PHYSICAL REVIEW D VOLUME 6, NUMBER 1 1 JULY 1972

Measuring π-π Phase Shifts in Colliding-Beam Experiments*

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(Received 7 February 1972)

The possibility of exploiting the unique features of the two-photon process $ee \to ee\pi\pi$ in colliding-beam experiments to measure the even partial-wave π-π phase shifts is examined in detail. The commonly used Weizsäcker-Williams approximation is shown to be inadequate. An exact procedure of phenomenological analysis for extracting the relevant phase shifts is formulated. Dynamical (model-dependent) calculations are performed separately to study the expected effects due to the final-state pion-pion interaction. The sensitivity of the measured cross sections to various prominent features of the s-wave π-π phase shifts (scattering length, existence of the scalar-meson resonance) is investigated.

I. INTRODUCTION

It has been generally recognized by now that in the forthcoming generation of e^+e^- and e^-e^- colliding-beam experiments at energies of 3 GeV or higher, the two-photon processes will play a dominant role. Calculations based on quantum electrodynamics indicate that the cross section for this new type of process becomes larger than that of the single-photon annihilation processes (which has dominated colliding-beam physics up to now) at about 1.5–2.0 GeV energy per beam.[1-3] Motivated by these developments, we studied, in a previous paper[4] (hereafter referred to as I), the general features of two-photon processes in colliding-beam experiments for arbitrary hadronic final states[5] and investigated the new possibilities offered by this type of process in studying hadron physics. We pointed out the possibility of probing the constituents of a near-mass-shell photon by the other one in the deep-inelastic region. This was also done by other groups and, in particular, in some detail by Brodsky *et al.*[6] We also emphasized the unique possibilities for studying the s-wave π-π interaction in the kinematically most favored exclusive channel, the two-pion production channel,[7] i.e., $ee \to ee\pi\pi$. It is toward a detailed study of this latter problem that this paper is devoted.

The uniqueness of this reaction (essentially $\gamma\gamma \to \pi\pi$) lies in two facts: (i) There is no other final-state hadron except the two pions (in contrast to all studies of the $\pi\pi$ system in pure hadronic reactions where a baryon is always present in the final state); (ii) because the initial state has positive charge conjugation, the two pions must have

$$J^P = 0^+, 2^+, \ldots \quad \text{and} \quad I^G = 0^+, 2^+$$

(in contrast to the corresponding one-photon annihilation process where $J^P = 1^-$, $I^G = 1^+$). The advantages are obvious: Because of (i), no theoretically ambiguous final-state correction effects (e.g., between the outgoing nucleon and pions) have to be taken into account in order to extract information on the π-π interaction; because of (ii), the dominant p-wave interaction (ρ meson) is absent, allowing an ideal chance to study the s-wave π-π interaction cleanly over a large energy range. In our previous work[4] (I), we outlined a phenomenological procedure to extract the s-wave π-π phase shifts δ_0^I and proposed a model to calculate the $\gamma\gamma \to \pi\pi$ amplitude when δ_0^I are given. Some related works[8] have also appeared in the last year. In all these works (including I) one assumes the standard equivalent-photon or Weizsäcker-Williams (WW) approximation formula which reduces the $ee \to ee\pi\pi$ process to that of $\gamma\gamma \to \pi\pi$ for on-shell photons. It is not hard to see, however, that although the WW approximation is appropriate for order-of-magnitude calculations (which were what the original investigations intended), it can induce errors of up to 50% in the differential cross section. This is comparable to, or, in certain regions, bigger than the effects due to π-π interactions which one tries to isolate in the present case. In order to extract useful information from this process one clearly needs to improve the approximation scheme.

In this paper we do three things: (i) We investigate the range of validity of the WW approximation and propose substitutes for it which are appropriate for the purpose of extracting information on π-π interactions; (ii) we propose phenomenological

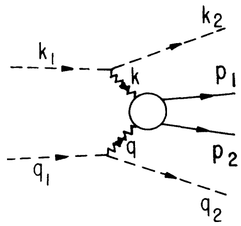

FIG. 1. The two-pion production process via two-photon exchange in e-e collision. The dashed lines are electrons, wavy lines are virtual photons, and solid lines are pions.

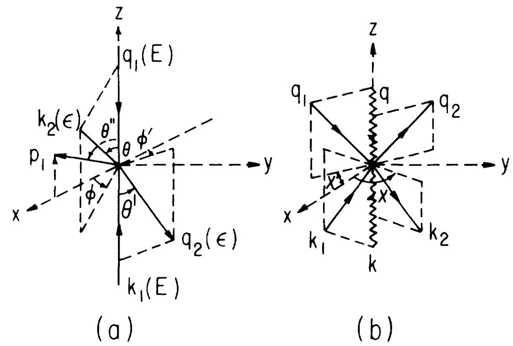

FIG. 2. (a) Kinematics of process (1) defined in the laboratory frame. The particle momenta correspond to those of Fig. 1. The quantities in parentheses after the momentum labels are the associated energy variables in the lab. (b) Kinematics of process (1) defined in the c.m. frame of the two outgoing pions. The pion momenta p_1 and p_2 are in the x-z plane and are not shown in the figure to avoid confusion. χ and χ' are the respective azimuthal angles of the planes of scattering for the two pairs of leptons. The variables ψ and ψ' cannot be given a simple geometrical interpretation in this figure (see Ref. 9).

procedures for extracting π-π phase shifts which are as model-independent as possible and discuss built-in checks; (iii) we present model calculations which illustrate the effects of the π-π interaction on the measured cross sections and test the sensitivity of the latter to certain qualitative features of the s-wave π-π phase shifts, for instance the value of the scattering length and the existence of the σ resonance.

An attempt is made to make this paper reasonably self-contained. We try to make the procedure and reasoning clear and the results explicit without giving too many details which either can be found in I or are relegated to the Appendixes.

II. EXACT FORMULATION OF THE SCATTERING AMPLITUDES AND CROSS SECTION

Consider the process

$$e+e\to e+e+\pi+\pi \tag{1}$$

via the two-photon-exchange mechanism. This is depicted in Fig. 1, which also specifies our momentum variables. The kinematics of this process is fully specified by 8 independent variables. In the laboratory (the c.m. system of the incoming leptons) these can be chosen as E, the incoming energy; $\epsilon, \theta, \phi, \epsilon', \theta', \phi'$, the energy and angles of two outgoing particles, say the two leptons; and θ'' the scattering angle of a third particle, say a pion, which also defines the x-z plane [Fig. 2(a)]. For calculational purposes, it is more convenient to choose as independent variables k^2 and q^2 (the two virtual-photon masses); the "hadronic variables" s (the total c.m. energy squared) and θ_π (the scattering angle) for the process $\gamma\gamma\to\pi\pi$ in its c.m. frame; and "lepton variables" (ψ, χ) and (ψ', χ') which specify, respectively, the configuration of each of the two pairs of lepton momenta in the "rest frame" of the corresponding virtual photon [i.e., the frame in which the photon 4-momentum has only one nonvanishing component, Fig. 2(b)]. These two sets of variables are defined in Fig. 2 and in I; the relations between them are given in Appendix A of I. We shall write out these relations explicitly when needed in subsequent discussions.

The exact amplitude for the above process corresponding to Fig. 1 can be written as

$$f=\langle k_2|\,j_\mu\,|k_1\rangle\frac{e^2}{k^2}\left(\int d^4x\,e^{ikx}\langle p_1p_2|\,T(J^\mu(x)J^\nu(0))\,|0\rangle\right)\frac{e^2}{q^2}\langle q_2|\,j_\nu\,|q_1\rangle\,. \tag{2}$$

Decomposing the virtual photons into their helicity components, each of the two vector products can be written as

$$j_\mu J^\mu=\sum_m j^{(m)}J_{(m)}=\sum_m (j\cdot\epsilon^{(m)*})(\epsilon_{(m)}\cdot J)\,, \tag{3}$$

where $\epsilon^\mu_{(m)}(k)$, $m=1,0,-1$, are the helicity polarization vectors associated with the virtual photon k. Substituting into (2), we obtain the following physically transparent expression for the scattering amplitude:

$$f=\langle k_2|\,j^{(m)}\,|k_1\rangle\frac{e^2}{k^2}\,T_{m,n}(k^2,q^2,s,\theta_\pi)\frac{e^2}{q^2}\langle q_2|\,j^{(n)}\,|q_1\rangle\ , \tag{4}$$

where

$$T_{m,n}=\int d^4x\, e^{ikx}\langle p_1p_2|T(J_\mu(x)J_\nu(0))|0\rangle\epsilon^\mu_{(m)}(k)\epsilon^\nu_{(n)}(q) \tag{5}$$

is clearly the helicity amplitude for the "hadronic process":

$$\gamma(k,m)+\gamma(q,n)\to\pi(p_1)+\pi(p_2)\,. \tag{6}$$

The cross section for $ee\to ee\pi\pi$ is

$$d\sigma_{ee}=\frac{\alpha^4}{2^8\pi^4E^2}\left(1-\frac{4\mu^2}{s}\right)^{1/2}\frac{1}{k^2q^2}\,d\rho\, d\Omega_\pi L^{lm}(\psi,\chi)\,W_{l,j;m,n}(k^2,q^2,s,\cos\theta_\pi)L^{jn}(\psi',\chi')\,, \tag{7}$$

where the same factorized form as in Eq. (4) is explicitly displayed. We have

$$W_{l,j;m,n}=T^*_{l,j}\,T_{m,n}\,,$$

$$L^{lm}(\psi,\chi)=\frac{1}{2k^2}\sum_{\lambda_1\lambda_2}\langle k_2\lambda_2|\,j^{(l)}|k_1\lambda_1\rangle^*\langle k_2\lambda_2|\,j^{(m)}|k_1\lambda_1\rangle\,,$$

$$L^{jn}(\psi',\chi')=\frac{1}{2q^2}\sum_{\lambda_1\lambda_2}\langle q_2\lambda_2|\,j^{(j)}|\,q_1\lambda_1\rangle^*\langle q_2\lambda_2|\,j^{(n)}|\,q_1\lambda_1\rangle\,, \tag{8}$$

$$d\rho=(d^3k_2/k_2^0)(d^3q_2/q_2^0)$$

= invariant phase-space element for the outgoing leptons,

$d\Omega_\pi$ = solid angle element in c.m. system of $\pi\pi$.

In Eq. (7), the leptonic matrix elements which are *known* functions[9] of the leptonic variables are completely factored from the hadron amplitudes which are *unknown* functions of k^2, q^2, and the other hadron variables. This formula gives the exact connection between the measured $ee\to ee\pi\pi$ cross section and the "density matrix" $W_{lj,mn}$ for the $\gamma\gamma\to\pi\pi$ process. The diagonal elements of this matrix $W_{mn,mn}$ are clearly just proportional to the cross section for $\gamma\gamma\to\pi\pi$, Eq. (6).

In principle, by measuring the distributions in the pure lepton variables (ψ, χ, ψ', χ' or their laboratory equivalents) one can isolate all the elements $W_{lj,mn}$ and from them determine the scattering amplitudes T_{mn} for $\gamma\gamma\to\pi\pi$. We emphasized in I that because the kinematics favor the low-energy region and because of the exclusion of the strong p wave by charge conjugation, the s partial wave plays the dominating role for this process. Since in the elastic unitary region the phases of the partial-wave amplitudes are just those of the π-π scattering,[10] one should be able to get very interesting information on the s-wave π-π phase shifts.

The exact relation Eq. (7) contains 20 distinct terms as given in (A9) of I. In practice, one needs to simplify the expression in order to make the problem more tractable, both experimentally and theoretically. One way is to average over the azimuthal angles χ and χ'. [These are the azimuthal angles of the scattering planes of the two pairs of leptons with respect to the scattering plane of the strong process $\gamma\gamma\to\pi\pi$ in the c.m. frame of the latter, Fig. 2(b).] We get

$$\begin{aligned}d\sigma=\frac{\alpha^4}{2^6\pi^2E^2}\left(1-\frac{4\mu^2}{s}\right)^{1/2}\frac{1}{k^2q^2}\,d\rho'd\Omega_\pi\\ \times[(\cosh^2\psi+1)(\cosh^2\psi'+1)\tfrac{1}{2}(W_{11,11}+W_{1\,-1,1\,-1})+(\cosh^2\psi+1)(\cosh^2\psi'-1)W_{10,10}\\ +(\cosh^2\psi-1)(\cosh^2\psi'+1)W_{01,01}+(\cosh^2\psi-1)(\cosh^2\psi'-1)W_{00,00}]\,,\end{aligned} \tag{9}$$

where

$$d\rho'=d\rho/d\chi d\chi'\,,$$

$$\cosh\psi=-q\cdot(k_1+k_2)/[(k\cdot q)^2-k^2q^2]^{1/2}\,, \tag{10}$$

$$\cosh\psi'=-k\cdot(q_1+q_2)/[(k\cdot q)^2-k^2q^2]^{1/2}\,.$$

Equation (9), which is still an exact expression for our process [Eq. (1) and Fig. 1], serves as the basis for our subsequent discussions.

So far, we have discussed only one diagram (i.e., Fig. 1) contributing to process (1). In principle there are other diagrams of the same order in α which could also contribute to this process, e.g., Figs. 3(a) and 3(b). It is clear, however, that diagrams of the type Fig. 3(a) are a factor α down from the single-photon annihilation processes which themselves are insignificant at high energies. Diagrams of the type Fig. 3(b) have been studied by several groups[2,11] and shown to be negligible as compared to the main diagram, Fig. 1,

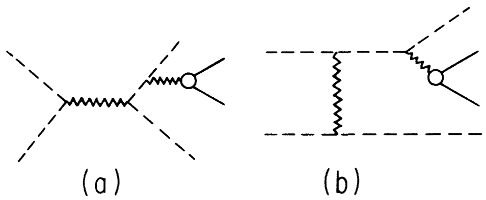

FIG. 3. Additional diagrams of the same order in α as Fig. 1 which may contribute to process (1).

as long as one does not restrict oneself to the large k^2, q^2 region. None of these should create a serious background problem for process (1) for energies of the order $E \gtrsim 3$ GeV.

III. THE WW APPROXIMATION AND ITS RANGE OF VALIDITY

It is customary to approximate Eq. (9) by noting that the small virtual-photon mass region $k^2 \simeq 0$, $q^2 \simeq 0$ is most important. Setting $k^2=0$, $q^2=0$ in the square bracket in Eq. (9), two significant simplifications occur: (i) The hadron amplitudes involving longitudinal photons vanish, thus only the first term survives; (ii) the kinematics simplify, thus the lepton lab scattering angles $\theta=\theta'=0$, and one can show

$$\begin{aligned} \cosh\psi &= (E+\epsilon)/(E-\epsilon)\,, \\ \cosh\psi' &= (E+\epsilon')/(E-\epsilon')\,. \end{aligned} \tag{11}$$

After integrating over all the lepton variables, one gets the standard formula[1-3,12]:

$$\frac{d\sigma^{(ee)}}{ds\,d(\cos\theta_\pi)} = 4\left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(\frac{E}{m}\right)\frac{1}{s}\left(\ln\frac{4E^2}{s}-\frac{3}{2}\right)\frac{d\sigma^{(\gamma\gamma)}}{d(\cos\theta_\pi)}\,, \tag{12}$$

where $d\sigma^{(\gamma\gamma)}/d(\cos\theta_\pi)$ is just the on-shell $\gamma\gamma\to\pi\pi$ differential cross section. The great majority of recent works on process (1) are based on the use of (12), which allows one to concentrate on the simpler process $\gamma\gamma\to\pi\pi$ with on-shell photons.

What is the range of validity of Eq. (12)? To answer this question, we have studied in some detail the case of pointlike pions, for which one knows the exact form of the matrix elements T_{mn} (and thus $W_{ij,mn}$).[13] One can then calculate the cross section from the exact formula, Eq. (9). The relevant Feynman diagrams are given in Fig. 4. The calculations are quite involved. We relegate the details to Appendix A and only discuss the results here.

Making use of some previous results of Baier and Fadin,[14] our detailed calculations yield

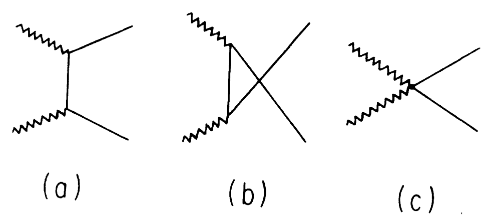

FIG. 4. Gauge-invariant "Born diagrams" for pointlike pions.

$$\begin{aligned} \frac{d\sigma^{(ee)}}{ds\,d(\cos\theta_\pi)} &= \frac{\alpha^4}{\pi s^2}\,\beta\left\{\left[L^2(L_s-\tfrac{3}{2}) - L_s^2(\tfrac{1}{3}L_s-\tfrac{3}{2}) - \tfrac{17}{3}LL_s\right]A \right. \\ &\qquad \left. +\tfrac{1}{3}LL_s B\right\}\,, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \beta^2 &= 1-4\mu^2/s\,, \\ L &= \ln(4E^2/m_e^2)\,, \\ L_s &= \ln(4E^2/s)\,, \\ A &= \frac{(4\mu^2/s)^2+\beta^4\sin^4\theta_\pi}{(1-\beta^2\cos^2\theta_\pi)^2}\,, \\ B &= 1+\frac{16\mu^2/s}{1-\beta^2\cos^2\theta_\pi}+\frac{2\beta^4\sin^2 2\theta_\pi}{(1-\beta^2\cos^2\theta_\pi)^2}\,, \\ \mu &= \text{pion mass}\,. \end{aligned} \tag{14}$$

Equation (13) is accurate up to terms which are quadratic in the large logarithmic factors L and L_s. Terms linear in these as well as terms of the form $(s/4E^2)L^2$ or $(s/4E^2)LL_s$ which are small in the energy range we consider are neglected. The standard WW formula, Eq. (12), corresponds to keeping only the term $L^2(L_s-\frac{3}{2})A$ on the right-hand side of Eq. (13) and approximating L^2 by

$$L^2 \simeq 4\ln^2(E/m)\,.$$

To get some feeling about Eq. (13), let us substitute some typical values of the variables. Thus, for $E=3$ GeV, $W=\sqrt{s}=2.25\mu$ we get

$$\begin{aligned} s/4E^2 &= 2.7\times10^{-3}\,, \\ L &= 18.7\,, \\ L_s &= 5.8\,. \end{aligned}$$

The coefficient of the A term in (13) comes out to be ~880 while the corresponding number obtained from the WW formula, Eq. (12), is ~1300. The difference is ~50% of the true answer. The main correction effect comes from the $-\frac{17}{3}LL_s$ term. The second term in the square brackets and the B term make relatively small corrections.

Previously, Brodsky *et al.*[3] investigated the validity of the WW approximation for the simpler case of single π^0 production. They found the exact result to be about 25% higher than that given by Eq. (12) and the effect is reduced by inserting a form factor for the π^0 vertex. The fact that the corrections to Eq. (12) in these two cases occur in opposite directions can be readily understood. This is explained in a footnote.[15] We shall show later on that strong-interaction effects are not likely to reduce the size of the corrections to Eq. (12) for our case.

Clearly, if one hopes to extract useful information from the two-photon process $ee \to ee\pi\pi$, 50% inaccuracy can not be tolerated and Eq. (12), as it stands, is useless. One must go back to the exact formula, Eq. (9).

This is not necessarily a setback as it might appear to be. A moment's reflection will reveal that the simple formula Eq. (12) cannot be used even if it were accurate. The reason is that to make certain that one is indeed measuring the $ee \to ee\pi\pi$ process, one has to tag one of the outgoing electrons in addition to detecting the pions. Only in this way can one exclude the one-photon annihilation background as well as eliminate the possibility of producing additional neutral particles. The WW approximation formula, Eq. (12), having integrated over all lepton variables, does not describe this situation. One will have to go back to the complete differential formula Eq. (7), or the partially integrated formula Eq. (9). We believe the latter is a more practical choice.

Two additional points are worth mentioning:

(i) The deviation from the WW approximation mainly comes from the large k^2 and/or q^2 region. Therefore, the equivalent photon approximation can be used with good success if one only integrates over the small k^2, q^2 region. A very interesting study in this respect is given by Kessler *et al.* in the last paper of Ref. 2.

(ii) Equation (9), which will be the basis of much of the subsequent discussions, is obtained by averaging over the azimuthal angles χ and χ' which are defined in the c.m. frame of the two outgoing pions. These variables are not to be confused with the lab azimuthal angles ϕ, ϕ' which most authors use. (The latter cannot be varied independently from s for fixed E, E'. Thus averaging over ϕ, ϕ' for fixed s and other variables as is implicitly done in most papers is not always a meaningful operation.) Our averaging procedure is done after the transformation to the π-π c.m. frame is carried out and for each fixed value of s. We note that for this colliding process, the lab and pion c.m. frames are not very much different, the transformation from the former to the latter does not significantly distort the kinematics of the individual events.

IV. EXTRACTION OF π-π PHASE SHIFTS-PHENOMENOLOGICAL ANALYSIS

We begin our phenomenological analysis by assuming the ideal situation where measurements of the process can be done with great accuracy and outline the model-independent procedure to extract the π-π phase shifts. Later on, we introduce the usual plausible dynamical assumptions which simplify the analysis with less than perfect data.

Consider the helicity amplitudes $T_{m,n}(k^2, q^2, s, \cos\theta_\pi)$ introduced earlier [Eq. (5)] for the process $\gamma\gamma \to \pi\pi$ [Eq. (6)]. We shall take the variables k^2, q^2, and s to be fixed and only explicitly display the θ_π dependence. Each of the helicity amplitudes has the partial-wave expansion

$$T_{mn}(\theta_\pi) = \sum_{l\ \text{even}} (2l+1) a_{mn}^{(l)}\, d_{m-n,0}^{(l)}(\theta_\pi)\,, \tag{15}$$

where $d^{(l)}(\theta_\pi)$ are the usual rotational matrices. The important features to notice are: (i) Because the charge-conjugation quantum number for the $\gamma\gamma$ state is +, the two pions can only be in states with even isospin and even angular momentum; (ii) to lowest order in α, the unitarity condition demands that the phase of each partial-wave amplitude $a_{mn}^{(l)}$ (for given isospin $I=0, 2$) is the same as the corresponding partial-wave amplitude for π-π scattering, at least when s is below the inelastic threshold, i.e.,

$$a_{mn}^{(l)I} = |a_{mn}^{(l)I}|\, e^{i\delta_l^I}\,, \tag{16}$$

where δ_l^I is the π-π phase shift for isospin I and angular momentum l.

Substituting (15) and (16) into Eq. (7) or (9) we get the connection between the measured cross section and the π-π phase shifts. To be more explicit, let us consider Eq. (9). The $W_{ij,mn}$ which enter into this formula can be written out (we omit the isospin index)

$$\begin{aligned} W_{11,11} &= |a_{11}^{(0)}|^2 + 10|a_{11}^{(0)}||a_{11}^{(2)}|\cos(\delta_0-\delta_2) d_{0,0}^{(2)}(\theta_\pi) \\ &\quad + 25|a_{11}^{(2)}|^2[d_{0,0}^{(2)}(\theta_\pi)]^2 + \cdots\,, \\ W_{1\,-1,1\,-1} &= 25|a_{1,-1}^{(2)}|^2[d_{2,0}^{(2)}(\theta_\pi)]^2 + \cdots\,, \\ W_{01,01} &= 25|a_{0,1}^{(2)}|^2[d_{-1,0}^{(2)}(\theta_\pi)]^2\,, \end{aligned} \tag{17}$$

with similar expressions for $W_{10,10}$ and $W_{00,00}$. If one can separate the various terms in the above formulas (by measuring the distributions in ψ and θ_π for fixed k^2, q^2, and s) one can eliminate the unknowns $|a_{mn}^{(l)}|$ and solve for $\cos(\delta_l - \delta_{l'})$, l, l' = even integers. A very important feature of this type of phase-shift analysis is the fact that the factors $\cos(\delta_l - \delta_{l'})$ enter linearly in the cross-section formula. Consequently, the well-known ambiguities associated with phase-shift analysis in hadron-hadron scattering do not arise at all in this case.

To simplify the parametrization, one can invoke the argument of angular momentum barriers for higher partial waves and assume that the final-state interaction is negligible except in the lowest angular momentum states. Further, since higher angular momentum corresponds to larger impact parameter where the interaction is dominated by the exchange of the least massive particles, one expects the higher partial waves to be dominated by the single-pion-exchange Born diagram, Fig. 4(a). Gauge invariance demands that the crossed Born diagram and seagull diagram, Figs. 4(b) and 4(c) be also included. One can, therefore, set in Eq. (17)

$$\begin{aligned} &\delta_l = 0\,, \\ &a_{mn}^{(l)} = a_{mn}^{(l)(B)}\,, \end{aligned} \qquad \text{for } l > l_0 \tag{18}$$

where $a_{mn}^{(l)(B)}$ are the partial-wave amplitudes calculated from the Born diagrams of Fig. 4 and l_0 is some small even integer.

More directly, one can write

$$\begin{aligned} W_{lj,mn} &= T_{lj}^{*}\, T_{mn}\,, \\ T_{mn} &= T_{mn}^{(B)} + \sum_{l=0}^{l_0} (2l+1)(|a_{mn}^{(l)}|\, e^{i\delta_l} - a_{mn}^{(l)(B)}) \\ &\qquad \times d_{m-n,0}^{(l)}(\theta_\pi) \end{aligned} \tag{19}$$

and substitute into Eq. (9) or Eq. (7). The phase shifts δ_l can then be extracted from data by systematically eliminating the unknowns $|a_{mn}^{(l)}|$, $l=0,\ldots,l_0$. For energies not too far from the threshold, one can simply take $l_0=0$ (we remind the reader again that the p wave is absent) and the procedure outlined above then reduces to that of I. The validity of the assumptions which go into this phenomenological procedure can, of course, be directly checked by experiment. As the quality of the data improves, one can increase the value of l_0 and see if the approximation (18) is valid or not. A similar procedure is known to be very useful in N-N phase-shift analysis[16] (where the situation is more complex due to nonlinear unitarity).

Except for Eq. (16), we have not included the isospin in our discussion. In principle, the only model-independent way of disentangling the $I=0$ and $I=2$ amplitudes is to measure both the $\pi^+\pi^-$ and $\pi^0\pi^0$ final states. However, there are both theoretical reasons and experimental evidence[17,18] that the $I=2$ final-state interaction is not important in the low-energy region. Therefore, as a first approximation, one can again use the gauge-invariant Born diagram result for the $I=2$ part. As the range and quality of data improve, this approximation can be removed.

In an actual experiment, one may not be able to completely average over the two variables χ and χ' as proposed. In that case, Eq. (7) still applies. With incomplete integration over χ and χ', some terms in addition to those in Eq. (9) will survive. They can be treated in the same manner as outlined in the above. An important point to bear in mind is that most additional amplitudes involve one or more longitudinal photons (helicity index 0) and give small contributions to the cross section.

In addition to the phenomenological analysis, discussed above, one can make dynamical assumptions which allow one to "calculate" the unknowns $|a_{mn}^{(l)}|$ and δ_l in the formulas given. One can then "predict" the measured $ee\to ee\pi\pi$ cross sections through Eq. (7) or (9). In particular, dispersion relations (which connect the real and imaginary parts of an analytic function) can be used to relate $|a_{mn}^{(l)}|$ to δ_l provided the latter is known for all energies. On the other hand, some predictions on δ_l itself can be made if one imposes some combinations of constraints due to crossing symmetry, analyticity, unitarity, partial conservation of axial-vector current (PCAC), and current algebra – to the best of one's ability. The dynamical theories involved, however, are far from perfected. A direct comparison of experimental data with such "predictions" which are obtained after a long chain of model-dependent calculations probably will not be particularly revealing. Since theoretically the $\pi\pi$ phase shifts δ_l^I are much more interesting than the absolute values of the partial-wave amplitudes $|a_{mn}^{(l)}|$, it is clearly desirable to extract δ_l from the data in as model-independent a way as possible and then compare the results with theoretical models for π-π scattering. This is the approach advocated here.

V. MODEL CALCULATIONS - SENSITIVITY OF MEASURED CROSS SECTION TO QUALITATIVE FEATURES OF THE s-WAVE π-π PHASE SHIFT

In spite of our caution against making theoretical predictions, if one wants to get some feeling of the effectiveness of extracting the π-π phase shifts from $e+e\to e+e+\pi+\pi$, it is useful to estimate the order of magnitude of correction effects due to π-π interaction on the measured cross section. Such studies could be useful in providing a guide for the planning of relevant experiments. The precise results may be somewhat model-dependent, but the qualitative features should be instructive if the dynamical assumptions made are reasonable.

Our model consists of using dispersion relations to calculate $|a_{mn}^{(0)}|$ from assumed phenomenological test phase shifts δ_0. We then test the sensitivity of the measured $ee\to ee\pi\pi$ cross section to variations of some important qualitative features of the

s-wave π-π phase shifts, such as the magnitude of the scattering length and the existence of the scalar meson around 700 MeV.

As mentioned before, in an actual experiment it is probably necessary to tag one or both of the outgoing electrons, thus specifying the full kinematics of $ee \to ee\pi\pi$. However, in order to understand the qualitative features, it is more transparent to present results for the case where the lepton variables are integrated. This we shall do in what follows. Our basic formula is therefore Eq. (9) with fixed s and $\cos\theta_\pi$ but all remaining lepton variables integrated over. We have discussed in some detail the results of such a calculation for the case of pointlike pions (Born approximation) in Sec. III and Appendix A. For the present case, we perform the same calculation with only one modification, namely, the s-wave Born amplitudes $a_{mn}^{(0)B}$ are replaced by the unitarity-corrected $|a_{mn}^{(0)}|e^{i\delta_0}$. [See Eq. (19) with $l_0=0$.]

The essential features of our model were described in our previous paper.[4] We shall outline them here.

(1) From our experience in Sec. III and Appendix A, we know that $W_{00,00}$, being proportional to k^2q^2, does not contribute significantly to the cross section. [This is because the factor k^2q^2 cancels the denominator $(1/k^2q^2)$ from the virtual-photon propagator, so the integrated cross section then loses two powers of $L=\ln(4E^2/m_e^2)$ which is the dominating factor.] We can write therefore

$$\frac{d\sigma}{ds\,d(\cos\theta_\pi)}=\frac{\alpha^4}{2^9\pi E^2}\left(1-\frac{4\mu^2}{s}\right)^{1/2}\int d\rho''\,\frac{1}{k^2q^2}\Big[(\cosh^2\psi+1)(\cosh^2\psi'+1)\tfrac{1}{2}(W_{11,11}+W_{1\,-1,1\,-1})$$
$$+(\cosh^2\psi+1)(\cosh^2\psi'-1)W_{10,10}+(\cosh^2\psi-1)(\cosh^2\psi'+1)W_{01,01}\Big],\tag{20}$$

where $d\rho''=d\rho/d\chi d\chi' ds$ and $\cosh\psi$ and $\cosh\psi'$ are given by (10).

(2) Now, we apply Eq. (19), which reads in this case

$$W_{mn,mn}=|T_{mn}|^2,$$
$$T_{11}=T_{11}^{(B)}+|a_{11}^{(0)}|e^{i\delta_0}-a_{11}^{(0)(B)},\tag{21}$$
$$T_{mn}=T_{mn}^{(B)}\ \text{ for }\ (m,n)=(1,-1),\ (0,1),\ \text{ and }(1,0).$$

The amplitudes $T_{mn}^{(B)}$ for charged pions are given in Appendix A. The only s-wave partial-wave amplitude [for $(m,n)=(1,1)$] is

$$a_B(s,k^2,q^2)=\frac{s}{s+k^2+q^2}\,a_B(s,0,0)+\frac{2(k^2+q^2)}{s+k^2+q^2},\tag{22}$$

where

$$a_B(s,0,0)=\frac{1-\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta},\quad \beta^2=1-4\mu^2/s.\tag{23}$$

The corresponding amplitudes for $\pi^0\pi^0$ production are obviously zero.

The amplitudes with definite isospin are related to the charged (c) and neutral (n) amplitudes by

$$T^{(I=0)}=\tfrac{2}{3}T^c-\tfrac{1}{3}T^n,\qquad T^{(I=2)}=(\sqrt{2}/3)(T^c+T^n)\tag{24}$$

or

$$T^c=T^{(0)}+(1/\sqrt{2})T^{(2)},\qquad T^n=-T^{(0)}+\sqrt{2}\,T^{(2)}.\tag{25}$$

(3) The unitarized s-wave amplitude $a(s)$ in Eq. (21) is calculated from the dispersion relation for the partial-wave amplitude. Assuming elastic unitarity, this dispersion relation becomes a standard Omnès-type integral equation. This equation can be solved in a standard way for any given set of π-π phase shifts. The form of this integral equation and explicit expressions for the solution are given in Appendix B. We only make two relevant remarks here:

(i) Strictly speaking, the solution to the Omnès equation is not unique; possible polynomial terms could be added. The arbitrariness can be fixed by additional physical requirements on the asymptotic behavior of the amplitude. In obtaining our solution, we have assumed that the s-wave partial wave does not rise linearly in s as $s\to\infty$.

(ii) In this model, the $\pi^0\pi^0$ production proceeds entirely through the s-wave partial amplitude and is isotropic in the π-π c.m. frame. The cross section for $ee\to ee\pi^0\pi^0$ can be obtained from Eq. (20) by setting

$$W_{10,10}^{(n)}=W_{01,01}^{(n)}=W_{1\,-1,1\,-1}^{(n)}=0,\qquad W_{11,11}^{(n)}=|T_{1,1}^{(n)}|^2=|-a^{(0)}+\sqrt{2}\,a^{(2)}|^2.\tag{26}$$

Now, we have everything expressed in terms of $\delta_0(s)$. We can therefore perform the calculation by assuming various possible sets of phenomenological phase shifts. Since we are interested only in the qualitative features of our model calculations, we choose simple analytic forms for the phase shifts which possess the desired properties. In choosing these phase shifts we obviously also bear

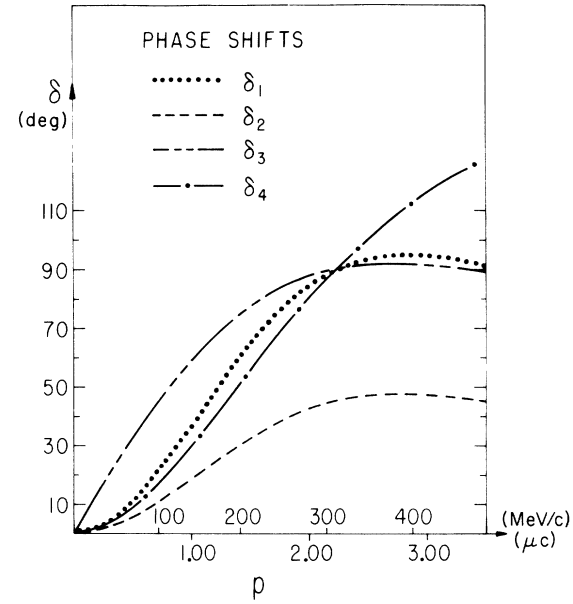

FIG. 5. Four sets of phase shifts used in the calculation of the magnitude of the s-wave amplitude for $\gamma\gamma \rightarrow \pi\pi$.

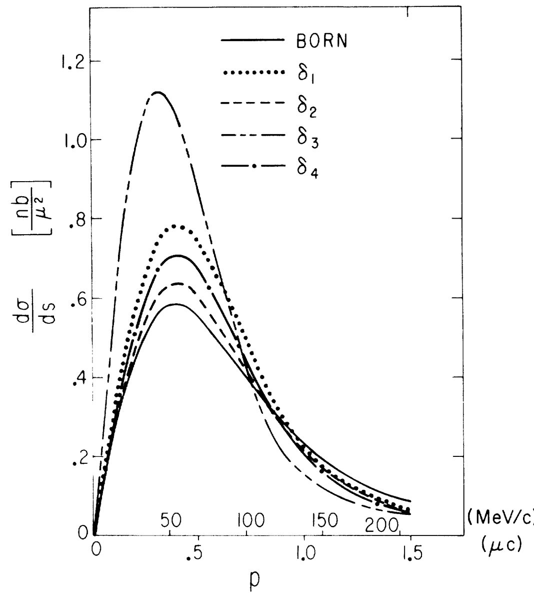

FIG. 6. Integrated cross section for process (1), $d\sigma/ds$, as a function of p, the magnitude of pion momentum in the dipion c.m. frame. The solid line is the cross section for pointlike pions. The other lines are unitarity-corrected using the 4 sets of phase shifts given in Fig. 5. The incident electron energy is $E = 3$ GeV.

in mind the available experimental information as well as theoretical models for $\delta_0(s)$. There are three features of the phase shift which determine its qualitative behavior over the elastic scattering region. These are (i) the scattering length; (ii) possible resonances ($\delta = \pi/2$); (iii) asymptotic behavior (δ increases to $n\pi$ or decreases to zero). We have chosen many sets of phase shifts which exhibit different types of behavior under these three categories and tested the effects on the measured cross section.

We present here results obtained from four sets of such phase shifts which illustrate the qualitative features of our model calculations. One of these is chosen to agree qualitatively with the phase shifts calculated by Kang and Lee[18] or Brown and Goble[18] using current algebra, unitarity, dispersion theory, and crossing symmetry. The features of this phase shift are its small scattering length, resonant behavior near $\sqrt{s} = 700$ MeV, and asymptotic return to zero. Each of the other three phase shifts differs from the first qualitatively in one respect, that is, one has no σ resonance, one has a large scattering length, and one approaches the value π as $s \rightarrow \infty$. These phase shifts, as functions of the pion c.m. momentum are depicted in Fig. 5. Analytic forms for these test phase shifts and the corresponding Omnès functions are given in Appendix B.

Some typical results are presented in Figs. 6 and 7. In Fig. 6, the integrated cross section $d\sigma/ds$ is plotted as a function of the pion momentum. The incident electron energy is taken to be 3 GeV. The solid curve is the cross section due to the Born diagrams without π-π interaction. The other curves are the unitarity-corrected cross sections with the 4 different sets of phase shifts mentioned previously. Note that (i) the π-π interaction can produce a very significant effect on the measured cross section; (ii) the cross section is concentrated in the near-threshold region, it becomes quite small beyond 200-MeV/c pion c.m. momentum.

In Fig. 7 we plot the differential cross section $d\sigma/dsd(\cos\theta_\pi)$ as a function of the pion c.m. scattering angle θ_π for four different values of s. The various lines have the same significance as in Fig. 6.

From these curves we can see that the measured cross section for $ee \rightarrow ee\pi\pi$ should be quite sensitive to the π-π interaction effect. In particular, the set of phase shift with scattering length of order unity gives rise to cross sections which are significantly different from the others. The other sets, all taken to have zero scattering lengths, tend to behave in qualitatively similar ways with 10–20% differences in the actual cross sections. Other phase shifts with small but nonzero scattering length give similar results. Stated explicitly, in the region near the threshold where our model should be fairly reliable (and where the bulk of the cross section lies) we see great sensitivity to the scattering length. We might point out that the absolute normalization for this process can be mea-

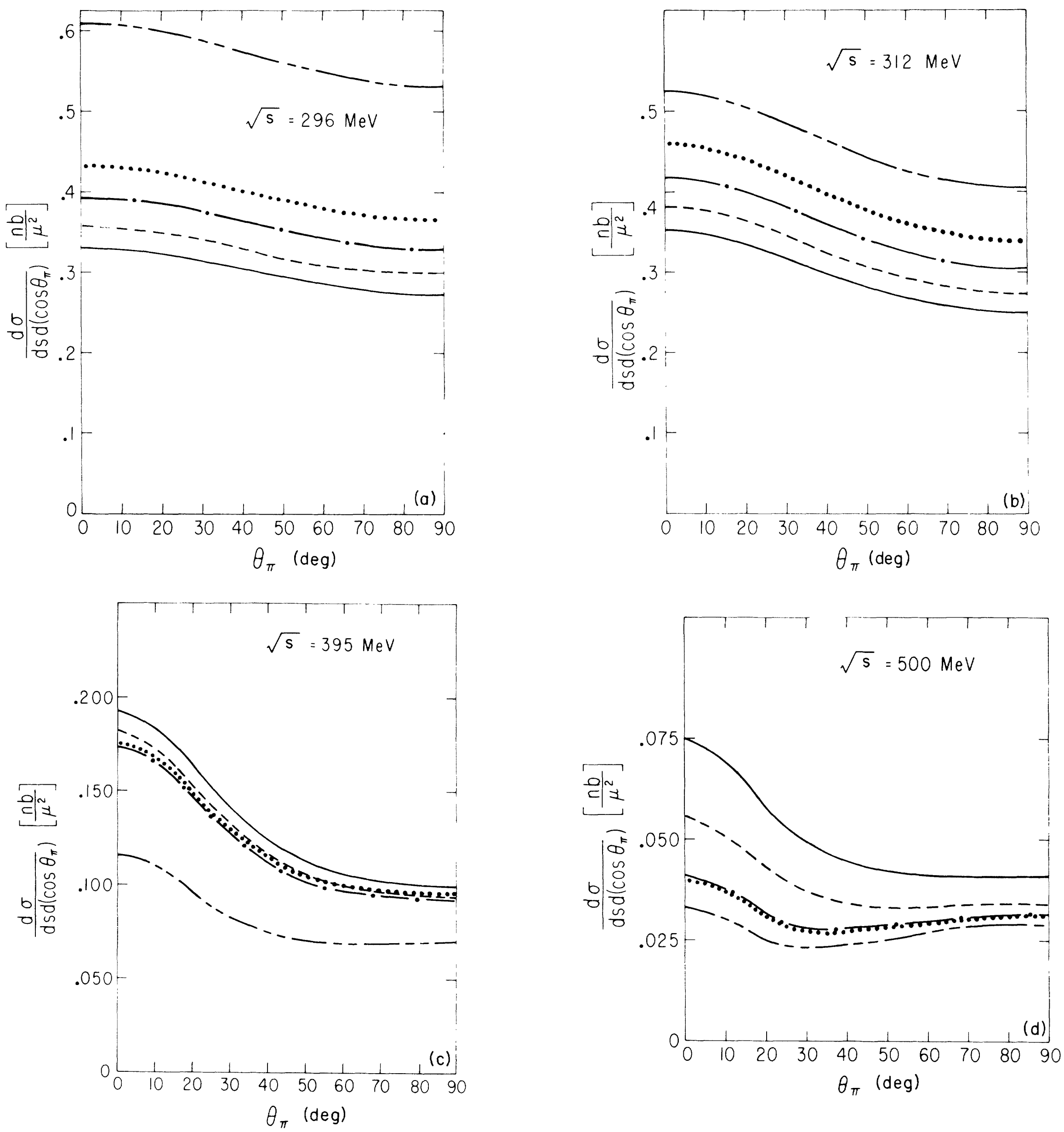

FIG. 7. Differential cross sections $d\sigma/dsd(\cos\theta_\pi)$ for fixed s plotted against the pion c.m. scattering angle θ_π. The various lines have the same meaning as in Fig. 6.

sured by comparing with the pure quantum-electrodynamic (QED) process $ee\rightarrow ee\mu\mu$.

In contrast, the model calculation yields small cross section and relative insensitivity to the various phase shifts in the 700-MeV region where the suspected scalar meson σ lies. In fact, the one phase shift which does not rise above 48° throughout yields cross sections almost indistinguishable from those from the other sets which do go through 90° near 700 MeV. In the context of our model, this happens because the unitarity-corrected s-wave amplitude becomes small as compared to the remaining Born amplitudes and the distinction between various phase shifts becomes insignificant. This fact is in turn related to the assumed asymptotic behaviors of the phase shifts and the s-wave amplitude. The predictions in this region can be easily changed by modifying these two factors. We see, therefore, in our model, the cross section has little sensitivity to the existence or nonexistence of the σ meson. Its effect is mixed up with the asymptotic behavior of the phase shift wherein lies the most ambiguous aspect of this model.

The failure of the model to demonstrate a clear signal for the σ meson does not mean, however, that this very interesting subject can not be studied in this reaction. The model is simply not reliable in this respect. *This circumstance only underlines the need of pursuing a model-independent procedure of extracting the π-π phase shifts as outlined in Sec. IV.* There exist in the literature other model calculations on this process either using a procedure similar to ours or involving some form of resonance parametrization for the relevant amplitudes. So far as we can tell, all suffer from ambiguities in this respect not less severe than those we explicitly spelled out.

After completing this work we received a revised version of the work by Goble and Rosner,[8] as well as the Cornell conference proceedings,[8] in which similar results obtained previously by the same authors were discussed. Since these results have considerable overlap with the present investigation, and since the conclusions with regard to the σ meson seem, on the surface, to differ, some specific comments are called for. The overlap is in their section on $\gamma\gamma\to\pi\pi$ and our Sec. V on the model calculations. The two models are essentially equivalent. Given the same π-π interaction as input, the solutions are also the same. The differences lie in emphasis and interpretation. Specifically:

(i) In Ref. 8, the input information on π-π scattering amplitudes is taken from model calculations based on current algebra while attention is focused on the effects of the σ meson in the process $\gamma\gamma\to\pi\pi$. In the present investigation, the π-π phase shifts are treated as phenomenological quantities to be extracted from the experiment; the current-algebra solution to the π-π phase shift is selected as one of several representative test phase shifts. It seems desirable that the results from this type of experiments are used both to extract the π-π scattering length, as an important test of current algebra ideas, and to investigate the σ meson problem.

(ii) Because of this difference in emphasis, the curves of Ref. 9 are plotted against the variable β, the pion c.m. velocity, which emphasizes the high-momentum region. The most striking feature is the appearance of a wiggle or shoulder around $\beta \simeq 0.8$–0.9 in $d\sigma/d\beta$ for $ee\to ee\pi\pi$ which is attributed to the existence of the σ meson. We have performed similar calculations using the variable β in our model. The results are almost the same as given in Ref. 8, if the phase shift corresponding to the current-algebra solution (δ_1 in Fig. 5) is used. However, the nonresonating phase shift (for instance, δ_2 of Fig. 5) also gives qualitatively similar results except the wiggle is less pronounced. In view of this fact and the inherent ambiguities of these model calculations in this energy region (stressed both in Ref. 8 and the present work), it should be clear that the interpretation of such a structure, if it were indeed found experimentally, must be treated with great care. We also note that the same curve when plotted versus the variable p (as is done in our Fig. 6) does not show such a structure at all.

ACKNOWLEDGMENTS

We wish to thank Professor Jack Sandweiss for several helpful comments. We are grateful to Professor Jonathan Rosner for discussions and for sending us copies of his work with Dr. R. Goble prior to publication. A correspondence with Dr. Tom Walsh was responsible for the addition of the comments at the end of Sec. III to the preprint version of this paper. We express our thanks to him also.

APPENDIX A

Here we outline the calculation of the differential cross section $d\sigma^{(B)}_{ee\to ee\pi^+\pi^-}/ds\,d(\cos\theta_\pi)$ for the case of pointlike pions. The result is used in the text both in Sec. III where the validity of the WW approximation is discussed and in Sec. V where the model calculation including unitarity corrections is reduced to a form similar to the present case.

For pointlike pions, the scattering amplitudes are given by the gauge-invariant Feynman diagrams, Figs. 4(a)–4(c). By neglecting terms proportional to (k^2q^2) (justification of this approximation is given later in this appendix), we get

$$\begin{aligned}
T^{(B)}_{1,1} &= \frac{2}{s+k^2+q^2}\left(\frac{4\mu^2}{1-\beta^2\cos^2\theta_\pi}+(k^2+q^2)\right) ,\\
T^{(B)}_{1,-1} &= \frac{2s}{s+k^2+q^2}\,\frac{\beta^2\sin^2\theta_\pi}{1-\beta^2\cos^2\theta_\pi} ,\\
T^{(B)}_{1,0} &= -(2sq^2)^{1/2}\,\frac{s-k^2+q^2}{s+k^2+q^2}\,\frac{\beta^2\sin 2\theta_\pi}{1-\beta^2\cos^2\theta_\pi} ,\\
T^{(B)}_{0,1} &= -(2sk^2)^{1/2}\,\frac{s+k^2-q^2}{s+k^2+q^2}\,\frac{\beta^2\sin 2\theta_\pi}{1-\beta^2\cos^2\theta_\pi} ,
\end{aligned} \tag{A1}$$

where $\beta=(1-4\mu^2/s)^{1/2}$ is the velocity of the pions in their c.m. frame. In the same approximation the remaining lepton variables ψ, ψ' in Eq. (9) are given by

$$\begin{aligned}
\cosh\psi &= \frac{8E(E-\epsilon')-s-k^2+q^2}{s+k^2+q^2} ,\\
\cosh\psi' &= \frac{8E(E-\epsilon)-s+k^2-q^2}{s+k^2+q^2} .
\end{aligned} \tag{A2}$$

Substituting (A1) and (A2) into Eq. (9) and integrating over all variables except s and $\cos\theta_\pi$ we get

$$\frac{d\sigma^{(B)}_{ee\to ee\pi\pi}}{ds\,d(\cos\theta_\pi)} = \frac{\alpha^4}{\pi s^2}\beta\sum_{i=1}^{4} z_i(s)Z_i(s,\cos\theta_\pi), \tag{A3}$$

where

$$\begin{aligned}
Z_1(s,\cos\theta_\pi) &= \frac{(1-\beta^2)^2+\beta^4\sin^4\theta_\pi}{(1-\beta^2\cos^2\theta_\pi)^2},\\
Z_2(s,\cos\theta_\pi) &= \frac{2(1-\beta^2)}{1-\beta^2\cos^2\theta_\pi},\\
Z_3(s,\cos\theta_\pi) &= 1,\\
Z_4(s,\cos\theta_\pi) &= \frac{\beta^4\sin^2 2\theta_\pi}{(1-\beta^2\cos^2\theta_\pi)^2},
\end{aligned} \tag{A4}$$

$$\begin{aligned}
z_i(s) &= \frac{s^2}{16E^2}\int\frac{d\rho''}{k^2q^2}\left[\left(\frac{8E(E-\epsilon')-s-k^2+q^2}{s+k^2+q^2}\right)^2+1\right]\left[\left(\frac{8E(E-\epsilon)-s+k^2+q^2}{s+k^2+q^2}\right)^2+1\right]\bar z_i(s,k^2,q^2), \quad i=1,2,3\\
\bar z_1 &= s^2/(s+k^2+q^2)^2, \quad \bar z_2 = s(k^2+q^2)/(s+k^2+q^2)^2, \quad \bar z_3=(k^2+q^2)^2/(s+k^2+q^2)^2,\\
z_4(s) &= \frac{s^2}{8E^2}\int\frac{d\rho''}{k^2q^2}\left[\left(\frac{8E(E-\epsilon')-s-k^2+q^2}{s+k^2+q^2}\right)^2+1\right]\left[\left(\frac{8E(E-\epsilon)-s+k^2-q^2}{s+k^2+q^2}\right)^2-1\right]\frac{sq^2(s+k^2-q^2)^2}{(s+k^2+q^2)^4},
\end{aligned} \tag{A5}$$

and

$$d\rho'' = \frac{d^3k_2}{k_2^0}\frac{d^3q_2}{q_2^0}\Big/d\chi d\chi' ds\,.$$

The integrals involved in evaluating $z_i(s)$, Eqs. (A5), are threefold integrals over, say, k^2, q^2, and ϵ (or ϵ'). Although very complicated, these integrals can be in principle done exactly. For dimensional reasons, the result can only involve quantities like β and

$$x = s/4E^2,$$

$$L = \ln\frac{4E^2}{m_e^2},$$

$$L_s = \ln\frac{4E^2}{s} = -\ln x\,.$$

For all practical purposes, it suffices to obtain results which are at least quadratic in the large logarithmic factors L, L_s and these only with constant coefficients (i.e., one can neglect terms like x^nL, x^nLL_s, $\ldots$, $n=1,2,\ldots$). The task then becomes considerably simpler. For instance, one immediately recognizes that the L factor can only come from the lower limit of the k^2 (or q^2) integral with the photon propagator factor $1/k^2$ in the integrand. Thus, if the propagator factor $1/k^2q^2$ is canceled by other factors in the integrand, then the result can, at most, be of order L_s (from the ϵ integration). Thus we are justified in neglecting terms proportional to k^2q^2 in our calculation. Using similar arguments, one can easily isolate the coefficients of the leading terms proportional to L^2L_s, L_s^3, LL_s^2, $\ldots$, etc.

Similar calculations for the cross section integrated over $\cos\theta_\pi$, i.e., $d\sigma^{(B)}_{ee\to ee\pi\pi}/ds$, have been done by Baier and Fadin[14] up to terms linear in the logarithmic factors. The integrals $z_i(s)$ can also be inferred from their results by integrating our Eq. (A3) over $\cos\theta_\pi$ and comparing the results. One then obtains

$$\begin{aligned}
z_1(s) &= L^2(L_s-\tfrac{3}{2}) - L_s^2(\tfrac{1}{3}L_s-\tfrac{3}{2})\\
&\quad -\tfrac{17}{3}LL_s - L(\tfrac{1}{6}\pi^2-\tfrac{815}{144}),\\
z_2(s) &= \tfrac{2}{3}L(L_s-\tfrac{11}{6}),\\
z_3(s) &= \tfrac{1}{3}L(L_s-\tfrac{7}{3}),\\
z_4(s) &= \tfrac{2}{3}L(L_s-\tfrac{25}{12}).
\end{aligned} \tag{A6}$$

If one only retains terms quadratic in L and L_s, then z_2, z_3, and z_4 become proportional. The corresponding terms in (A3) can be combined and we get the result quoted in Sec. III, Eq. (13). In our direct calculations of the integrals $z_i(s)$, Eqs. (A5), we have checked all the coefficients except those of LL_s and L_s.

APPENDIX B

In this appendix we show some of the formulas relevant to our dispersion (Omnès equation) treatment of the s-wave strong-interaction modified amplitude, including a list of the test phase shifts that we employed.

In writing down the dispersion relation for $a^I(s)$ (we shall omit the angular momentum and helicity labels l, m, n), we assume that the right-hand cut is dominated by elastic unitarity and that the only important contribution to the left-hand cut comes from the Born term. The relation is

$$a^I(s,k^2,q^2) = a^{I(B)}(s,k^2,q^2) + \frac{s}{\pi}\int_{4\mu^2}^{\infty} ds'\,\frac{a^I(s,k^2,q^2)\sin\delta_0^I e^{-i\delta_0^I}}{s'(s-s')}\,. \tag{B1}$$

This standard Omnès equation has the solution

$$a^I(s,k^2,q^2)=a^{I(B)}(s,k^2,q^2)+\frac{se^{\Delta^I(s)}}{\pi}\int_{4\mu^2}^{\infty}ds'\,\frac{a^{I(B)}(s',k^2,q^2)\sin\delta_0^I\,e^{-\mathrm{Re}\Delta^I(s')}}{s'(s'-s)}\,, \tag{B2}$$

with

$$\Delta^I(s)=\frac{s}{\pi}\int_{4\mu^2}^{\infty}ds'\,\frac{\delta_0^I(s')}{s'(s'-s)}\,. \tag{B3}$$

For the test phase shifts we use, the ambiguity associated with the Omnès solution is fixed by requiring the s-wave amplitude not to rise linearly in energy as $s\to\infty$.

The Born amplitude over the elastic unitarity region can be well approximated by one or two poles on the left-hand cut. For example, for the charged amplitude

$$a_B(s,0,0)=\frac{14.7\,\mu^2}{s+3.35\,\mu^2} \tag{B4}$$

is accurate to 4% over the range $4\mu^2\leqslant s\leqslant 16\mu^2$, and

$$a_B(s,0,0)=\frac{7.05\,\mu^2}{s+1.86\,\mu^2}+\frac{9.15\,\mu^2}{s+66.4\,\mu^2} \tag{B5}$$

is accurate to 1% for $4\mu^2\leqslant s\leqslant 100\,\mu^2$. If the pole approximation is made, the Omnès solution can be integrated analytically.

Thus, writing

$$\begin{aligned}
&a^{I(B)}(s,k^2,q^2)=\frac{s}{s+k^2+q^2}\,a_0^{I(B)}(s)+\frac{k^2+q^2}{s+k^2+q^2}\,a_1^{I(B)}\,,\\
&a_0^{I(B)}(s)=a^{I(B)}(s,0,0)=\sum_i\frac{b_i^I}{s+c_i}\,,\\
&a_1^{I(B)}=\tfrac{4}{3}\ \text{ for } I=0 \text{ and } (2\sqrt{2}/3) \text{ for } I=2\,,
\end{aligned} \tag{B6}$$

we get

$$a^I(s,k^2,q^2)=\frac{s}{s+k^2+q^2}\,a_0^I(s)+\frac{k^2+q^2}{s+k^2+q^2}\,a_1^I(s)\,,$$

where

$$\begin{aligned}
&a_0^I(s)=e^{\Delta^I(s)}\times\sum_i\frac{b_i^I}{c_i}\left(1-\frac{s}{s+c_i}\,e^{-\Delta^I(-c_i)}\right)\,,\\
&a_1^I(s)=e^{\Delta^I(s)}\left\{a_1^{I(B)}[1-s\Delta'^I(0)]+\sum_i\frac{sb_i^I}{c_i}\,[1+c_i\Delta'^I(0)-e^{-\Delta^I(-c_i)}]\right\}\,.
\end{aligned} \tag{B7}$$

In obtaining (B7) we have again neglected terms of higher order in k^2 and q^2 for reasons already mentioned. The most important feature of Eq. (B7) is that $a^I(s,k^2,q^2)$ is written in the same form as the Born amplitude, with only the substitutions $a_0^{I(B)}(s)\to a_0^I(s)$ and $a_1^{I(B)}\to a_1^I(s)$. Since the integration involved in evaluating Eq. (20) is done for fixed s, there are no changes in any of the integrals that one needs to evaluate. Thus one needs only to isolate the s-wave part of the Born calculation discussed in Appendix A, make the appropriate substitutions of $a^I(s,k^2,q^2)$ for $a^{I(B)}(s,k^2,q^2)$, and then one can write down the integrated answer directly. The substitutions affect only the Z_i of Appendix A, and we shall give just the modified forms of these Z_i here:

$$\begin{aligned}
&Z_1\to Z_1+\frac{1-\beta^2}{D}\,\mathrm{Re}\left(-a_0^{(B)}+a_0^0+\frac{1}{\sqrt{2}}\,a_0^2\right)+\tfrac{1}{4}\|-a_0^{(B)}+a_0^0+a_0^2\|^2\,,\\
&Z_2\to\tfrac{1}{2}\mathrm{Re}\left(2\frac{1-\beta^2}{D}-a_0^{(B)}+a_0^0+\frac{1}{\sqrt{2}}\,a_0^2\right)\left(a_1^0+\frac{1}{\sqrt{2}}\,a_1^2\right)^*\,,\\
&Z_3\to\tfrac{1}{4}\|a_1^0+(1/\sqrt{2})a_1^2\|^2\,,\\
&Z_4\to Z_4\,,
\end{aligned} \tag{B8}$$

with $D=1-\beta^2\cos^2\theta_\pi$. We would like to call the reader's attention to the fact that the large corrections to the WW approximation exhibited in Eq. (13) remain also in this case. This is important because these corrections are in the opposite direction to the unitarity corrections and of comparable magnitude in a wide region of the variables.

For completeness, we give the analytic forms for the test phase shifts and the corresponding Omnès functions. The isospin-zero phase shifts are

$$\begin{aligned}
\delta_1(s)&=67.6\pi(s-4)/(s+28)^2\,,\\
\delta_2(s)&=\tfrac{1}{2}\delta_1(s)\,,\\
\delta_3(s)&=\tfrac{5}{2}\sqrt{5}\,\pi(s-4)^{1/2}/(s+26)\,,\\
\delta_4(s)&=\pi(s-4)/(s+16)\,.
\end{aligned}\tag{B9}$$

Three of these phase shifts, δ_1, δ_3, and δ_4, pass through 90° at $s=24$. The Omnès functions are

$$\begin{aligned}
\Delta_1(s)&=67.6\times\left\{\frac{s}{196}\,\frac{s+252}{(s+28)^2}\ln 32-\frac{s-4}{(s+28)^2}\ln(s-4)-\frac{1}{28}\,\frac{s}{s+28}-\frac{1}{196}\ln 4\right\}\,,\\
\Delta_2(s)&=\tfrac{1}{2}\Delta_1(s)\,,\\
\Delta_3(s)&=\frac{5}{4}\,\frac{s}{s+16}\ln 20-\frac{s-4}{s+16}\ln(s-4)-\tfrac{1}{4}\ln 4\,,\\
\Delta_4(s)&=\frac{5\sqrt{5}}{52}\,\pi\left(2-\frac{s\sqrt{30}}{(s+26)}\right)\,.
\end{aligned}\tag{B10}$$

For the isospin-2 corrections, which are small, we always took the $I=2$ phase shift to be $-\frac{1}{4}\delta_1(s)$.

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*Supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-264.

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$$L^{lm}(\psi,\chi)=e^{-i(l-m)\chi}[\bar d(\psi)_1^l\,\bar d(\psi)_1^m+\bar d(\psi)_{-1}^l\,\bar d(\psi)_{-1}^m]\,,$$

where

$$\bar d(\psi)=\begin{pmatrix}\frac{1}{2}(\cosh\psi+1) & -\sinh\psi/\sqrt{2} & -\frac{1}{2}(\cosh\psi-1)\\ -\sinh\psi/\sqrt{2} & \cosh\psi & \sinh\psi/\sqrt{2}\\ -\frac{1}{2}(\cosh\psi-1) & \sinh\psi/\sqrt{2} & \frac{1}{2}(\cosh\psi+1)\end{pmatrix}.$$

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for $\gamma\gamma \to \pi^0$ the matrix element is proportional to $[(s+k^2+q^2)^2 - 4K^2q^2]^{1/2}$ which cancels part of the damping factors in the k^2 and q^2 integrations mentioned above and it is easy to demonstrate that the $LL_s{}^2$ term does occur with a positive contribution. This then becomes the main correction effect. When a form factor is put in, however, the upper ranges of the k^2, q^2 integrations are again damped, thus reducing the correction effect for the single π^0 production.

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$\bar{p}p$ Backward Scattering and s-Channel Resonances: Upper Limit to the $\bar{p}p$ Coupling of the $S(1929)$ Meson

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(Received 13 September 1971)

From the energy dependence of the $\bar{p}p$ and $\bar{p}n$ inelastic cross sections we deduce an upper limit to the resonant contribution in $\bar{p}p$ backward scattering for c.m. energies between 1915 and 1950 MeV. This limit is smaller than the expected contribution from diffraction scattering. The energy dependence of the 180° $\bar{p}p$ elastic cross section in this energy range cannot therefore be directly related to the formation of s-channel resonances.

I. INTRODUCTION

The observation with the CERN missing-mass spectrometer of a very narrow resonance ($\Gamma \leq 35$ MeV) at a mass of 1929 MeV[1] has stimulated the search for its possible coupling to the $\bar{p}p$ system. However, the low-energy $\bar{p}p$ elastic scattering cross section is dominated by a conspicuous forward peak apparently due to diffraction, and consequently the detection of a highly inelastic resonance from the energy dependence of the total elastic cross section appears very difficult. It has since been pointed out by Cline *et al.*[2] that a better sensitivity to a possible resonant $\bar{p}p$ interaction could be achieved by looking at the energy dependence of the backward scattering cross section, away from the forward diffraction peak.

It is the purpose of this paper to show that published data on the $\bar{p}p$ cross section, together with new data on the $\bar{p}n$ annihilation cross section in the momentum range 350–580 MeV/c, allow one to place a significant upper limit on the possible contribution of a single resonant interaction to the backward $\bar{p}p$ elastic cross section if the resonance has a mass between 1915 and 1945 MeV and a width between 10 and 40 MeV. At the same time we will point out that at these low momenta, even in the backward direction, diffraction scattering is capable of masking possible effects of s-channel resonances.

II. DATA

The values of the $\bar{p}p$ and $\bar{p}n$ inelastic cross sections in this energy interval are shown in Fig. 1. The $\bar{p}p$ data are those of Loken and Derrick[3] and Amaldi *et al.*[4] The $\bar{p}n$ data are new and we shall briefly describe how they have been obtained.

The experiment is very similar to that of Ref. 4. The deuterium-filled 81-cm Saclay bubble chamber has been exposed to a separated antiproton beam from the CERN proton synchrotron. Three exposures at beam momenta of 467, 549, and 612 MeV/c (at the entrance of the illuminated region of the chamber) have been made. All $\bar{p}$ interactions have been measured if they occur in a re-