown in Fig. 12. In Table II, we indicate each type of diagram and state which criterion eliminates it.[36] (The diagrams are labeled as in Fig. 12.)

From Table II, we see, for instance, that $(a\bar{c})$ exotic eliminates the contribution which has a Pomeranchukon in $(b\bar{b})$ and a Reggeon in $(a\bar{a})$. This is a leading secondary and behaves like $s_{ab}^{\alpha_{a\bar{a}}/2-1/2}$ in the pionization limit.[37] So the condition to eliminate both leading secondaries is $(a\bar{c})$ and $(b\bar{c})$ exotic. The condition that eliminates all secondaries [including terms with Reggeons in both $(a\bar{a})$ and $(b\bar{b})$] is (ab), $(a\bar{c})$, and $(b\bar{c})$ exotic.

These conditions for the absence of secondaries in fragmentation and pionization may be seen in a general way without examining particular diagrams as mentioned earlier. We make use of the following properties of all duality diagrams for an n-particle inclusive reaction, $a+b \to c_1+c_2+\cdots+c_{n-2}+X$ (The relevant diagrams are for the $a+b+\bar{c}_1+\bar{c}_2+\cdots+\bar{c}_{n-2} \to \bar{a}+\bar{b}+c_1+c_2+\cdots+c_{n-2}$ forward amplitude.):

(a) Each diagram consists of the external particles (made of quark-antiquark pairs) attached to quark loops.

(b) There is a strict ordering of particles around each quark loop such that all incoming particles can be separated from all outgoing particles as in Fig. 13(a).

(c) Any group of particles which have low relative momenta and are separated from other particles by a Pomeranchukon or Reggeon must be adjacent on a quark loop or on separate quark loops. For example, in Fig. 13(b), the particles $b\bar{b}\bar{c}c$ are arranged so that this diagram contributes to the region of b fragmenting into c.

From these simple rules, the general prescription for the absence of secondary contributions in an arbitrary process is manifest: *No secondary Regge poles contribute to an inclusive reaction if, and only if, each channel whose subenergy becomes large is exotic.*

VI. RELATION TO DUAL PERTURBATION THEORY AND MUELLER'S HYPOTHESIS

We first wish to clarify the connection between our model and dual perturbation theory (DPT) based on Feynman-like diagrams.[20]

Although the classification of diagrams is exactly the same, the philosophy of our approach is rather different. In DPT, at least in principle, one has a

TABLE II. Terms contributing to the pionization cross section. The labelling in the first column corresponds to that of Fig. 12. The last column indicates the conditions that force specific terms to be absent.

Type of contribution (labeled by Fig. 12)	Exchanged Regge singularity in $(a\bar{a})$	in $(b\bar{b})$	Condition for absence of contribution
(a)	P	P	
(b)	R	P	$(a\bar{c})$ exotic
(c)	P	R	$(b\bar{c})$ exotic
(d)	R	R	(ab) exotic
(e)	R	R	$(a\bar{c})$ or $(b\bar{c})$ or $(ab\bar{c})$ exotic

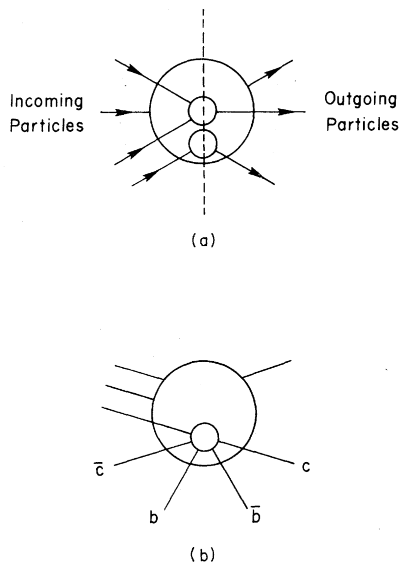

FIG. 13. (a) Illustration of the ordering of particles implied by duality. The dashed line can always be drawn to separate the incoming from outgoing particles. (b) Diagram contributing to the region of b fragmenting into c in $a+b \rightarrow c+d_1+d_2+d_n+X$, where d_i are also detected.

definite prescription[38] for calculating the diagrams based on unitarity and the assumption that the Veneziano amplitude is the Born approximation. However, since the Veneziano formula does not correspond too well to the physical world, it is hard to believe in this prescription. Our approach is to abstract from the DPT certain properties that we expect will be solved by a more realistic model and we feel free to modify all other aspects. To this we add certain phenomenological information, essentially the HF conjecture and some features of absorption. Finally, we take the optimistic attitude that the whole scheme is consistent with known rigorous constraints (such as sum rules[35]). This is certainly nontrivial since, for example, our manner of implementing the HF conjecture has been very simple minded and is surely not the only possible one (see, for example, Ref. 12).

To illustrate our approach consider Fig. 2(b) which we have identified with the Pomeranchukon and have assumed behaves up to logarithms like $s^{\alpha(t)}$ as $s\rightarrow\infty$ for fixed t and that $\alpha(0)=1$. In fact Freund[22] argued in favor of this identification before the prescription for calculating loops was established, solely on the basis of duality. DPT shows a suggestive J-plane[23,24] singularity for this diagram but with a different intercept. Furthermore, for a physical J the singularity becomes a cut in s that violates unitarity. However, it is more or less accepted that while the existence of such a singularity is model-independent (i.e., the topology arguments of Ref. 39), its intercept and the fact that it is a cut are not.[24] Therefore, we only use DPT to tell us which classes of diagrams can have a Pomeranchukonlike singularity and in this way we are led to rule 3.

However, DPT also shows that whenever a diagram has the Pomeranchukon in a certain channel that the same diagram will have the f or f' pole in that channel. We have argued that this feature would break exchange degeneracy (EXD) and the HF hypothesis and therefore have established rule 2. This is not necessarily incompatible with DPT if we consider that the diagrams of the skeleton model correspond to an infinite set of diagrams in DPT.

An interesting and rather controversial diagram is the one of Fig. 8(a) which has been used by some authors[40] as evidence in favor of the vanishing of the triple-Pomeranchukon vertex. Contrary to common myth, a thorough calculation of this diagram following the methods of Ref. 23 reveals that it *does* have a triple-Pomeranchukon singularity and the Pomeranchukon in the $b\bar{b}$ channel is dual to *resonances* in the $ab\bar{c}$ channel.

Of course, as is well known, in a unitary theory the triple-Pomeranchukon vertex must vanish when $s_{a\bar{c}}=s_{\bar{a}c}=0$ if the Pomeranchukon is a pole with unit intercept. This would only be true in DPT for the completely summed series.

The observation that resonances build the Pomeranchukon in diffraction dissociation in DPT is very important and independent of the manner of implementation of the HF hypothesis. It is discussed further in III.[15]

A more difficult point to understand in the context of DPT is the fourth rule which states that diagrams with handles have discontinuities in the same channels as ones without them. We are not sure if this rule is too strong or even consistent with unitarity. For the four-point function, as discussed earlier, the rule is true if the Pomeranchukon is purely imaginary. We would hope that in any case it would be true to at least the leading power of s although this is very much an open question.

We now turn to a discussion of the validity of Mueller's hypothesis that the Pomeranchukon and secondary Reggeons dominate inclusive cross sections. Our whole analysis has assumed this to be true at laboratory energies. The dual-resonance model indicates a possible source of failure of this assumption. Recall that the Veneziano model is the sum of three beta functions,

$$V=B_{st}+B_{ut}+B_{su}.$$

As $s\to\infty$, for fixed t, the first two terms lead to Regge behavior (at least outside of a wedge) and the third term, B_{su}, has indefinite oscillating phase. It is universally assumed that in the fully unitarized theory the resonances will develop widths in such a manner as to lead to Regge behavior along the real s axis and lead to exponential damping of the third term.

An analogous situation holds in inclusive processes[41] for which diagrams in which Reggeons carry large mass (analogous to B_{su} for the four-point function) are damped by an exponential which depends on the imaginary part of the trajectory function. If these terms contributed significantly at present laboratory energies, they would undermine the whole notion of Reggeon-resonance duality as well as the applicability of Mueller's ansatz of Regge behavior of the six-point function. Whereas the usual correlation length is determined from the difference between the intercepts of the Pomeranchukon and the leading Reggeon, these extra terms might lead to an entirely different kind of correlation. Even if resonances have widths that grow with mass squared, these terms may lead to significant correlation at present energies since the actual rate of decrease of the exponential is unknown. This would be an unhappy situation for most Regge theorists.

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[1]For reviews of duality see, e.g., H. Harari, in *Summer School in Elementary Particle Physics, Theories of Strong Interactions at High Energies,* held at Brookhaven National Laboratory, 1969, edited by R. F. Peierls (Brookhaven National Laboratory, Upton, N.Y., 1970); in Lectures at the 1970 International School of Physics "Ettore Majorana," edited by A. Zichichi (Academic, New York, to be published); M. Jacob, in Lectures at the Herczeg-Novi Summer School, Yugoslavia, 1970 (unpublished); J. D. Jackson, invited talk in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics,* edited by E. Gotsman (Weizmann Science Press, Jerusalem, 1971).

[2]See, e.g., C. Michael, in *Springer Tracts in Modern Physics,* edited by G. Höhler (Springer, New York, 1970), Vol. 55.

[3]C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1969).

[4]H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[5]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968).

[6]In this paper, the terms Reggeons and Regge pole should be understood to exclude the Pomeranchukon.

[7]H. Harari, invited talk in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics,* Ref. 1; H. Harari and A. Schwimmer, Phys. Rev. D 5, 2780 (1972), and references therein. See also the review by R. J. N. Phillips, Rapporteur's talk, in Proceedings of the Amsterdam International Conference on Elementary Particles, 1971 (unpublished).

[8]Here the symbol P is meant to represent the full Pomeranchukon contribution which, in many models (including the one under discussion in this paper), is made of a primordial Pomeranchukon plus some kind of absorption. Symbolically "$P \equiv P + P\otimes P + \cdots$."

[9]A. H. Mueller, Phys. Rev. D 2, 2963 (1970). See also, H. P. Stapp, LBL Report No. UCRL 20623 (unpublished); C. -I Tan, Phys. Rev. D 4, 2412 (1971); J. C. Polkinghorne, Nuovo Cimento 7A, 555 (1972).

[10]D. Gordon and G. Veneziano, Phys. Rev. D 3, 2116 (1971).

[11]G. Veneziano, Lett. Nuovo Cimento 1, 681 (1971).

[12]S.-H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972).

[13]M. B. Einhorn, M. B. Green, and M. A. Virasoro, Phys. Letters 37B, 292 (1971).

[14]Although this paper repeats some of I, we shall at times refer back to I for the sake of brevity.

[15]M. B. Einhorn, M. B. Green, and M. A. Virasoro, LBL Report No. LBL-768, 1972 (unpublished).

[16]H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, *ibid.* 22, 689 (1969); T. Matsuoka, K. Ninomiya, and S. Sawada, Progr. Theoret. Phys. (Kyoto) 42, 56 (1969).

[17]D. Neville, Phys. Rev. Letters 22, 494 (1969).

[18]A. DiGiacomo, S. Fubini, L. Sertorio, and G. Veneziano, Phys. Letters 33B, 171 (1970). G. Veneziano, invited talk in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics,* Ref. 1.

[19]We need never consider "non-orientable" diagrams. These diagrams contain states with forbidden quark orientations. See J. Shapiro, Phys. Rev. D 4, 1249 (1971), for a rigorous proof that such diagrams never contribute.

[20]K. Kikkawa, B. Sakita, and M. A. Virasoro, Phys. Rev. 184, 1701 (1969); K. Kikkawa, S. A. Klein, B. Sakita, and M. A. Virasoro, Phys. Rev. D 1, 3258 (1970).

[21]By vanish, we mean of course suppressed compared with other diagrams which do contribute.

[22]P.G.O. Freund, Lett. Nuovo Cimento 4, 147 (1970); P. G. O. Freund and R. J. Rivers, Phys. Letters 29B, 510 (1969).

[23]D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwarz, Phys. Rev. D 2, 697 (1970); C. S. Hsue, B. Sakita, and M. A. Virasoro, *ibid.* 2, 2857 (1970); G. Frye and L. Susskind, Phys. Letters 31B, 589 (1970).

[24]C. Lovelace, Phys. Letters 34B, 500 (1971).

[25]In I, we first noticed this fact and then deduced that

Fig. 2(a) has to vanish. For variety, we chose to emphasize the converse here. The correspondence between the couplings of the Pomeranchukon and those of the f and f' Regge poles [see R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. D 4, 3439 (1971)] is maintained by Fig. 2(b) even if we assume that the f and f' poles in Fig. 2(b) have very small residues.

[26]For a discussion of Regge-Regge cuts in phenomenology, see H. Harari, Phys. Rev. Letters 26, 1079 (1971).

[27]This property is shared by Reggeon-multiple Pomeranchukon cuts ($R\times P\times P$, etc.) obtained by adding several handles.

[28]Harari (Ref. 7) also assumes the Pomeranchukon to be purely imaginary but for very different reasons.

[29]This notation was introduced in R. Cahn and M. B. Einhorn, Phys. Rev. D 4, 3337 (1971). $(a{:}c_1|c_2|c_3{:}b)$ means the scattering process $a+b\rightarrow c_1+c_2+c_3+X$ in the region of a fragmenting into c_1, b fragmenting into c_3 and with c_2 a pionization product.

[30]Chan Hong-Mo, C. S. Hsue, C. Quigg, and J.-M. Wang, Phys. Rev. Letters 26, 672 (1971).

[31]Chan Hong-Mo and P. Hoyer, Phys. Letters 36B, 79 (1971).

[32]See, for example, R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. Letters 26, 1515 (1971).

[33]For a summary of total cross-section data, see the Particle Data Tables. The most recent data from Serpukhov are in S. P. Denisov *et al.*, Phys. Letters 36B, 528 (1971).

[34]J. Ellis, J. Finkelstein, P. H. Frampton, and M. Jacob, Phys. Letters 35B, 227 (1971).

[35]G. Veneziano, seminar (unpublished) at the International School of Physics "Enrico Fermi," Varenna, 1971; L. Caneschi, CERN Report No. Ref-Th-1416, 1971 (unpublished).

[36]M.-S. Chen and F. E. Paige, Phys. Rev. D 5, 2760 (1972).

[37]H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971).

[38]C. Lovelace, Phys. Letters 32B, 703 (1970); V. Alessandrini, Nuovo Cimento 2A, 321 (1971); M. Kaku and L. P. Yu, Phys. Letters 33B, 166 (1970).

[39]V. Alessandrini and D. Amati, Nuovo Cimento 4A, 793 (1971).

[40]P. H. Frampton, Phys. Letters 36B, 591 (1971); R. C. Brower and R. E. Waltz, CERN Report No. CERN-TH-1335, 1971 (unpublished).

[41]M. A. Virasoro, Phys. Rev. D 3, 2834 (1971); C. E. DeTar, K. Kang, C.-I Tan, and J. H. Weis, *ibid.* 4, 4251 (1971).

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Spectrum of Physical States in a Dual-Resonance Model

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The dual-resonance model entails an exponentially growing resonance spectrum for the hadrons. It is well known that not all the states which achieve the factorization are physical ones; there are spurious and ghost states among them. We give a general formula for counting the ghost states at any given mass value and also an asymptotic expression for the ghost spectrum. The Virasoro model, in which the leading trajectory intercept is unity, is not considered here. A method for constructing the subspace of real states, which is orthogonal to the space of spurious states, is also given. After removing the ghost states from the real states, the remaining ones are taken as physical states, which then constitute the resonance spectrum. Implications of this resonance spectrum in the statistical approach are briefly discussed.

I. INTRODUCTION

After the n-point dual-resonance amplitudes were shown to be factorizable in terms of harmonic-oscillator states,[1,2] there was brought to light the unexpectedly rich level structure of resonances making up the amplitudes. It has been shown[1-3] that the multiplicity of independent states at a certain mass value increases exponentially with the mass. However, it is immediately apparent from the method employed that many of the states included in the factorization cannot be interpreted as physical or resonance states since they have negative norms (ghost states), and would give rise to negative-pole residues. Moreover, there are spurious states[4] due to linear dependence which do not couple to the external-particle states at all. In order to reach some understanding of