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[10]This is to be compared with the corresponding spin-0 result, which is the same as Eq. (21), except that the constant term is changed to

$$-\tfrac{47}{36}+\tfrac{31}{18}\cos\theta-\tfrac{47}{36}\cos^2\theta+\tfrac{4}{3}\cos^3\theta .$$

See J. Schwinger, Particles, Sources, and Fields (Addison-Wesley, Reading, Mass., to be published), Vol. II.

[11]Ref. 1, Eq. (39).

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[16]P. Bock, Phys. Letters 30B, 628 (1969).

[17]Another way to derive this result is to explicitly represent the spin state in terms of the helicity states (in the lab frame); for example

$$\langle \text{out} \,|\, \text{up}, \lambda_i\rangle = \frac{1}{\sqrt{2}}[f(+,\lambda_i;\text{out}) + e^{i(\pi/2-\phi)}f(-,\lambda_i;\text{out})] .$$

Squaring and summing over undetected helicities yields Eq. (26).

[18]Another way to evaluate Eq. (24) is to note that it can be simplified [by using Eq. (2)] to the form

$$N=-\frac{4\pi\alpha(-td)^{1/2}}{\kappa T}[-4t\,\mathrm{Im}\,M_3-(\kappa-\tfrac{1}{2}t)d\,\mathrm{Im}\,M_5+2d\,\mathrm{Im}\,M_6] .$$

The imaginary part of M_3, M_5, and M_6 can be easily extracted from the integrated form of the M_i's given in Appendix C.

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Deep-Inelastic Contribution to Baryon Electromagnetic Mass Differences*

Hiroshi Katsumori

Department of Applied Physics, Chubu Institute of Technology, Kasugai, Nagoya-Sub., Japan

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Unsubtracted dispersion relations for $t_1(q^2,\nu)$ and $t_2(q^2,\nu)$ are proposed even in the case of the $\Delta I=1$ mass difference, with the requirement of the absence of divergences worse than logarithmic ones. By the use of the experimental data on the inelastic nucleon structure functions, the possibility is shown that the deep-inelastic effect leads to the correct sign of the observed $n-p$ mass difference, under the condition that

$$\lim_{q^2\to\infty}\int_0^{2M}(1-2q^2R/\omega^2)G_2\,d\omega>0 ,$$

where $\omega=q^2/\nu$, $R=\sigma_l/\sigma_t$ is the ratio of virtual-photon cross sections, and G_2 stands for $\nu W_2(q^2,\nu)$ in the Bjorken limit. The sufficient condition is then found to be either $R\simeq k\omega^2/q^2$ ($k<\frac{1}{2}$) or $R\propto 1/q^{2+\delta}$ ($\delta>0$), as $q^2\to\infty$. In consideration of the experimental fact that the ratio of structure functions, $W_{2,n}/W_{2,p}$, in the range where the greater part of the contribution to the relevant integral results is not far away from the threshold value as $\omega/2M\to 1$ predicted by Bloom and Gilman, the deep-inelastic part of the $n-p$ mass difference is effectively written in the form of the magnetic-moment-type self-energy. It is also shown that if this is similarly applicable to other baryons, and if the SU(3) magnetic-moment relations hold, the correct signs and right orders of magnitude of the mass differences $\Sigma^- - \Sigma^+$, $\Xi^- - \Xi^0$, as well as $\frac{1}{2}(\Sigma^+ + \Sigma^-) - \Sigma^0$, are reproduced by the theory with no adjustable parameter except an input of the observed $n-p$ mass difference.

I. INTRODUCTION

It was first pointed out by the author[1] in 1953 that the observed $n-p$ mass difference might be explained in terms of the predominance of the magnetic-moment self-energy over the electric-charge self-energy. The mass difference was given by

$$\Delta M(n-p)\simeq\Delta(Q^2)\langle e^2/r\rangle-\Delta(\mu^2)\langle(e/2M)^2/r^3\rangle , \tag{1.1}$$

where Q is the electric charge of the nucleon in units of the elementary charge e, μ is the magnetic moment of the nucleon in units of the nucleon magneton $e/2M$, M is the nucleon mass, and then $\Delta(Q^2)=Q_n^{\,2}-Q_p^{\,2}=-1$, $\Delta(\mu^2)=\mu_n^{\,2}-\mu_p^{\,2}=-4.14$ for the $n-p$ mass difference. The field-theoretic calculation of the mass difference at an early stage[1,2] seemed to support the above possibility, because cutoff factors could at that time be suitably chosen to make the magnetic self-energy dominant. Afterwards, however, as the experimental data on the nucleon structure were accumulated, the nucleon elastic form factors turned out to fall off very rapidly with the momentum transfer squared, q^2.[3] By the use of empirical form factors such as the dipole-fit form factors, the contribution to the magnetic self-energy at large q^2 was strongly suppressed, so the notorious wrong sign for the $n-p$ mass difference was obtained.[4] It was also made clear that analogous attempts to explain the $\Sigma^- - \Sigma^+$ and $\Xi^- - \Xi^0$ mass differences resulted in discouragingly small values compared with the observed ones owing to the same situation as in the $n-p$ mass difference, while only for the $\frac{1}{2}(\Sigma^+ + \Sigma^-) - \Sigma^0$ mass difference was the result satisfactory.[5] Although many interesting attempts have successively been made to overcome the difficulty, the conclusions obtained so far do not appear promising.[6] It has been noted,[7] however, that when we once have a dominant contribution to the $n-p$ mass difference with the correct sign in the form of the magnetic self-energy [like the second term of (1.1)], we may easily explain the correct signs and right orders of magnitude of the $\Sigma^- - \Sigma^+$ and $\Xi^- - \Xi^0$ mass differences by the use of the SU(3) magnetic-moment relations,[8]

$$\mu_n = 2\mu_\Lambda = -2\mu_{\Sigma^0} = \mu_{\Xi^0} = -(2/\sqrt{3})\mu_{\Lambda\Sigma^0}\,, \qquad \mu_p = \mu_{\Sigma^+}\,, \quad \mu_n + \mu_p = -\mu_{\Sigma^-} = -\mu_{\Xi^-}\,. \tag{1.2}$$

The main purpose of the present work is to investigate whether the deep-inelastic effect on the electromagnetic self-energy can give the correct sign of the observed $n-p$ mass difference under certain conditions, with the aid of recent experimental data on the inelastic electron-nucleon scattering. To carry out the calculation, we make assumptions concerning the validity of the Cottingham formula[9] and the nontrivial Bjorken limit.[10] Unlike Harari's proposal[11] or Pagels's treatment,[12] we propose unsubtracted dispersion relations[13] for both the functions $t_1(q^2, \nu)$ and $t_2(q^2, \nu)$ even in case of the $\Delta I = 1$ mass difference, with the requirement of the absence of divergences worse than logarithmic ones. Thanks to the experimental fact that the difference of the inelastic structure functions of the proton and neutron, $\nu W_{2,p} - \nu W_{2,n}$,[14,15] is clearly positive in the whole range of $\omega(=q^2/\nu)$, $0 \lesssim \omega/2M \lesssim 1$, the sufficient condition leading to the correct sign of the $n-p$ mass difference is found to be either $R \simeq k\omega^2/q^2$ ($k < \frac{1}{2}$) or $R \propto 1/q^{2+\delta}$ ($\delta > 0$) as $q^2 \to \infty$, where $R = \sigma_l/\sigma_t$ is the ratio of virtual photon cross sections.

Furthermore, considering that the experimental ratio $W_{2,n}/W_{2,p}$ in the region[14,15] $0.2 \lesssim \omega/2M \lesssim 0.7$ (where the greater part of the contribution to the relevant integral comes in) may be approximately equal to the threshold value (as $\omega/2M \to 1$) predicted by Bloom and Gilman,[16] we can effectively write the deep-inelastic part of the $n-p$ mass difference in the form of a magnetic-moment-type self-energy like the second term of (1.1). It is also shown that if such an approximation is similarly good for other baryons, and if the baryon magnetic moment relations in the SU(3) symmetric limit (1.2) hold, the correct signs and right orders of magnitude of the observed hyperon mass differences can be explained without any adjustable parameter except an input of the observed $n-p$ mass difference. Here the input means that the calculated mass difference containing a logarithmic divergence is identified with the corresponding observed quantity by analogy with the conventional renormalization.

Section II is devoted to discussion of a logarithmic divergence and unsubtracted dispersion relations. In Sec. III we calculate the deep-inelastic contribution under the prescribed conditions and describe how the magnetic-moment-type self-energy arises. In Sec. IV, we numerically estimate the hyperon mass differences $\Sigma^- - \Sigma^+$, $\frac{1}{2}(\Sigma^+ + \Sigma^-) - \Sigma^0$, $\Xi^- - \Xi^0$. We close with a few concluding remarks in Sec. V.

II. LOGARITHMIC DIVERGENCE AND UNSUBTRACTED DISPERSION RELATIONS

We will first require that the electromagnetic self-energy of the baryon must be at most logarithmically divergent, and that any logarithmically divergent self-energy (if it arises) must not be eliminated simply because it is unwanted but must be adequately renormalized in terms of the observed quantities such as the $n-p$ mass difference. The requirement seems plausible, insofar as we rely on the conventional field theory. Let us imagine a fictitious world where all the strong interactions are switched off. The (charged) physical baryon would then become a Dirac core, which would behave just as a positive or negative electron in the theory of quantum electrodynamics apart from the bare mass. That is, only the minimal electromagnetic interaction would work, and hence the logarithmically divergent self-mass δM being treated by means of the mass-renormalization procedure would occur. Now we return to the real world where the strong interactions are switched on. Since the physical baryon thus gets hadronic radiative reaction effects due to virtual emission and absorption of various hadrons, it must have the spread-out distribution of charge and magnetic moment. Consequently it appears difficult to admit that the physical baryon could have a stronger singular distribution than the original Dirac core, even if it may maintain a part of the original singularity. This is the reason why we want to require the absence of divergences worse than the logarithmic one.

Next we will assume unsubtracted dispersion relations for both the functions $t_1(q^2,\nu)$ and $t_2(q^2,\nu)$ which are related to the forward Compton amplitude,[13]

$$T_{\mu\nu}(q^2,\nu)=t_1(q^2,\nu)(q^2g_{\mu\nu}-q_\mu q_\nu)+t_2(q^2,\nu)[\nu^2g_{\mu\nu}+q^2p_\mu p_\nu/M^2+\nu(p_\mu q_\nu+p_\nu q_\mu)/M]\,, \tag{2.1}$$

where $-q^2$ is the virtual photon mass squared, $\nu=-p\cdot q/M$ is the photon laboratory energy, p is the baryon momentum, and M is the baryon mass. According to Cottingham's work,[9] the electromagnetic mass of a baryon is written in terms of these functions:

$$\delta M=-\frac{1}{4\pi}\int_0^\infty\frac{dq^2}{q^2}\int_{-q}^{q}d\nu(q^2-\nu^2)^{1/2}[3q^2t_1(q^2,i\nu)-(q^2+2\nu^2)t_2(q^2,i\nu)]\,. \tag{2.2}$$

The unsubtracted dispersion relations which the functions $t_i(q^2,\nu)$, $i=1,2$ obey are

$$t_i(q^2,\nu)=\frac{4Mq^2}{q^4-4M^2\nu^2}f_i(q^2)+\frac{1}{\pi}\int_{\nu_0}^\infty\frac{\operatorname{Im}t_i(q^2,\nu')2\nu'd\nu'}{\nu'^2-\nu^2}\,, \tag{2.3}$$

where $\nu_0=(2Mm_\pi+m_\pi{}^2+q^2)/2M$, m_π is the pion mass, and $f_i(q^2)$ are related to the Sachs form factors,

$$f_1(q^2)=\frac{\alpha}{\pi}\frac{G_M{}^2(q^2)-G_E{}^2(q^2)}{4M^2[1+(q^2/4M^2)]}\,,$$
$$f_2(q^2)=\frac{\alpha}{\pi}\frac{(q^2/4M^2)G_M{}^2(q^2)+G_E{}^2(q^2)}{q^2[1+(q^2/4M^2)]}\,. \tag{2.4}$$

The reasons we assume the unsubtracted dispersion relations are as follows. We are concerned about the order of divergences, so we wish to avoid not only an ambiguous subtraction constant but any technique which is in danger of making the order of intrinsic divergence obscure. In addition, since we are looking for the deep-inelastic contribution as $q^2\to\infty$, $\nu\to\infty$, q^2/ν fixed, we are not necessarily bound to Harari's argument[11] based on the Regge-pole theory, in which he has asserted the necessity of a subtraction in order that the fixed-q^2 dispersion integral in ν may converge as $\nu\to\infty$.

Before discussing the deep-inelastic contribution, we make a remark on the elastic contribution to the self-energy δM^E at large q^2. By the use of unsubtracted dispersion relations, we obtain the leading term at large q^2,

$$\delta M^E\simeq\frac{3\alpha}{2\pi}M\int_{\neq 0}^\infty\frac{dq^2}{q^2}[G_E{}^2(q^2)-\tfrac{1}{2}G_M{}^2(q^2)]\,, \tag{2.5}$$

where α is the fine-structure constant, and G_E and G_M are the Sachs form factors. The requirement of the absence of divergences worse than logarithmic ones thus implies that both $G_E(q^2)$ and $G_M(q^2)$ may be constant or zero as $q^2\to\infty$.[17] The desirable signs and approximate ratios of all the baryon mass differences immediately follow from (2.5), in the event of $G(q^2)$ at large q^2 dominating the integral. A function $G(q^2)$ common to all the baryons is defined by the scaling relations,[18]

$$G_E^i(q^2)=Q_iG(q^2),\qquad G_M^i(q^2)=\mu_iG(q^2)\,, \tag{2.6}$$

where i refers to a member of the octet baryons or represents the $\Lambda\Sigma^0$ transition. The rapid falloff of the elastic form factors is experimentally known up to the available q^2, and so even a very small constant core in $G(q^2)$ at large q^2 is unlikely to exist. However, such a possibility could not be excluded,[7] because we are not sure whether the available experimental data have already disclosed the entire asymptotic characteristics. As for the Dirac and Pauli form factors F_1, F_2, the requirement means that $F_1(q^2)$ approaches a constant or zero in any way, while $F_2(q^2)$ tends to zero not slower than $1/q^2$, as $q^2\to\infty$.

III. DEEP-INELASTIC CONTRIBUTION

Let us now consider the deep-inelastic contribution to the baryon self-energy. Assuming unsubtracted dispersion relations, we have the inelastic part of the self-energy,[9]

$$\delta M^I=\frac{1}{2\pi}\int_0^\infty dq^2\int_{\nu_0}^\infty\nu\,d\nu\left\{-3\left[\left(1+\frac{q^2}{\nu^2}\right)^{1/2}-1\right]\operatorname{Im}t_1(q^2,\nu)+\left[\left(1+\frac{q^2}{\nu^2}\right)^{1/2}\left(1-2\frac{\nu^2}{q^2}\right)+2\frac{\nu^2}{q^2}\right]\operatorname{Im}t_2(q^2,\nu)\right\}. \tag{3.1}$$

The usual structure functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ are related to the functions t_1 and t_2 as follows:

$$\operatorname{Im}t_1(q^2,\nu)=\frac{\alpha}{q^2}\left[W_1(q^2,\nu)-\frac{\nu^2}{q^2}W_2(q^2,\nu)\right],\qquad \operatorname{Im}t_2(q^2,\nu)=\alpha W_2(q^2,\nu)/q^2\,, \tag{3.2}$$

and the ratio W_1/W_2 is bounded,

$$\frac{W_1}{W_2}=\frac{1+(\nu^2/q^2)}{1+R}, \tag{3.3}$$

where $R=\sigma_l/\sigma_t$, the ratio of the absorption cross sections for longitudinal and transverse photons. Bjorken[10] has suggested that the structure functions should scale for large q^2 and ν with $\omega=q^2/\nu$ fixed,

$$\lim_{\substack{q^2\to\infty\\ q^2/\nu \text{ fixed}}} W_1(q^2,\nu)=G_1(\omega), \qquad \lim_{\substack{q^2\to\infty\\ q^2/\nu \text{ fixed}}} \nu W_2(q^2,\nu)=G_2(\omega). \tag{3.4}$$

Experiments[19] have shown the remarkable fact that the Bjorken scaling takes place with reasonable accuracy even for small q^2 and ν. Thus we may approximately estimate the main inelastic contribution by assuming a nontrivial Bjorken limit in the deep-inelastic region.

Introducing the variable ω into (3.1) instead of ν, and combining the relation (3.3) with the Bjorken limit (3.4), we have

$$\delta M^I \simeq -\frac{3\alpha}{8\pi}\int_{\neq 0}^{\infty}\frac{dq^2}{q^2}\int_0^{2M} d\omega\left[2\left(1-\frac{Rq^2}{\omega^2}\right)(1+R)^{-1}\left(1-\frac{\omega^2}{4q^2}+\cdots\right)-\left(1-\frac{\omega^2}{6q^2}+\cdots\right)\right]G_2(\omega). \tag{3.5}$$

The condition for the absence of divergences worse than a logarithmic one is seen to be that the ratio R tends to zero not slower than $1/q^2$ as $q^2\to\infty$.[20] In order to obtain a dominant δM^I with the negative sign, however, the somewhat stronger condition is necessary:

$$\lim_{q^2\to\infty}\int_0^{2M}(1-2\,q^2R/\omega^2)G_2(\omega)d\omega>0. \tag{3.6}$$

Some parametrizations of R which are consistent with the measurements over the measured range have been reported[14,15]:

$$R=0.18\pm 0.10,$$
$$R=0.031q^2/M^2, \tag{3.7}$$
$$R\approx q^2/\nu^2(=\omega^2/q^2).$$

Neither the first choice nor the second is compatible with (3.6). The third choice satisfies the inequality (3.6) only in the case of a coefficient less than $\frac{1}{2}$. We thus find the sufficient condition to be either

$$R\simeq k\omega^2/q^2 \quad (k<\tfrac{1}{2}) \tag{3.8}$$

or

$$R\propto 1/q^{2+\delta} \quad (\delta>0) \tag{3.9}$$

as $q^2\to\infty$.

The experimental data for the inelastic electron-nucleon scattering[14,15] indicate that the difference of the structure functions $\nu W_{2,p}-\nu W_{2,n}$ is evidently positive in the whole range of ω. From the visual-fit curve through the experimental points of $\nu W_{2,p}-\nu W_{2,n}$ versus $2M/\omega$,[21] we may roughly estimate

$$\frac{1}{2M}\int_0^{2M}(\nu W_{2,p}-\nu W_{2,n})d\omega\simeq 0.05>0. \tag{3.10}$$

Putting the condition (3.8) or (3.9) into the formula (3.5), and using the numerical value (3.10), we have the deep-inelastic part of the $n-p$ mass difference

$$\Delta M^I(n-p)\simeq 0.08\int_{\neq 0}^{\infty}\frac{dq^2}{q^2}\ (\text{in MeV})>0, \tag{3.11}$$

where an additional factor $(1-2k)$ is needed in the case of (3.8).

Bloom and Gilman[16] have derived relations between the elastic and inelastic form factors and have predicted that the threshold value of the ratio

$$\frac{W_{2,n}}{W_{2,p}}\to\frac{\mu_n^{\,2}}{\mu_p^{\,2}}\simeq 0.47 \tag{3.12}$$

as $\omega/2M\to 1$ for large q^2. The extrapolation intercept at $\omega/2M=1$ obtained from the visual-fit curve of the data points $W_{2,n}/W_{2,p}$ versus $(\omega/2M)^{-1}$ has appeared to be consistent with the threshold value of (3.12).[14] Very recent experiments[15] suggest not only the presence of an isospin-dependent part of the photon-nucleon amplitude but the unexpected possibility that the neutron's scattering may vanish in the limit $\omega/2M\to 1$. In spite of such unexpected circumstances, we want to pay attention to the following. The major contribution to the $n-p$ mass difference integral with respect to ω comes out in the intermediate range $0.2\lesssim\omega/2M\lesssim 0.7$ where the experimental value of the ratio $W_{2,n}/W_{2,p}$ decreases slowly from ~0.6 to ~0.4, namely it deviates a little from the threshold value of (3.12); the contribution to the mass difference in the region $0\lesssim\omega/2M\ll 1$ becomes very small because $W_{2,n}/W_{2,p}\simeq 1$; the contribution in the region $\omega/2M\approx 1$ also becomes negligible owing to vanishingly small $W_{2,n}$ and $W_{2,p}$. We may therefore effectively estimate the integral with respect to ω by assuming

$$G_2(\omega) \simeq \mu^2 g(x), \quad x_0 \lesssim x = \omega/2M \lesssim 1, \tag{3.13}$$

where $x_0 = \omega_{\min}/2M$ and $g(x)$ is a certain common function determined experimentally. The lower limit $\omega_{\min}$ or x_0 should be defined by

$$\int_{x_0}^{1} g(x)\,dx \simeq \frac{1}{2M}\int_0^{2M} \frac{(W_{2,p}-W_{2,n})d\omega}{\mu_p{}^2-\mu_n{}^2} \simeq 0.012\,, \tag{3.14}$$

but the numerical value of the integral is not very sensitive to x_0.

Hoping that the above situation for the nucleon may similarly be realized for other baryons, we assume that (3.13) gives a fairly good approximation to estimate the analogous integral for all the baryons, though we have neither experimental nor theoretical basis for this assumption at the present time. We then obtain the result

$$\frac{\Delta M_i}{M_i} \simeq -\frac{3\alpha}{4\pi}\,\Delta(\mu_i{}^2)\int_{\neq 0}^{\infty}\frac{dq^2}{q^2}\int_{x_{0i}}^{1} g(x)\,dx \tag{3.15}$$

$$\simeq -(0.21\times 10^{-4})\Delta(\mu_i{}^2)\int_{\neq 0}^{\infty}\frac{dq^2}{q^2}\,, \tag{3.16}$$

where the subscript i refers to a certain isomultiplet within which we are considering the mass difference, and the lower limit x_{0i} may be approximated to x_0. The expression (3.15) or (3.16) is obviously in the form of a magnetic-moment-type self-energy like the second term of (1.1).

IV. NUMERICAL ESTIMATE OF MASS DIFFERENCES

As stated in Sec. II, the appearance of a logarithmic divergence in (3.11) or (3.16) should be ascribed to the same origin as in quantum electrodynamics. We may expect that the logarithmic divergence difficulty here inherited from recent field theory should be solved by future theory which could essentially remove the existing divergences contained in the renormalization constants in quantum electrodynamics.

Extending the idea of the conventional renormalization to the difference of self-energies, $\Delta M(n-p) = \delta M_n - \delta M_p$, we will identify the divergent $n-p$ mass difference with the observed quantity $\Delta M^{\text{obs}}(n-p) = 1.29$ MeV. Now we postulate that the deep-inelastic contribution $\Delta M^I(n-p)$ given by (3.11) together with the usual elastic contribution $\Delta M^E(n-p) = -0.79$ MeV constitutes the major part of the observed $n-p$ mass difference. In other words, we may replace the divergent integral $\int_{\neq 0}^{\infty} dq^2/q^2$ by the numerical value $(\Delta M^{\text{obs}} - \Delta M^E)/0.08 \simeq 26$ as an input for evaluating the other baryon mass differences. Here we understand that in the case of (3.8) an additional factor $(1-2k)^{-1}$ is included. Assuming that the divergent integral in (3.16) is common to all the baryon octet members, we can estimate the deep-inelastic contribution to the $\Sigma^- - \Sigma^+$, $\frac{1}{2}(\Sigma^+ + \Sigma^-) - \Sigma^0$, and $\Xi^- - \Xi^0$ mass differences with no adjustable parameter.

The deep-inelastic part is now combined with the elastic one, so that the resultant figures are to be compared with the experimental data. Table I shows how our result improves the situation. In particular, for the $\Sigma^- - \Sigma^+$ and $\Xi^- - \Xi^0$ mass differences, the agreement between the calculated and observed values is considerably improved. In evaluating ΔM^E and ΔM^I in Table I, the SU(3) magnetic moment relations (1.4) have been assumed, taking into account the strong mass-splitting effect. For these mass differences, we have used the numerical values of $\Delta(\mu^2)$:

$$\mu_n{}^2 - \mu_p{}^2 = -4.14\,,$$

$$\mu_{\Sigma^-}{}^2 - \mu_{\Sigma^+}{}^2 = -11.35\ (-7.03)\,,$$

$$\tfrac{1}{2}(\mu_{\Sigma^+}{}^2 + \mu_{\Sigma^-}{}^2) - (\mu_{\Sigma^0}{}^2 + \mu_{\Lambda\Sigma^0}{}^2) = 1.01\ (0.63)\,,$$

$$\mu_{\Xi^-}{}^2 - \mu_{\Xi^0}{}^2 = -5.69\ (-2.89)\,.$$

The numbers in parentheses above as well as those in Table I indicate the corresponding values if we ignore the difference between the nucleon and hyperon magnetons.[22] The dipole form factor adopted to evaluate ΔM^E has been assumed universal with respect to a nondimensional variable $q/2M_i$,[23] i.e.,

$$G(q^2) = 1/[1 + (4M_N{}^2/0.71)(q/2M_i)^2]^2\,, \tag{4.1}$$

with $M_N = 0.939$ in units of GeV. It should be noted in Table I that the Coleman-Glashow electromagnetic mass relation[8] still holds fairly well, though the strong mass-splitting effect is taken into account.

We make a comment on the above prescription which has let the logarithmically divergent integral be identified with the observed quantity. Let us now look back on Bethe's original work about the

TABLE I. The elastic and deep-inelastic parts of the calculated mass difference ΔM^E, ΔM^I are listed in units of MeV. The sum $\Delta M^E + \Delta M^I = \Delta M^{\text{calc}}$ is compared with the observed mass difference ΔM^{obs}. The numerical figures in parentheses indicate the corresponding values in case one ignores the difference between the nucleon and hyperon magnetons.

	ΔM^E (MeV)	ΔM^I (MeV)	ΔM^{calc} (MeV)	ΔM^{obs} (MeV)
$n-p$	−0.8	2.1(input)	1.3	1.3± 0.0
$\Sigma^- - \Sigma^+$	0.6(0.4)	7.3(4.5)	7.9(4.9)	7.9± 0.1
$\frac{1}{2}(\Sigma^+ + \Sigma^-) - \Sigma^0$	1.2(1.2)	−0.6(−0.4)	0.6(0.8)	0.9± 0.1
$\Xi^- - \Xi^0$	1.7(1.5)	4.0(2.0)	5.7(3.5)	6.6± 0.7

Lamb shift,[24] which may give us a typical and heuristic example. In his calculation, he encountered a logarithmic divergence owing to the nonrelativistic treatment, but the result evaluated by means of a suitable cutoff was in remarkably good agreement with experiment. Furthermore the logarithmic divergence that appeared in his work was completely settled by the renormalization theory of quantum electrodynamics. Can we expect to solve analogously our divergence difficulty in the future? We should remember, of course, that "good luck does not always repeat itself." Yet we are tempted to conjecture that our logarithmic divergence also has an appropriate physical meaning when a suitable cutoff is made, if one hopes that the divergence difficulty will be solved by the future field theory.

We will roughly estimate the size of cutoff momentum which provides an adequate value to the logarithmically divergent integral. Let the upper and lower limits of the integration variable q in (3.11) be Λ and Λ_0; then we have

$$\Lambda/\Lambda_0 \simeq e^{13} \simeq \exp[(f^2/4\pi)^{-1}], \tag{4.2}$$

where f is the strong pseudovector coupling constant between pion and nucleon. It seems quite interesting to compare (4.2) with a similar formula in quantum electrodynamics. As is well known, if we ascribe most of the mass of an electron to its electromagnetic self-energy, we obtain[25]

$$\Lambda/m_e \simeq e^{137} \simeq \exp[(e^2/4\pi)^{-1}]. \tag{4.3}$$

V. CONCLUDING REMARKS

We have shown in this paper that the deep-inelastic effect leads to a corelike self-energy with the negative sign which implies a kind of magnetic self-energy, under some prescribed conditions. As was emphasized in a previous paper,[7] the existence of such a corelike magnetic self-energy seems favorable for the understanding of the electromagnetic mass differences of all the baryons. It appears unlikely, however, that the necessary core arises from the elastic contribution alone, in view of the rapid falloff of the elastic form factors. Thanks to the nontrivial Bjorken limit, we have been able to obtain the corelike contribution from the deep-inelastic effect.

Among several assumptions proposed or conditions prescribed in the present work, the crucial ones seem to be

(i) an unsubtracted dispersion relation for $t_1(q^2, \nu)$ in the case of the $\Delta I = 1$ mass difference;

(ii) an asymptotic behavior of R in the limit $q^2 \to \infty$;

(iii) an approximation (3.13) for the structure function of all the baryons.

The unsubtracted dispersion relation has been proposed from a plausible but rather speculative conjecture, so we need some justifiable basis. The argument given by Jackiw and others[13] seems to be in favor of our proposal. As for the asymptotic behavior of R, the sufficient condition (3.8) or (3.9) has been presented to guarantee the negative δM^I. Although the condition does not seem to be incompatible with the available measurements, the justifiable asymptotic behavior should be determined by the future experiments. The effective approximation (3.13) has been assumed in consideration of recent experimental data.[14,15] This approximation is not always needed to explain the correct sign of the $n-p$ mass difference. It has been used not only to express the mass difference in the form of the magnetic-moment-type self-energy, but to connect the $n-p$ mass difference with all the hyperon mass differences through the SU(3) magnetic moment relations. According to Kendall's report,[15] the threshold value of $W_{2,n}/W_{2,p}$ as $q^2 \to \infty$, $\omega \to 2M$ appears to be smaller than the prediction by Bloom and Gilman.[16] However, as far as the integral of the structure function difference is concerned, the main contribution comes from the region where the approximation is good enough. Whether a similar relation holds for all the baryons must be controversial, of course. If this is not the case, the content of this paper following (3.13) is empty.

Even if our crude result is not far from reality at least semiquantitatively, we should also take into account every possible contribution ignored in this work which might come out constructively. There have been several investigations about the low-energy resonance contribution[26] and the Regge-pole contribution,[27] both of which have turned out to be small.

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[1]H. Katsumori, Memoirs of Osaka Gakugei Univ. B,

No. 2, 28 (1953); H. Katsumori, Soryushiron-Kenkyu (mimeographed circular in Japanese) 6, 1242 (1954).

[2]Similar attempts were independently made by several authors: R. P. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954); Y. Oishi and H. Katsumori, Progr. Theoret. Phys. (Kyoto) 12, 109 (1954); A. Petermann, Helv. Phys. Acta 27, 441 (1954). Notice that in the paper of Feynman and Speisman, their statement on the sign of the magnetic moment self-energy was wrong.

[3]See, for example, W. K. H. Panofsky, in *Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967,* edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 371.

[4]The earlier experimental data seemed to be reconciled with the $n-p$ mass difference: H. Katsumori and M. Shimada, Phys. Rev. 124, 1203 (1961). However, as experiments disclosed the nucleon structure in greater and greater detail, the notorious wrong sign was confirmed.

[5]See, for example, Table I in D. J. Gross and H. Pagels, Phys. Rev. 172, 1381 (1968), which contains a list of the electromagnetic mass differences of all the baryons calculated in terms of the dipole-fit form factors.

[6]See, for example, M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970).

[7]H. Katsumori and M. Morita, Memoirs of Chubu Inst. of Tech. 3, 137 (1967); H. Katsumori, *ibid.* 6, 105 (1970).

[8]S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

[9]W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[10]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[11]H. Harari, Phys. Rev. Letters 17, 1303 (1966).

[12]H. Pagels, Phys. Rev. 185, 1990 (1969).

[13]R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970). Since Cottingham's $t_1(q^2,\nu)$ used in our paper is equivalent to their $T_L(q^2,\nu)/q^2$, the argument on the structure of the current-commutation function seems to favor our proposal for unsubtracted dispersion relations.

[14]J. Drees, invited talk presented at the Frühjahrstagung Kern- und Hochenergiephysik, Hamburg, 1971 (unpublished).

[15]After the completion of the present work, new experimental data on the deep-inelastic electron scattering have come to our notice. H. W. Kendall, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972), p. 248.

[16]E. B. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970); see also S. M. Berman and M. Jacob, *ibid.* 25, 1683 (1970).

[17]In the case of $G(q^2) \lesssim C/|q^2|^\delta$ at large q^2, C being a constant and δ a very small positive number, we find difficulty in discriminating it in practice from a pure constant. Cf. R. Acharya, H. H. Aly, and K. Schilcher, Phys. Rev. 170, 1597 (1968).

[18]The scaling relations for the elastic form factors should be thought of as exact only in the symmetry limit, e.g., SU(6)$_W$: K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965). Recent experiments have shown a deviation from the scaling. See W. Bartel *et al.*, Phys. Letters 33B, 245 (1970); L. E. Price *et al.*, Phys. Rev. D 4, 45 (1971); Ch. Berger *et al.*, Phys. Letters 35B, 87 (1971).

[19]F. J. Gilman, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 177; R. E. Taylor, *ibid.* p. 251. See also Refs. 14 and 15.

[20]This prediction is obviously in contradiction to Sakurai's vector-meson-dominance model: J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969).

[21]See Fig. 17 of Ref. 14 and Fig. 14 of Ref. 15.

[22]There may be a controversy on the symmetry breaking of the baryon magnetic moments: H. Katsumori (unpublished).

[23]H. Katsumori, Progr. Theoret. Phys. (Kyoto) 45, 1988 (1971).

[24]H. A. Bethe, Phys. Rev. 72, 339 (1947).

[25]See, for example, S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1961), p. 541.

[26]See, for example, T. Muta, Phys. Rev. 171, 1661 (1968).

[27]See, for example, Y. Hara, Nucl. Phys. B8, 441 (1968).