Comments and Addenda

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Comment on a Limiting Temperature in Hadron Physics*

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It has been pointed out by Yang that if hadron physics were indeed governed by a limiting temperature $T_0 \sim 160$ MeV, then such a far-reaching conclusion would constitute no less than the establishment of a fourth law of thermodynamics. Here we comment briefly but critically on the basis for the limiting-temperature concept with particular emphasis on its implications for the production of massive exotic hadron states.

I. INTRODUCTION

Ever since Fermi's early application of thermodynamics methods[1] to the study of hadron matter and interaction, there has been much further discussion in the literature concerning the usefulness of the concept in high-energy collisions. One such approach[2] is to ask how large the production cross section of massive exotic hadron states (quarks, etc.) can be in hadron collisions if the temperature introduced by Fermi is allowed to attain as high a value as reasonably possible (say ~1–2 BeV) for given incident energies. More precisely, the assumptions are that the statistical-thermodynamics model of Fermi is meaningful for a discussion of problems of strong interactions, secondly that all the available energy goes into the interaction process, and finally that following a high-energy p-p collision, only hadrons stable in strong interactions participate in the statistical equilibrium. The last assumption, which states in practice that only a finite-hadron-mass spectrum contributes to the equilibrium, leads essentially to a Stefan-Boltzmann-type law between the energy of the system and its temperature ($E = \sigma T^4$, where σ depends on the mass spectrum). Clearly, for large E, large T can be expected. Typically, for $T \sim 15 m_\pi$ and quark mass $m_q = 10$ GeV each, the production cross section can be of the order of a few mb for incident lab energy ~10^{14} eV – a not unwelcome feature in terms of their possible experimental discovery. However, the model in its simplest form leads to isotropic angular distribution for the production of known hadron secondaries (π, K, etc.) as well as excessive production of $p\bar{p}$ pairs in disagreement with data.

A second thermodynamics approach to hadron collisions at high energies has received much attention recently.[3] This formulation assumes that resonances must also be included in the mass spectrum determining the statistical equilibrium. It is argued that to include only hadrons stable in strong interactions[2] in the equilibrium consideration, because they have a lifetime much longer than the reaction time scale of strong interactions and hence can escape from the interaction volume $\Omega = \gamma \Omega_0 \cong \gamma \frac{4}{3}\pi (1/m_\pi)^3$ (here γ is the Lorentz contraction factor), need not always be relevant. Hagedorn[4] stressed that in very-high-energy collisions the collision time $\Delta t \approx 1/\gamma m_\pi$ becomes very short and, with respect to this short time, resonances live very long. Inclusion of both stable and unstable hadron states in a discussion of equilibrium (which sets in "instantaneously") is then analogous to the thermodynamic establishment of chemical equilibrium[5] in, for instance, $2H_2 + O_2 \rightleftharpoons 2H_2O$.

The inclusion of all resonant states in a bootstrap picture leads to an exponential mass spectrum as emphasized especially by Frautschi[3] and Hagedorn.[6] The exponential spectrum is the statement that the number of species of hadron particles with mass between m and $m + dm$ increases as $m \to \infty$ as

$$\rho(m)dm \sim Cm^{-B} e^{\beta_0 m} dm, \quad B, \beta_0 > 0. \tag{1}$$

The partition function is then given by

$$Z(\Omega, T) = \int d^3k \int_{m_0}^{\infty} \frac{dm\, C e^{\beta_0 m - \beta(k^2+m^2)^{1/2}}}{m^B} , \tag{2}$$

where $\beta = 1/T$ (in units in which the Boltzmann constant $k = \hbar = c = 1$) and m_0 is a cutoff mass whose value is not relevant at high temperature ($\beta \to 0$). The Ω dependence of Z indicates that the level spectrum of the system depends on its interaction volume Ω. Clearly (2) exists only if $\beta > \beta_0$, or

$$T < T_0 , \tag{3}$$

where $T_0 = 1/\beta_0$. The quantity $T_0 \equiv 1/\beta_0$ is thus a limiting (maximal) temperature for any system in thermal equilibrium.

Frautschi and Hagedorn,[3] and Huang and Weinberg,[7] cited evidence for the limiting-temperature model as coming from two principal directions:

(1) The Veneziano model, which incorporates such desirable theoretical features as duality, crossing symmetry, and factorization, leads to a spectrum[8] of form (1), with

$$\beta_0 = 2\pi(\tfrac{1}{6}D\alpha')^{1/2}, \quad B = \tfrac{1}{2}(D+1) , \tag{4}$$

where $\alpha' \approx 1\ \mathrm{GeV}^{-2}$, $D = 4$ or 5; hence $1/\beta_0$ is in the range of 170–180 MeV.

(2) The statistical bootstrap model leads to the prediction that secondary particles boil off in high-energy hadron collisions with weight factor $\exp[-(M^2 + p_\parallel{}^2 + p_\perp{}^2)^{1/2}/T]$, T being close to but always below T_0. Hagedorn and collaborators[3] have made detailed fits on this basis. The most impressive fits are those at large $p_\perp$, where the weight factor is approximately

$$\exp(-p_\perp/T) \approx \exp(-p_\perp/T_0) \tag{5}$$

and the fits at large M (production of $\overline{K}$, $\overline{p}$, $\overline{d}$, $\overline{\mathrm{He}}{}^3$, etc.) where the factor is approximately

$$\exp(-M/T) \approx \exp(-M/T_0) . \tag{6}$$

$M = m_1 + m_2$ for associated pair production. From these fits with $B = \frac{5}{2}$, the value

$$T_0 \sim 160\ \mathrm{MeV} \tag{7}$$

is obtained.

That both the Veneziano model[7,8] and the statistical bootstrap model[3,6] lead to essentially the same conclusion concerning the existence of an exponentially rising mass spectrum with compatible values of T_0 and B is of course very encouraging to the limiting-temperature concept for hadron physics. In fact the two model approaches need not be orthogonal to one another, as pointed out by several authors.[3,9]

One striking prediction of the limiting-temperature concept (3) is that the strong pair-production rate of massive hadron states (e.g., quarks) does not depend on the available energy but only on the temperature once one is fairly high above threshold. If massive hadron states $m_X > 4m_p$ exist they will not be seen in strong-interaction collisions,[10] no matter how high the energy is pushed: The counting rates determined largely by exponential factor (6) will go down by roughly a factor 10^5 for each increment of m_X by 1 GeV. Hence if one subscribes to the viewpoint advanced by Hagedorn[3] that T_0 is the universal maximum temperature for all matter, the production of quarks and other massive exotic states in both accelerator and cosmic ray experiments becomes an academic question indeed. We shall come back to this point again below.

II. A PROVISIONAL FORMULATION OF THE FOURTH LAW OF THERMODYNAMICS

It is evident that if T_0 is the universal limiting temperature for all hadron matter (let alone for all matter), its profound implications transcend detailed models and constitute no less than the establishment of a fourth law of thermodynamics,[9] as first emphasized by Yang.[11] An equivalent (provisional) formulation of the fourth law[11] might read as follows:

(i) The density of hadron mass levels is of increasing exponential form given by (1).

(ii) The natural interaction volume Ω_0 of hadron excited states is fixed (apart from the Lorentz contraction factor γ). Typically strong interactions are confined to a fixed volume $\Omega_0 = \frac{4}{3}\pi(1/m_\pi)^3$.

Inputs (i) and (ii) imply a finite limiting temperature T_0 [cf. Eqs. (1)–(3)]. Thus in first measure a fourth law of thermodynamics must be the postulates (i) and (ii). An equivalent statement is that a finite limiting temperature T_0 exists which can be derived from inputs (i) plus (ii).

Given the existence of an infinity of degrees of freedom in a system which is amenable to the methods of statistical thermodynamics, there is then no sharp distinction between the applicability of a fourth law to macroscopic phenomena (e.g., big-bang theory, neutron stars,[7,12] etc.) or to microscopic phenomena as in nuclear reactions.[3]

III. CRITICAL APPRAISAL AND TESTS

It is evidently going to be of substantial importance to test the universality of such a "fourth" law. We delineate below some key tests of a limiting-temperature concept, with special reference to the currently popular assertion that $T_0 \sim 160$–180 MeV.[3,7]

A. The Nuclear-Structure Problem

Bethe showed many years back[13] that even in nuclear-structure problems (another area of strong interactions), the nuclear energy levels have a level density of exponential form

$$\rho(m)dm \propto e^{b(Am)^{1/2}}\,, \tag{8}$$

where b is a constant, A is the mass number of the nucleus, and m is the excitation energy about the ground state for that particular A. The Bethe formula (8) holds when the excitation energy *m per nucleon* is less than the depth of the nuclear potential well (< 30 MeV). Hence the temperature T (per nucleon) is less than 30 MeV and thus, understood in this way, does not ostensibly contradict a limiting temperature $T_0 \sim 160$ MeV given by (7). The production rates of $\bar{d}$, $\overline{\mathrm{He}}^3$, etc., in p-p collisions also appear to follow the Hagedorn fits (6) and (7) experimentally.[3]

However, irrespective of the details of the spectrum (whether exponential or not), the natural volume $\frac{4}{3}\pi A r_0^3$ ($r_0 \sim 1$ F) for nuclei is constantly expanding for increasing A [in contrast to postulate (ii)]. As stressed by Yang,[11] one can think of an extreme gedanken experiment in which $A = 10\,000$ and the energy of the nucleus is ~9000 GeV (including, say, Coulomb corrections); the nuclear size is then ~$10\,000(\frac{4}{3}\pi r_0^3)$. In this case the temperature T_0 can be high or low – in fact, we do not care.

The fact that nuclei and all their excited states do not fit into a *fixed* volume $\Omega_0 = \frac{4}{3}\pi r_0^3$ (but rather have expanding volume $A\Omega_0$) raises a question about their relevance for inclusion in the bootstrap considerations of Frautschi and Hagedorn.[3]

B. Behavior of Secondaries for Large Transverse Momentum

It is relevant to ask whether the limiting-temperature prediction that secondaries from, say,

$$p + p \to X_a + X_b \tag{9}$$

will continue to be produced following the exponential-type falloff depicted by Eq. (5) with $T_0 \sim 160$ MeV, for arbitrarily large $p_\perp$. We suspect not, because experience with deep-inelastic electron scattering $e^- + p \to e^- +$ "anything" already hints persuasively that the vertex $\gamma + p \to$ "anything" is substantial (absence of large form-factor suppression). Hence the scale of cross section for (9) from electromagnetic exchange could be of the form[14,15]

$$\frac{4\pi\alpha^2}{p_\perp^{\,4}}\,(\text{numerical factor}). \tag{10}$$

Beyond a crossover point $p_\perp \sim 5-10$ GeV in process (9), the electromagnetic deep-inelastic contribution (10) dominates the limiting-temperature distribution (5). Of course one may argue that electromagnetic contributions to inelastic hadron processes do not necessarily vitiate the significance of a fourth law applied strictly to strong interactions. However, the successful Wu-Yang[16] picture of elastic pp scattering visualized as the exchange of some pointlike vector particle (gluon) coupled strongly to protons can be used for inelastic collisions (9) as well. Since we invoke here a strong deep-inelastic process the confrontation with (5) is more severe. The gluon exchange contribution is expected then to lead to a secondary particle production cross section[14] of form (10) with $\alpha = 1$, namely

$$\frac{4\pi}{p_\perp^{\,4}}\,(\text{numerical factor}). \tag{11}$$

Of course, (11) will then inundate electromagnetic exchange (10) by a factor of 10^4. Both (10) and (11) exhibit power-law behaviors, and hence there will not be a limiting temperature for these processes.

C. Production of Heavy Hadron Pairs

The constraints (6) and (7) in the limiting-temperature picture rule out meaningful production of massive hadron pairs $m_X + m_{\bar{X}} > 8$ BeV in both particle accelerator experiments and cosmic-ray collision of primaries. Recent evidence on the cosmic-ray muon anomaly[17] suggests on the other hand that massive X hadrons[18] with masses up to 40 GeV each and strong production cross section $\sigma \sim \frac{1}{3}$ mb in $p + p \to X + \bar{X} + \cdots$ collisions are needed for an adequate explanation.

To summarize, a restricted fourth law of thermodynamics applicable to the usual low-lying hadrons and their interactions at moderate values of $p_\perp$ may be in evidence. Its universal relevance to all hadron matter remains to be seen. The existence of an ultimate T_0 has often been contrasted with boiling; the exponentially increasing number of particle states makes it more favorable to increase the creation of particles (without raising temperature) rather than increase the kinetic energy of existing particles (hence temperature), upon adding energy to the system. Analogous to the existence of both latent heats of fusion (melting) and vaporization (boiling) for H_2O, one might speculate on the possible existence of a higher temperature T_0' (in the GeV range) governing the massive X hadrons (quarks, heavy triplets, etc.) if these should bootstrap among themselves suitably to yield the correct type of exponential mass spectrum. Production cross sections with weight factor $\exp[-(m_X + m_{\bar{X}})/T_0']$ can then be detectable for low-lying members of the X set. Again it is possible that thermodynamics fails completely for these exotic X hadrons. Certainly such favored production mechanisms as diffraction dissociation for generating large cross sections of heavy pairs[2,18] do not lend themselves easily to a temperature interpretation.

IV. A SUBJECTIVE VIEW OF QUARK SEARCH

The situation relative to quark search is indeed a paradoxical one. On the one hand a *light*-quark model has been successful in describing hadron spectroscopy, deep-inelastic electroproduction,[19] and current matrix elements.[20] Yet materialization of real quarks with the same quantum numbers is possible only if they are now *heavy*. This is not necessarily disastrous and in fact is somewhat reminiscent of the effective mass m_e^* of electrons in crystals compared with the free-electron mass m_e. For the case of high diamagnetic susceptibility such as in bismuth and gamma-brass $(m_e^*/m_e) \ll 1$.

One way of looking at this picture[21] is to regard the "light" quarks as quanta of H_0; the heavy quarks which experimentalists look for are quanta of the total Hamiltonian H. There is not necessarily any direct connection between them.[22] For example, if heavy quarks are found (and some such massive set may be needed to saturate the Adler sum rule[23]), a target of them can be assembled. One would scatter electrons from them and no doubt find they have an elastic form factor $G(q^2)$ much like nucleon form factors, with a radius $\sim 0.5-0.8$ F. The inelastic form factor W_2 would probably look like the nucleon's, with a scaling behavior and a description in terms of partons (with quark quantum numbers). According to this viewpoint characteristics of quarks have closer affinity with the nucleon than with heavy nuclei of comparable mass (e.g., C^{12}, or even U^{238} for that matter). There is then no strong reason why their production characteristics in hadron-hadron collisions should necessarily follow the miniscule expectations for heavy nuclei from Eqs. (6) and (7).

Note added. It has been called to our attention that an energy-density vs temperature relation $E/N \sim T^4$ (cf. Ref. 2) for quark production has also been suggested by Zeldovich and Novikov.[24]

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[3]See the excellent summaries by S. Frautschi [Phys. Rev. D 3, 2821 (1971)] and R. Hagedorn [CERN Report No. 71-12, 1971 (unpublished)]. Complete lists of references to past literature are given in these works.

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[12]It has been suggested by Frautschi (Ref. 3), however, that at the high density involved in astrophysics problems, the hadrons are squeezed together and overlapping if confined in an isolated box of dimension of a fermi. This may create difficulties with the bootstrap derivation of the spectrum, especially if hard cores are involved in hadron-hadron interaction.

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Comment on the Possible Relation Between Prominent Resonances and Limiting Scaling Data in Inelastic Electroproduction on Nucleons

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A recent parametrization of a relation between prominent resonances and limiting scaling data in inelastic electroproduction on nucleons is shown to be incompatible with experiment, unless the electromagnetic current has an isotensor component.

The observation made by Bloom and Gilman[1] that there is a possible relation between prominent resonances and the limiting scaling curve in inelastic electroproduction on nucleons can be made more quantitative by the following expression[2]:

$$\nu W_{2N}(Q^2, \nu) = F_{2N}(\omega')[R_N(W) + B_N(W)]\,, \tag{1}$$

where $R_N(W)$ is a sum of Breit-Wigner forms for the prominent resonances and $B_N(W)$ is a smooth "background" factor, such that $R_N(W) \approx 0$ and $B_N(W) \approx 1$ when $W > 2$ GeV, i.e., when we are outside the region of prominent resonances. The subscript N is for either p or n. $F_{2N}(\omega')$ is the scaling function with

$$\omega' = \frac{2M\nu + M^2}{Q^2} = 1 + \frac{W^2}{Q^2} \quad (Q^2 \geq 1\ \mathrm{GeV}^2/c^2)\ . \tag{2}$$

Observe that up to now Eq. (1) has only been compared with experimental data for inelastic electroproduction on protons but *not* on neutrons.

Now we state a *theorem:* Equation (1) implies

$$\frac{F_{2n}(\omega')}{F_{2p}(\omega')} = \mathrm{const} \qquad (\omega' < 5)\,. \tag{3}$$

Note that the theorem is valid for any scaling variable, i.e., we could have[2]

$$\omega = \frac{2M\nu}{Q^2}\,, \quad \omega^L = \frac{M}{|\vec{q}| - q_0}\,,$$

etc., replacing ω' in Eq. (1).

Proof. Isospin invariance of strong interactions and the assumption that the electromagnetic current has *no* isotensor component implies

$$\nu W_{2p}(I = \tfrac{3}{2}) = \nu W_{2n}(I = \tfrac{3}{2})\ , \tag{4}$$

where $\nu W_{2N}(I = \frac{3}{2})$ means the electroproduction on nucleons of the $I = \frac{3}{2}$ final states and similarly for $I = \frac{1}{2}$;

$$\nu W_{2N} \equiv \nu W_{2N}(I = \tfrac{1}{2}) + \nu W_{2N}(I = \tfrac{3}{2})\ . \tag{5}$$

We can write Eq. (1), at least when $W < 2$ GeV, as

$$\nu W_{2N} = F_{2N}(\omega')[R_N(W; I = \tfrac{1}{2}) + B_N(W; I = \tfrac{1}{2}) + R_N(W; I = \tfrac{3}{2}) + B_N(W; I = \tfrac{3}{2})] \tag{6}$$

in an obvious notation. Now Eqs. (4) and (6) imply that, for *fixed W*,

$$F_{2p}(\omega')[R_p(W; I = \tfrac{3}{2}) + B_p(W; I = \tfrac{3}{2})] = F_{2n}(\omega')[R_n(W; I = \tfrac{3}{2}) + B_n(W; I = \tfrac{3}{2})]\,, \tag{7}$$

which gives immediately Eq. (3). The best place to look for a large contribution to the $I = \frac{3}{2}$ channel is near the $\Delta(1236)$.

Discussion. Equation (3) will fail

(a) if there is an isotensor component to the electromagnetic current, but this seems unlikely