

Multiparticle Production-Feynman Fluid Analogy*

Myron Bander

Department of Physics, University of California at Irvine, Irvine, California 92664
(Received 6 January 1972)

The analogy between high-energy events with many particles in the final state and a distribution of particles in a fluid is examined in detail. It is shown that many theoretical models lend themselves to such an interpretation. Using experimental data on prong cross sections and one-particle inclusive distributions, some properties of this analogous fluid are computed. The possibility and significance of a "phase transition" are discussed and predictions based on such considerations are made for higher energies.

I. CLASSICAL-FLUID-MULTIPARTICLE-PRODUCTION ANALOGY

One of the current ways of viewing high-energy inclusive and exclusive multiparticle production is Feynman's^{1,2} analogy of such reactions with statistical properties of fluids. This analogy permits us to translate some of the intuition we have about classical fluids³ bounded by finite walls to the distribution of produced particles in the multiparticle phase space of high-energy processes.

In this article we shall exploit this analogy seriously and compute, from experimental data, quantities such as the grand canonical partition function, pressure, distribution function, etc., and compare these with general features implied by multiparticle-production models.⁴ The class of models we shall consider are those that are consistent with this analogy and that more specifically imply a nontrivial thermodynamic limit. This procedure, coupled with theoretical prejudice as to the limit of certain quantities at infinite energy, permits us to predict the detailed behavior of these quantities at high but finite energies. For example we shall make a prediction for the topological n -prong cross sections, σ_n , as a function of n for energies higher than those at which data are presently available.

In order to define the notation used in this article let us review the procedure described in Ref. 1 for establishing the aforementioned analogy.⁴ We are interested in the production, at high energies, of n_1 particles of type 1, n_2 of type 2, etc., by two incident particles with center-of-mass energy \sqrt{s} . Instead of using the momenta of the secondary particles we shall describe a production process by means of the longitudinal rapidity⁵

$$y = \frac{1}{2} \ln \frac{E + p_{||}}{E - p_{||}} \quad (1)$$

and the transverse momentum \vec{p}_{\perp} . Following Wilson (Ref. 1) let us denote (y, \vec{p}_{\perp}) by the collective

variable \vec{r} . Instead of the energy of the incident system we use

$$Y = \ln s. \quad (2)$$

In the center-of-mass system the y 's are confined to essentially $-\frac{1}{2}Y < y < \frac{1}{2}Y$, and the \vec{p}_{\perp} 's are sharply bounded. We may thus assume that all points \vec{r} fall into a cylinder of length Y in the longitudinal or y direction and bounded radius of the order of 300 MeV in the transverse direction. An event is specified by fixing the \vec{r} 's of all produced particles. We thus note the analogy between a specific production configuration and a configuration of particles in a fluid bounded by the above cylinder. The probability of obtaining such a configuration is (up to constants)

$$\rho(\{n_i\}; \{\vec{r}\}; Y) = e^{-2Y} |M(\{n_i\}; \{\vec{r}\}; Y)|^2 \delta^4(P - \sum p). \quad (3)$$

In the above $\{\vec{r}\}$ denotes collectively the coordinates necessary to describe the final state and $M(\{n_i\}; \{\vec{r}\}; Y)$ is the matrix element for this transition. $\rho(\{n_i\}; \{\vec{r}\}; Y)$ may likewise be considered the distribution function for a fluid with n_i particles of type i at points $\vec{r}_{i,j}$, $j=1, \dots, n_i$ in a volume with longitudinal length Y . The cross section for producing n_i particles of type i ,

$$\sigma(\{n_i\}; Y) = \frac{\int d\{\vec{r}\} \rho(\{n_i\}; \{\vec{r}\}; Y)}{n_1! n_2! \dots}, \quad (4)$$

corresponds to the canonical partition function for the analogous fluid. A one-particle distribution in an n -particle configuration for a particle of type 1 for instance is

$$\rho^{(1)}(\{n_i\}; \vec{r}; Y) = \frac{\int' d\{\vec{r}\} \rho(\{n_i\}; \{\vec{r}\}; Y)}{(n_1 - 1)! n_2! \dots}, \quad (5)$$

where the prime on the integration denotes that we do not integrate over one element of volume of type 1. Again one may draw the required analogy with a fluid.

It is a maxim of statistical mechanics that life

is simpler in the grand canonical ensemble than it is in the canonical ensemble. With this in mind let us obtain the grand canonical partition function

$$Q(\{z_i\}; Y) = \sum z_1^{n_1} z_2^{n_2} \cdots \sigma(\{n_i\}; Y), \quad (6)$$

where z_i is the fugacity of the i th particle type (component). The one-particle distribution in the grand canonical ensemble is

$$\rho(\{z_i\}; \{\bar{r}\}; Y) = \sum z_1^{n_1} z_2^{n_2} \cdots \rho(\{n_i\}; \{\bar{r}\}; Y). \quad (7)$$

For fluids one assumes the existence of the thermodynamic limit from which one may obtain the pressure and surface tension

$$\ln Q(\{z_i\}; Y) \underset{Y \rightarrow \infty}{\sim} p(\{z_i\})Y + s(\{z_i\}). \quad (8)$$

(We take the thermodynamic limit solely in the longitudinal direction as dimensions in the transverse directions are bounded.)

The existence of this limit will be investigated in the context of the multiperipheral models and the diffractive dissociation model for multiparticle production. In the multiperipheral model the "pressure" has an interesting interpretation. In all that follows we assume that certain interpretations and hypotheses on experimental observations as valid. The most important of these are: constancy of total cross sections (mild, i.e., logarithmic, variation could be accommodated by dividing all partial cross sections by the total cross section), Feynman scaling,⁶ and short-range correlation in rapidity.¹

II. MULTIPERIPHERAL AND DIFFRACTIVE MODELS

A. Multiperipheral Models

Under this category⁷ we shall include any of the multiexchange mechanisms.⁸ The critical assumptions we shall make on these models is that for a wide range of coupling constants of the produced particles to the exchanged ones we obtain Regge behavior of the total cross section; the inclusive spectrum obtained from these models has a central plateau in the rapidity distribution. For ease of discussion let us concentrate on only one type of produced particle.

Following Fubini⁹ and Mueller² note that the grand partition function Eq. (6) is just the total cross section for a multiperipheral theory in which all the coupling constants of the produced particles have been scaled by a factor of \sqrt{z} . Using the assumptions indicated in the beginning of this section we obtain for large Y

$$Q(z; Y) \approx \sum \beta_r(z) \exp\{[\alpha_r(z) - 1]Y\}. \quad (9)$$

The sum extends over several Regge trajectories. We assume that at $z=1$ the leading trajectory passes through $\alpha(z)=1$. The "pressure" Eq. (8) may now be evaluated and we obtain

$$p(z) = \alpha(z) - 1, \quad (10)$$

where $\alpha(z)$ denotes the $t=0$ intercept of the leading trajectory as a function of z . We thus note that the multiperipheral models allow for the existence of a nontrivial thermodynamic limit.

Based on detailed calculations^{5,10} and on general arguments we expect $\alpha(z)$ to be a smooth monotonic function of z with $\alpha(z=1)=1$. As a consequence the pressure is expected likewise to be smooth and $p(z=1)$ should be zero. We shall soon note that this behavior is indicated by experiment.

The structure of the one-particle distribution may likewise be obtained. Without delving into the details of the model^{5,11} we find that

$$\rho(z; \bar{r}; Y) \approx f(z; p_1) \exp\{[\alpha(z) - 1]Y\}, \quad (11)$$

where for large Y , $\alpha(z)$ is again the leading trajectory. Contingent on the validity of this model we expect to be able to recover the pressure from the one-particle distribution and compare it to the pressure obtained by looking at the grand partition function.

B. Diffractive-Dissociation Models

In this section we shall consider the general properties of models¹² based on the exchange of a vacuum trajectory, geometry, or any other diffractive mechanism that is responsible for the production of many-particle systems. The features of these models are that each $\sigma(n; Y)$ approaches a limit as Y tends to infinity and that the one-particle distribution function is strongly correlated to the beam or target particle. The inclusive spectrum is either the beam or target fragmentation.¹³ There is no central plateau and the particle multiplicities behave as constants with energy. (There is a special version of this type of models¹⁴ which does give a logarithmic rise of the multiplicity; we shall not explicitly discuss this model as the partition function would not exist for $z > 1$.) The partition function, Eq. (6), is independent of Y (the intercept of the leading trajectory is fixed at $\alpha=1$ independent of the coupling constant) and thus the pressure is identically zero. From the preceding discussion we expect all particles to be correlated with the incident momenta; thus it is not surprising that the pressure is zero as all the particles in the "fluid" tend to cling to the "container walls."

C. Hybrid Models

It may be expected that both of the above mechanisms are operative and contribute to the production of many-particle systems. A diffractive mechanism is expected to exist due to the fact that there appear to be exclusive cross sections that survive (up to logarithms) for large energies. That there is another mechanism may be inferred from the fact that the diffractive contributions do not seem to be sufficient to build up the whole of the total cross section and that the multiplicity does grow logarithmically¹⁵ with increasing energy. We shall assume that this other mechanism is of the multiperipheral variety.

Based on discussions of Secs. II A and II B we can outline our expectation on the pressure and on the one-particle distribution in this hybrid model. For $z > 1$ the leading trajectory from the multiperipheral mechanism will dominate and the pressure will be a smooth function $p(z) > 0$, reducing to zero for $z = 1$. For $z < 1$ we expect the diffractive mechanism to become dominant and the pressure should remain equal to zero. In statistical mechanics these are just the features of a phase transition.

The one-particle distribution will likewise behave discontinuously as z passes through the value one. For $z > 1$ the multiperipheral process should dominate and the distribution should be flat in the rapidity variable with some dependence on the rapidity near the edges of phase space. For $z < 1$ the diffractive process will come to play and the distribution should concentrate near the edges of the rapidity plot. As indicated by Eq. (11) the pressure may be determined from the one-particle distribution. In the central region where the multiperipheral mechanism governs the distribution the pressure should be smooth and not indicate any phase transition at $z = 1$. In the wings of the rapidity plot the diffractive process will play a role and we should either obtain a pressure curve with a phase break in it or perhaps find no energy dependence at all in $\rho(z, \bar{Y}, Y)$ and thus have essentially zero pressure for all z . Although the pressure may be defined, in analogy with statistical mechanics, only in the central region, for subsequent discussion we shall extend this concept to the fragmentation region through an application of Eq. (11) to this region.

III. EXPERIMENTAL DETERMINATION

A. Grand Canonical Partition Function

Because of obvious experimental limitations, the only data on $\sigma(n_i; Y)$ that are available for a large

range of energies are on various charged-prong-number cross sections. We shall evaluate the partition function as a function of the fugacity z for charged particles and with the fugacity for neutral particles being kept equal to one. The pressure was obtained as follows. Directly from experimental data we may evaluate

$$Q(z; Y) = \sum z^{2n} \sigma(2n; Y), \quad (12)$$

where $\sigma(2n; Y)$ is the $2n$ -prong topological cross section. From previous discussion we expect that $\ln Q$ will have the following structure for sufficiently large Y :

$$\ln Q(z; Y) = p(z)Y + s(z). \quad (13)$$

A plot of $(\ln Q)/Y$ vs $1/Y$ extrapolated to $1/Y = 0$ will yield the pressure, $p(z)$. This procedure is illustrated for actual p - p data in Fig. 1.^{15,16} Some remarks are in order about the details of this graph. For higher energies the elastic cross section is not included in the report of the experimental data.¹⁵ As it was assumed that the total cross section is a constant an elastic p - p cross section was assumed to have a value yielding a total cross section of around 39 mb. (The magnitude of the elastic cross section necessary to achieve this was always about 10 mb.) Thus the fact that $p(z = 1)$ is equal to zero is not a triumph of this analysis but is built into it from the outset. For lower energies the definition of Y was modified somewhat from that of Eq. (2) in accordance with the procedure used in Ref. 15, and we took

$$Y = 2 \ln(\sqrt{s} - 2m_p). \quad (14)$$

For even moderate s the difference between Eq. (2) and Eq. (13) is slight. After the data are plotted, the best straight line was drawn for each value of z . The intercept gave the value of the pressure. The results of the best linear extrapolation are indicated in Fig. 2. There is no sign of a phase transition.

As may be noted from Fig. 1 a straight line appears to be a credible extrapolation for $z > 1$; for $z < 1$, however, even a cursory glance at Fig. 1 will show that there is more structure. Based on the prejudice acquired in the previous section, a quadratic extrapolation constrained to yield $p(z < 1) = 0$ was made. The details are indicated by the dashed lines of Fig. 1. The utility of this extrapolation will be discussed in Sec. IV. If this extrapolation turns out to be the valid one then the pressure curve would be expected to follow that of Fig. 2 for $z > 1$ and to level off and stay at zero for $z < 1$. In Sec. IV and in Fig. 4 we refer to the best linear extrapolation as extrapolation A and to the one yielding a phase transition at $z = 1$ as

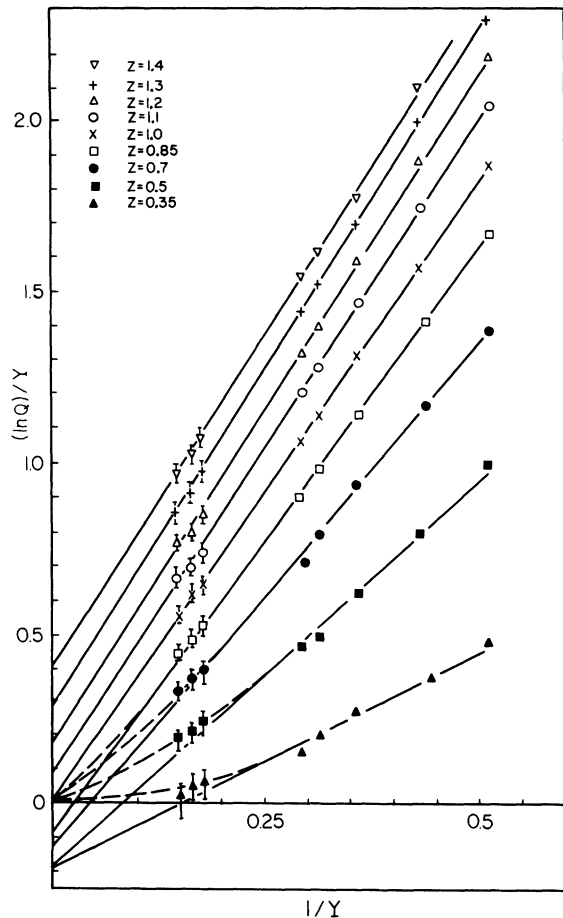


FIG. 1. Logarithm of the partition function divided by Y for $pp \rightarrow$ charged prongs vs $1/Y$. The low-energy points are from Ref. 16 and the three high-energy points are from Ref. 15. The two extrapolations discussed in the text are indicated.

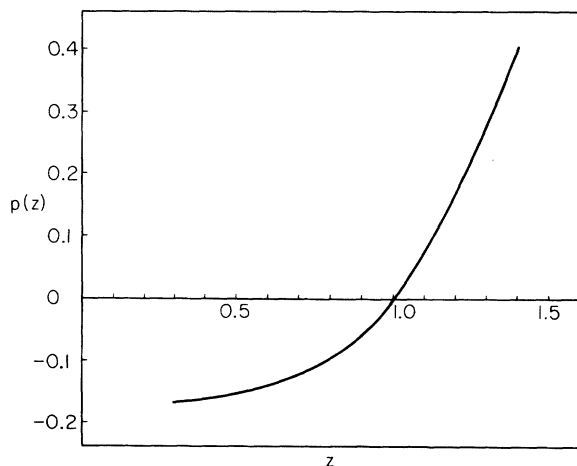


FIG. 2. Partial pressure due to charged particles in $p-p$ collisions as a function of the charged-particle fugacity.

extrapolation B .

B. One-Particle Distribution

In a similar fashion to the one described above, one may form $\rho(z; \bar{r}, Y)$ from data on one-particle distributions from n -prong final states. At present we are limited to machine energies below 30 GeV, where scaling may have set in but where we have as yet seen no evidence for the central plateau. Thus all subsequent comparisons with experiment are suspect and a more definitive analysis will have to await the results of higher-energy experiments. Undaunted, we have looked at various existing inclusive data. Due to experimental difficulties much of the data is presented without absolute normalization. In the spirit of our assumptions discussed in Sec. I the one-particle distributions from n -particle final states were normalized in a way to give scaling for the inclusive distribution at $x=0$. (x is Feynman's⁶ scaling variable, $x = 2p_{||}/\sqrt{s}$.) The processes that were studied were $pp \rightarrow \pi^{\pm} + \text{anything}$,¹⁷ $\pi^{-} p \rightarrow \pi^{-} + \text{anything}$ ¹⁸ and the process on which we shall report the details $\pi^{+} p \rightarrow \pi^{-} + \text{anything}$ at 7 GeV/c^{19,20} and 18.5 GeV/c.¹⁸ Note: this is an exotic inclusive channel²¹ and perhaps scaling will set in at lower energies.

As in Sec. III A we evaluate

$$\rho(z; x; Y) = \sum z^{2n} \frac{d\sigma}{dx} (2n; x; Y) \quad (15)$$

and from this the pressure via

$$\frac{\ln \rho(z; x; Y)}{Y} \underset{Y \rightarrow \infty}{\sim} p(z; x). \quad (16)$$

(Instead of the rapidity in this section we will use Feynman's longitudinal fraction x .⁶) In general we have very few points in Y to work with, and a straight-line extrapolation, though probably unreliable, fits very well. The results of this procedure for the process $\pi^{+} p \rightarrow \pi^{-} + \text{anything}$ for various ranges of x is shown in Fig. 3. For convenience the pressure obtained from the prong distribution is repeated and labeled "topological."

We note that the general trends discussed in Sec. II C are evident. For $z > 1$ the pressure obtained from the prong distributions and that from $x \sim 0$ (hopefully the start of the central plateau) are similar. For $z < 1$ the $x=0$ pressure has much less of an indication of flattening out than does the pressure from the prong distributions. It has the features of a pressure or Regge trajectory characteristic of multiperipheral mechanisms. In the fragmentation regions ($x \sim \pm 0.2$) the pressures show much less variation with z and are in general smaller than the pressure at $x=0$. This is more indicative of a diffractive process. For example

the fragmentation of a π^+ into a π^- is expected to occur in a final state of few particles in an energy-independent way (diffractive production of an A_1 or A_2 resonance). With the appropriate warnings as to the use of low-energy inclusive data the rough features of the hybrid model seem to be supported.

IV. CONCLUSION AND PREDICTIONS

In an erudite way these calculations may be viewed as the computation of the equation of state for the "Feynman gas." More concretely it indicates that the limits discussed do exist and that multiparticle production proceeds via a mixture of mechanisms which may well be the diffractive and multiperipheral ones.

We do not at present have any real indication of a phase transition from the pressure obtained from the prong cross section and very flimsy suspect evidence from the pressure seen in the one-particle distribution. (Admittedly, we have presented the best case here though the results obtained from the other processes discussed in Sec. III C are not markedly different.) However, it is an appealing idea, at least to this author, to have distinct production mechanisms with the consequent phase transition. If such a discontinuity does occur it is interesting to speculate on its nature. As discussed earlier for large fugacities, Q will be dominated by states with a large number of particles more or less uniformly distributed in rapidity. Below $z=1$ it will be the states with fewer particles that will dominate and the produced particles will concentrate at the ends of the rapid-

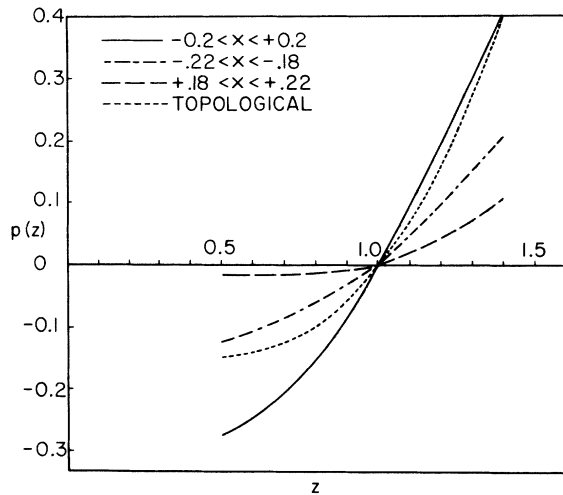


FIG. 3. Pressure as a function of fugacity obtained from the reaction $\pi^+p \rightarrow \pi^- + \text{anything}$ (Refs. 18, 19, and 20). For convenience the pressure from the topological cross sections (Fig. 2) has been replotted.

ity space; the pressure will go to zero and we will have only surface effects. The density at which such a transition may be expected can be estimated from Fig. 2 and the expression relating density of particles to the fugacity,

$$\frac{n}{Y} = z \frac{\partial p}{\partial z}. \quad (17)$$

This derivative evaluated at $z=1$ suggests that the critical n/Y is around 0.75. If these speculations are valid then events with $n/Y < 0.75$ will have the produced particles cluster in the fragmentation regions while for $n/Y > 0.75$ we expect a more uniform distribution.

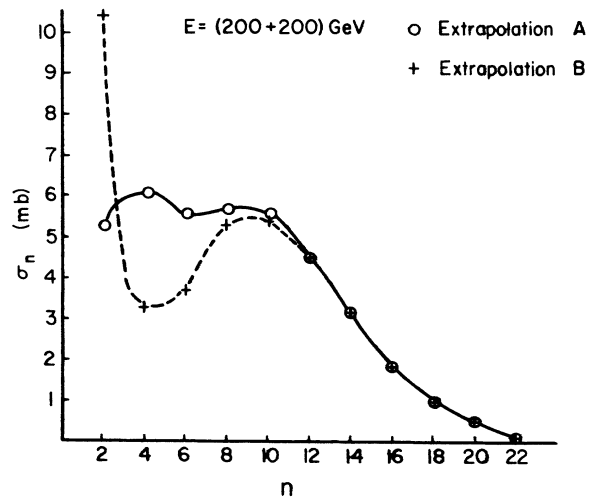
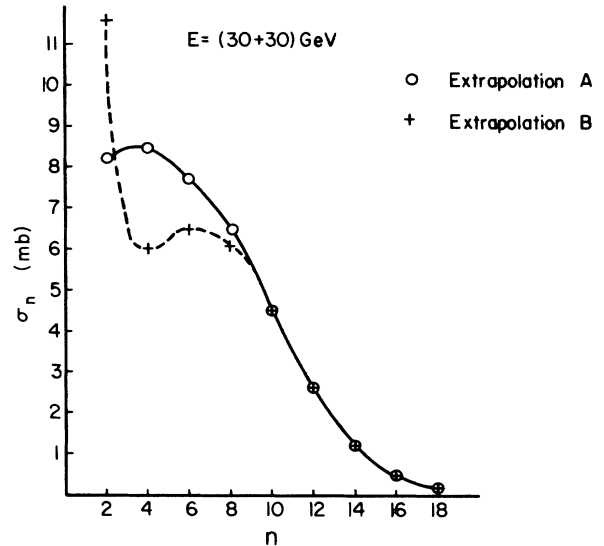


FIG. 4. Prediction of charged-prong cross section as a function of the number of prongs for very high energies. The curves are drawn to guide the eye.

Can we make predictions for higher energies (at least higher than those used in the analysis presented here)? Based on the prejudice with which we extrapolate the data of Fig. 1 in order to obtain the pressure, we may determine $Q(z; Y)$ for Y larger than that of any data points presented. From the $Q(z; Y)$ we may reconstruct the $\sigma(n; Y)$ which would yield such a partition function. Based on the two extrapolation methods discussed earlier, the best linear method (extrapolation A) and the one constrained to give $p(z < 0) = 0$ (extrapolation B), the prong cross sections have been computed for an intersecting storage ring energy of (30 + 30) GeV in the center-of-mass system and for a hoped-for energy of (200 + 200) GeV. The re-

sults are presented in Fig. 4. We note the dramatic dip in the prong cross section in the situation B. The existence of such a dip was emphasized by Wilson.¹

ACKNOWLEDGMENTS

For the central ideas of using the grand canonical ensemble and many other discussions and suggestions I wish to thank Dr. J. D. Bjorken. I am very grateful to Dr. A. Erwin, Dr. T. Ferbel, and Dr. S. Stone for very kindly providing me with details of their experimental data in a form useful for the present analysis. I have benefited from many discussions with Dr. H. Chen, Dr. G. Shaw, and Dr. D. Silverman.

*Supported in part by the National Science Foundation.

¹K. Wilson, Cornell Report No. CLNS-131 (unpublished); J. Bjorken, in *Particles and Fields-1971*, proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971).

²A. H. Mueller, *Phys. Rev. D* **4**, 150 (1971).

³I. Z. Fisher, in *Statistical Theory of Liquids* (Univ. of Chicago Press, Chicago, 1964); T. L. Hill, *Statistical Mechanics* (McGraw-Hill, New York, 1956); K. Huang, *Statistical Mechanics* (Wiley, New York, 1963).

⁴For a review of high-energy multiparticle reactions, see W. R. Frazer *et al.*, University of California, San Diego Report No. UCSD 10P10-83 (unpublished).

⁵C. E. DeTar, *Phys. Rev. D* **3**, 128 (1971).

⁶R. P. Feynman, *Phys. Rev. Letters* **23**, 1415 (1969), and in *High Energy Collisions*, Third International Conference held at the State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

⁷Some of the results obtained in this subsection are applicable to more general situations than just to the multiperipheral model. See Ref. 2.

⁸The essential features of these models are described in Refs. 4 and 5 and in G. F. Chew and A. Pignotti, *Phys.*

Rev. **176**, 2112 (1968).

⁹S. Fubini, in *Strong Interactions and High Energy Physics*, edited by R. Moorhouse (Plenum, New York, 1964).

¹⁰D. Silverman and C.-I Tan, *Phys. Rev. D* **3**, 991 (1971).

¹¹A. H. Mueller, *Phys. Rev. D* **2**, 2963 (1970).

¹²J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, *Phys. Rev.* **188**, 2159 (1969).

¹³D. Silverman, P. D. Ting, and H. J. Yesian, *Phys. Letters* **35B**, 427 (1971).

¹⁴R. Hwa, *Phys. Rev. Letters* **26**, 1143 (1971).

¹⁵L. W. Jones *et al.*, *Phys. Rev. Letters* **26**, 728 (1971).

¹⁶J. D. Hansen, D. R. O. Morrison, N. Tovey, and E. Flaminio, CERN Report No. CERN/HERA 70-2, 1970 (unpublished).

¹⁷D. B. Smith, R. J. Sprafka, and J. A. Anderson, *Phys. Rev. Letters* **23**, 1064 (1969); D. B. Smith, LBL Report No. UCRL-20632, 1971 (unpublished).

¹⁸N. N. Biswas *et al.*, *Phys. Rev. Letters* **26**, 1589 (1971); W. D. Shephard *et al.*, *ibid.* **27**, 1164 (1971); A. Erwin (private communication).

¹⁹M.-S. Chen *et al.*, *Phys. Rev. Letters* **26**, 1585 (1971).

²⁰T. Ferbel and S. Stone (private communication).

²¹Chan Hong-Mo, C. S. Hsue, C. Quigg, and J.-M. Wang, *Phys. Rev. Letters* **26**, 672 (1971).