Compton Scattering. II. Differential Cross Sections and Left-Right Asymmetry*

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The differential cross sections for polarized electron-photon scattering are calculated to order e^6 and the unpolarized result of Brown and Feynman is rederived. The left-right asymmetry is calculated for a polarized target electron in agreement with Miller and Wilcox. This leaves the disagreement between theory and experiment unresolved. These results are obtained by using the six invariant amplitudes in spectral form, which were calculated in the preceding communication.

I. INTRODUCTION

The differential cross section for unpolarized Compton scattering, to order e^6, was calculated by Brown and Feynman[1] (hereafter we refer to this paper as BF) about 20 years ago; their result has not been independently confirmed. The left-right asymmetry (for polarized target electron) was computed a decade ago by Miller and Wilcox[2] and by Frolov,[3] who obtained different answers.[4] However, the general case has not previously been calculated.

In the preceding paper[5] (called paper I) the six invariant amplitudes are obtained in spectral form to order e^4. Here, we will use those amplitudes to evaluate the physically more interesting helicity amplitudes, which contain all polarization information. As particular applications, the helicity differential cross sections are calculated, and the unpolarized cross section and the asymmetry parameter are rederived.

In Sec. II, we show how the helicity amplitudes for the Compton processes are obtained from the M_i's of paper I. In Sec. III, the (polarized and unpolarized) differential cross sections are calculated. The low-energy limit of the unpolarized cross section (including the constant term) is presented. Also, there are some remarks on the pair-creation case. The left-right asymmetry for a polarized electron target is discussed in Sec. IV. Appendixes A and B contain the integrals required in Sec. III, while in Appendix C we list the M_i's in integrated form.

II. HELICITY AMPLITUDES

In this section, we express the helicity amplitudes for the Compton processes in terms of the invariant amplitudes M_i. This can be done by applying Eq. (I52) (or its crossed version) to the appropriate helicity states. (Here I refers to equations in paper I.)

For Compton scattering, we start from

$$\mathfrak{M} = 2mu^*_{p_1\sigma_1}\gamma^0 e^{*\mu}_{k_1\lambda_1}\sum_{i=1}^{6}\left(\frac{1}{2m^4}M_i\right)(\mathfrak{L}_i)_{\mu\nu}e^{\nu}_{k_2\lambda_2}u_{p_2\sigma_2}$$
$$\equiv f(\sigma_2\lambda_2;\sigma_1\lambda_1), \quad (1)$$

where σ_2 (σ_1) and λ_2 (λ_1) are the incoming (outgoing) helicities of the electron and photon, respectively. To find these helicity amplitudes in terms of the M_i's, it is convenient to work in the center-of-mass (c.m.) system with $\vec{p}_2$ in the z direction and $\vec{p}_1$ in the x-z plane, and to use the following explicit representations for the polarization vectors and the Dirac spinors[6]:

$$u_{p\sigma} = \left[\left(\frac{p^0+m}{2m}\right)^{1/2} + \left(\frac{p^0-m}{2m}\right)^{1/2} i\gamma_5\sigma\right]v_\sigma,$$

$$\gamma^0 v_\sigma = v_\sigma, \quad v_{2+} = \begin{pmatrix}1\\0\end{pmatrix}, \quad v_{2-} = \begin{pmatrix}0\\1\end{pmatrix},$$

$$v^*_{1+} = (\cos\tfrac{1}{2}\theta, \;\; \sin\tfrac{1}{2}\theta), \quad v^*_{1-} = (-\sin\tfrac{1}{2}\theta, \;\; \cos\tfrac{1}{2}\theta),$$

$$\vec{e}_{2+} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}, \quad \vec{e}_{2-} = -\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\\0\end{pmatrix},$$

$$\vec{e}^{\,*}_{1+} = \frac{1}{\sqrt{2}}(\cos\theta,\; i,\; -\sin\theta),$$

$$\vec{e}^{\,*}_{1-} = -\frac{1}{\sqrt{2}}(\cos\theta,\; -i,\; -\sin\theta),$$

where θ is the angle between $\vec{p}_2$ and $\vec{p}_1$. Then, with the help of the definitions

$$m^2 s = -(p_1+k_1)^2, \quad m^2 t = -(p_1-p_2)^2,$$
$$m^2 u = -(p_1-k_2)^2, \quad \kappa = 1-s, \quad \tau = 1-u,$$

and the identities

$$d \equiv 1-su = \kappa^2\cos^2(\tfrac{1}{2}\theta), \; -st = \kappa^2\sin^2(\tfrac{1}{2}\theta),$$
$$\gamma^0\gamma_k = i\gamma_5\sigma_k, \quad \sigma_{ij} = \epsilon_{ijk}\sigma^k,$$

we obtain

$$f(--;++)=\frac{(-t)^{1/2}}{8\kappa}[(\kappa-2)tM_1-\kappa tM_2-2(\kappa^2+d)M_3],$$

$$f(++;+-)=-\frac{td^{1/2}}{4\kappa}(M_1+M_3),$$

$$f(-+;+-)=\frac{(-t)^{3/2}}{8\kappa}[(2-\kappa)M_1-\kappa M_2+2M_3], \qquad (2)$$

$$f(++;++)=-\frac{d^{1/2}}{8\kappa}[2(\kappa^2+t)M_4+dM_5-(\kappa^2-t)M_6],$$

$$f(-+;++)=-\frac{d(-t)^{1/2}}{8\kappa}[2M_4+\tfrac{1}{2}(2-\kappa)M_5+M_6],$$

$$f(+-;+-)=-\frac{d^{3/2}}{8\kappa}(2M_4+M_5+M_6).$$

In the total count of 16 amplitudes, these six occur 2, 4, 2, 2, 4, and 2 times, respectively (apart from phase factors resulting from parity conservation and time-reversal invariance).[7] As mentioned in paper I, we see that M_1, M_2, and M_3 describe photon helicity flip, and M_4, M_5, and M_6 describe photon helicity nonflip amplitudes. These are the same results given by Bardeen and Tung,[8] except for some sign errors in their paper.

For the pair-creation case, again it is convenient to work in the c.m. system with $\vec{k}_2$ in the z direction and $\vec{p}_1$ in the x-z plane, and to use the following explicit representations:

$$\vec{e}_{2'-}=\vec{e}_{2+}^{\,T}=-\frac{1}{\sqrt{2}}(1,i,0),$$

$$\vec{e}_{2'+}=\vec{e}_{2-}^{\,T}=\frac{1}{\sqrt{2}}(1,-i,0),$$

$$v_{1+}^{*}=v_{1'-}^{T}=(\cos\tfrac{1}{2}\psi,\ \sin\tfrac{1}{2}\psi),$$

$$v_{1-}^{*}=v_{1'+}^{T}=(-\sin\tfrac{1}{2}\psi,\ \cos\tfrac{1}{2}\psi),$$

where ψ is the angle between $\vec{k}_2$ and $\vec{p}_1$. Then, with the help of the definitions and identity,

$$m^2s=-(p_1-k_2')^2,\quad m^2t=-(p_1+p_1')^2,$$

$$m^2u=-(p_1-k_2)^2,\quad 2p^0=t^{1/2},$$

$$2p=(t-4)^{1/2},\quad u-s=4pp^0\cos\psi,$$

we obtain the pair-creation helicity amplitudes, $f'(\lambda_2\lambda_{2'};\ \sigma_1\sigma_{1'})$:

$$f'(++;++)=\tfrac{1}{4}t[-pM_1+p^0M_2+(p^0+p)\cos\psi\, M_3],$$

$$f'(--;++)=\tfrac{1}{4}t[-pM_1-p^0M_2+(p^0-p)\cos\psi\, M_3],$$

$$f'(++;+-)=\tfrac{1}{4}t\sin\psi\, M_3,$$

$$f'(+-;++)=-\tfrac{1}{4}pt\sin^2\psi(M_4-\tfrac{1}{2}p^2M_5), \qquad (3)$$

$$f'(+-;+-)=\tfrac{1}{8}pt\sin\psi(1+\cos\psi)(2p^0M_4-pM_6),$$

$$f'(-+;+-)=-\tfrac{1}{8}pt\sin\psi(1-\cos\psi)(2p^0M_4+pM_6).$$

By taking linear combinations of these amplitudes, we reproduce Bardeen and Tung's set except, again, for some sign errors.

III. DIFFERENTIAL CROSS SECTIONS

A. Helicity Cross Sections

From the probability amplitude, Eq. (I51), we obtain the expression for the helicity differential cross section (in the c.m. system),

$$\left(\frac{d\sigma}{d\Omega}\right)_{\sigma_2\lambda_2\to\sigma_1\lambda_1}=[64\pi^2(1-\kappa)m^2]^{-1}|f(\sigma_2\lambda_2;\sigma_1\lambda_1)|^2,$$

which yields, to order e^6, the differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\sigma_2\lambda_2\to\sigma_1\lambda_1}=[64\pi^2(1-\kappa)m^2]^{-1}\{[f^{(2)}(\sigma_2\lambda_2;\ \sigma_1\lambda_1)]^2+2f^{(2)}(\sigma_2\lambda_2;\sigma_1\lambda_1)\,\mathrm{Re}f^{(4)}(\sigma_2\lambda_2;\sigma_1\lambda_1)\}. \qquad (4)$$

The second-order helicity amplitudes can be easily obtained from Eq. (2) by using the e^2 part of M_i [Eq. (I53)], which only comes from the D_i terms. They are

$$f^{(2)}(--;++)=-8\pi\alpha\frac{(-t)^{3/2}}{\kappa^2\tau},\qquad f^{(2)}(++;++)=-8\pi\alpha(\kappa^2+t)\frac{d^{1/2}}{\kappa^2\tau},$$

$$f^{(2)}(++;+-)=8\pi\alpha\frac{td^{1/2}}{\kappa^2\tau},\qquad f^{(2)}(-+;++)=8\pi\alpha\frac{d(-t)^{1/2}}{\kappa^2\tau}, \qquad (5)$$

$$f^{(2)}(-+;+-)=-8\pi\alpha\frac{(\kappa-1)(-t)^{3/2}}{\kappa^2\tau},\qquad f^{(2)}(+-;+-)=-8\pi\alpha\frac{d^{3/2}}{\kappa^2\tau}.$$

The fourth-order helicity amplitudes can be obtained from Eq. (2) by using the α^2 part of Eq. (I53). There, the M_i's are given in spectral form so we have to carry out the spectral integration. The necessary integrals can be easily done (see Appendixes A and B), and the integrated form of the M_i's is given in Appendix C. The final forms of the helicity amplitudes are (apart from a factor of α^2)

$$f^{(4)}(--;++)=\frac{4(-t)^{1/2}}{\kappa}\left[-\frac{t}{\kappa\tau}I(\lambda)-\frac{4}{t}y^2-1-2\left(\frac{1}{\kappa}+\frac{1}{\tau}\right)+G_0(\kappa)-G_0(\tau)+\left(\frac{2(2\kappa+\tau-2)}{\kappa}y\,\mathrm{csch}2y+\frac{\kappa-2}{\kappa(\kappa-1)}\right)\ln\kappa\right.$$
$$\left.+\left(\frac{2(\kappa-2)}{\tau}y\,\mathrm{csch}2y+\frac{3\tau-2}{\tau(\tau-1)}\right)\ln\tau\right], \tag{6}$$

$$f^{(4)}(++;+-)=-\frac{4d^{1/2}}{\kappa}\left[\frac{t}{\kappa\tau}I(\lambda)+\frac{1}{2}+\frac{2}{\kappa}-\frac{\kappa-4}{2\tau}+\frac{2}{td}[\kappa(\kappa+2)-\tau(\kappa-2)]y^2+\frac{t(\kappa-1)}{2d}G_0(\kappa)\right.$$
$$+\frac{t[\kappa-\tau(\kappa-2)]}{2\tau d}G_0(\tau)+\left(\frac{1}{\tau d}[\kappa\tau^2+\tau(\kappa^2-6\kappa+4)-2\kappa(\kappa-2)]y\,\mathrm{csch}2y-\frac{\kappa+3\tau-2}{\tau(\tau-1)}\right)\ln\tau$$
$$\left.+\left(-\frac{2}{\kappa}+\frac{1}{\kappa d}[2\tau^2(\kappa-1)+\tau(3\kappa^2-10\kappa+4)+\kappa(\kappa^2-4\kappa+4)]y\,\mathrm{csch}2y\right)\ln\kappa\right], \tag{7}$$

$$f^{(4)}(-+;+-)=-\frac{4(-t)^{1/2}}{\kappa}\left[\frac{t(\kappa-1)}{\kappa\tau}I(\lambda)+\frac{3\kappa-2}{\tau}-\frac{2}{\kappa}+2-\frac{4}{t}y^2-\frac{t}{\tau}G_0(\tau)\right.$$
$$+\left(-\frac{2}{\kappa}[\kappa^2-4\kappa+2+\tau(\kappa-1)]y\,\mathrm{csch}2y-\frac{3\kappa-2}{\kappa}\right)\ln\kappa$$
$$\left.-\left(\frac{2}{\tau}(\kappa^2-3\kappa+2+\kappa\tau)y\,\mathrm{csch}2y+\frac{3\tau(\kappa-1)-4\kappa+2}{\tau(\tau-1)}\right)\ln\tau\right], \tag{8}$$

$$f^{(4)}(++;++)=-\frac{4d^{1/2}}{\kappa}\left[-\frac{\kappa^2+t}{\kappa\tau}I(\lambda)-2y\tanh y-\frac{2}{d}[\kappa+\tau(2\kappa+1)+\tau^2]yh(y)\coth y-\frac{1}{2(\kappa-1)}-\frac{\kappa+1}{2(\tau-1)}\right.$$
$$-\frac{\kappa}{\tau}-\frac{2}{\kappa}-\frac{2}{\tau}-\frac{1}{d}[\tau^2+\tau(2\kappa+1)+2\kappa^2-\kappa-8]y^2+\frac{\kappa}{2d}(-\kappa^2-2\kappa+4-\tau)G_0(\kappa)$$
$$+\frac{\tau}{2d}(-\tau^2-2\kappa\tau-\kappa^2+3\kappa+4)\;G_0(\tau)$$
$$+\left(\frac{1}{\kappa d}[\tau^2(2-\kappa)+\tau(-\kappa^3-3\kappa^2+8\kappa-4)+\kappa(-\kappa^3+10\kappa-4)]y\,\mathrm{csch}2y+\frac{5\kappa^2-8\kappa+4}{2\kappa(\kappa-1)^2}\right)\ln\kappa$$
$$+\left(\frac{1}{d\tau}[-\kappa^3(\tau-2)-\kappa^2(\tau-1)(3\tau-2)-\kappa(\tau-1)(3\tau^2-4\tau-4)+\tau(-\tau^3+2\tau^2+6\tau-4)]y\,\mathrm{csch}2y\right.$$
$$\left.\left.-\frac{1}{2\tau(\tau-1)^2}[\kappa(-3\tau^2+6\tau-4)-2\tau^3+\tau^2+4\tau-4]\right)\ln\tau\right], \tag{9}$$

$$f^{(4)}(-+;++)$$
$$=-\frac{4(-t)^{1/2}}{\kappa}\left[-\frac{d}{\kappa\tau}I(\lambda)-\frac{d}{2\tau}\left(\frac{4}{\kappa}+\frac{1}{\tau-1}\right)+\frac{2}{d}[\kappa(2\kappa-1)-\tau(\tau+1)]yh(y)\coth y-(\kappa-\tau)y\,\mathrm{csch}2y\right.$$
$$-\frac{1}{d}[\tau^2-\tau(2\kappa-1)-2\kappa^2+9\kappa-8]y^2-\frac{\kappa}{2d}[-\tau(\kappa-1)-2\kappa^2+7\kappa-4]G_0(\kappa)+\frac{\tau}{2d}[\kappa(\tau-2)-\tau^2+4]G_0(\tau)$$
$$+\left(\frac{1}{\kappa d}[(\kappa^2-3\kappa+2)\tau^2+\tau(3\kappa^3-10\kappa^2+12\kappa-4)+\kappa(2\kappa^3-11\kappa^2+14\kappa-4)]y\,\mathrm{csch}2y-\frac{2(\kappa-1)}{\kappa}\right)\ln\kappa$$
$$+\left(\frac{1}{\tau d}[\kappa^2(\tau^2-2\tau+2)-4\kappa(\tau^2-\tau+1)-\tau(\tau^3-2\tau^2-6\tau+4)]y\,\mathrm{csch}2y\right.$$
$$\left.\left.-\frac{1}{2\tau(\tau-1)^2}[\tau(\tau-1)\kappa-2\tau^3+\tau^2+4\tau-4]\right)\ln\tau\right], \tag{10}$$

$$f^{(4)}(+-;+-)=-\frac{4}{\kappa d^{1/2}}\left[-\frac{d^2}{\kappa\tau}I(\lambda)-\frac{d}{2}\left(-4+\frac{4}{\kappa}+\frac{4}{\tau}+\frac{\tau}{\tau-1}\right)\right.$$

$$+\frac{2}{d}[-\kappa^2(\kappa^2-3\kappa+1)-\kappa\tau(2\kappa^2-5\kappa+2)+\tau^2(\kappa-1)-\tau^3]yh(y)\coth y-2[\kappa(\kappa-3)+\tau]y\tanh y$$

$$+\frac{1}{d}[-\kappa^4+\kappa^3(5-2\tau)+\kappa^2(-2\tau^2+13\tau-11)+\kappa(5\tau^2-20\tau+8)+\tau(-\tau^2-\tau+8)]y^2$$

$$+\frac{\kappa}{2d}[-\tau^2(\kappa-1)^2-\tau(2\kappa^3-10\kappa^2+13\kappa-4)-\kappa(\kappa^3-5\kappa^2+8\kappa-4)]G_0(\kappa)$$

$$+\frac{\tau}{2d}[-\kappa^2(\tau^2-3\tau+3)+\kappa(3\tau^2-7\tau+4)-\tau(\tau^2-4)]G_0(\tau)$$

$$+\left(\frac{1}{\kappa d}[-\tau^3(\kappa-1)^2(\kappa-2)+\tau^2(-3\kappa^4+14\kappa^3-24\kappa^2+18\kappa-4)\right.$$

$$+\kappa\tau(-3\kappa^4+19\kappa^3-39\kappa^2+30\kappa-8)+\kappa^2(-\kappa^4+7\kappa^3-16\kappa^2+14\kappa-4)]y\,\mathrm{csch}2y$$

$$\left.+\frac{\kappa-2}{2\kappa(\kappa-1)}[\tau(3\kappa^2-5\kappa+2)+2\kappa^3-3\kappa^2+2\kappa]\right)\ln\kappa$$

$$+\left(\frac{1}{\tau d}[\kappa^3(-\tau^3+3\tau^2-3\tau+2)+\kappa^2(-\tau^4+8\tau^3-14\tau^2+10\tau-4)\right.$$

$$+\kappa\tau(2\tau^3-13\tau^2+14\tau-8)-\tau^2(\tau^3-2\tau^2-6\tau+4)]y\,\mathrm{csch}2y$$

$$\left.\left.+\frac{1}{2\tau(\tau-1)^2}[\tau(2\tau^3-\tau^2-4\tau+4)-\kappa(\tau-1)(3\tau^2-4\tau+4)]\right)\ln\tau\right]. \quad (11)$$

In the above, we have adopted BF's notation for the special functions and the parameter y (*not* to be confused with the spectral variable y). We have

$$t=\kappa+\tau,\quad 4\sinh^2 y=-(\kappa+\tau),\quad \ln\kappa=\ln|\kappa|-i\pi,$$

$$h(y)=\ln(2\sinh y)-\tfrac{1}{2}y+\frac{1}{2y}[\tfrac{1}{6}\pi^2-f(e^{-2y})],$$

$$I(\lambda)=2(1-2y\coth 2y)\ln\lambda-4y\coth 2y[2h(y)-h(2y)], \quad (12)$$

$$\tfrac{1}{2}zG_0(z)=f\left(\frac{1}{1-z}\right)+\tfrac{1}{2}\ln^2|1-z|+\begin{cases}-\tfrac{1}{6}\pi^2-i\pi\ln|1-z|, & z<0\\ -\tfrac{1}{6}\pi^2, & 0<z<1\\ +\tfrac{1}{3}\pi^2, & z>1\end{cases}$$

where $f(x)$ is the Spence function[9]

$$f(x)=-\int_0^x\frac{dt}{t}\ln|1-t|\,.$$

Equations (4), (5), and (6)–(11) then give a complete description of the helicity cross sections. The infrared divergence is considered in part C of this section.

B. Unpolarized Cross Section

The unpolarized cross section can be calculated from the helicity cross sections by summing them with the appropriate weight factors and dividing by 4 (thus averaging over initial helicities). For the fourth-order cross section it is easy to show from Eqs. (4) and (5) that

$$\left(\frac{d\sigma}{d\Omega}\right)^{(4)}_{\rm unpol}=+\frac{\alpha^2}{2(1-\kappa)m^2}U, \quad (13)$$

where

$$U=4\left(\frac{1}{\kappa}+\frac{1}{\tau}\right)^2-4\left(\frac{1}{\kappa}+\frac{1}{\tau}\right)-\left(\frac{\kappa}{\tau}+\frac{\tau}{\kappa}\right). \tag{14}$$

For the sixth-order cross section it is straightforward but quite tedious to show that

$$\begin{aligned}
-\frac{2\pi(1-\kappa)m^2}{\alpha^3}\left(\frac{d\sigma}{d\Omega}\right)^{(6)}_{\text{unpol}} = {} & I(\lambda)U-4\left(1-\frac{1}{\kappa}-\frac{1}{\tau}\right)y\tanh y+2\left(4+\frac{\tau+2}{\kappa}+\frac{\kappa+2}{\tau}\right)yh(y)\coth y+\left[3\left(\frac{\kappa}{\tau}+\frac{\tau}{\kappa}\right)-\frac{8}{\kappa\tau}+4\right]y^2\\
&+\left\{\left[\left(4\kappa+3\tau+2+\frac{\tau^2+2\tau-24}{\kappa}+2\frac{\kappa^2-12}{\tau}+\frac{16}{\kappa\tau}-\frac{8(\tau-2)}{\kappa^2}\right)y\,\text{csch}2y\right.\right.\\
&\left.\left.+1+\frac{3}{\tau}-\frac{7}{\kappa\tau}+\frac{3\tau+16}{2\kappa}+\frac{3\tau-16}{2\kappa^2}+\frac{2\kappa-\tau^2-\kappa^2\tau}{2\kappa^2\tau(\kappa-1)}-\frac{1}{2\tau}\frac{2\kappa^2+\tau}{(\kappa-1)^2}\right]\ln\kappa+(\kappa\leftrightarrow\tau)\right\}\\
&+\frac{\tau+2}{2\tau(\kappa-1)}+\frac{\kappa+2}{2\kappa(\tau-1)}-\frac{2(\tau-4)}{\kappa^2}-\frac{2(\kappa-4)}{\tau^2}-\frac{3\tau+22}{2\kappa}-\frac{3\kappa+22}{2\tau}+\frac{16}{\kappa\tau}\\
&+\left[\left(\frac{\kappa^2+\kappa-3}{\tau}+\frac{2}{\kappa}+\frac{\tau}{\kappa^2}+\kappa+\frac{\tau}{2}-1\right)G_0(\kappa)+(\kappa\leftrightarrow\tau)\right],
\end{aligned} \tag{15}$$

where the real parts of $\ln\kappa$ and $G_0(\kappa)$ are understood. Equation (15) is exactly the result given by BF except that we have expressed it in explicitly symmetrized form.

C. Soft-Photon Processes

The cross sections given in parts A and B of this section are incomplete, for we must include the double Compton effect, where an undetectably soft photon accompanies the usual final state. This is indicated by the appearance of $\ln\lambda$. Here we will briefly sketch the calculation of these processes. (For further details see the work cited in Ref. 10.)

The skeletal action term describing the interaction of three photons and two electrons is

$$W_{2,3}=\tfrac{1}{2}e^3\int\psi(x)\gamma^0 q\gamma A(x)G_+(x-x')q\gamma A(x')G_+(x'-x'')q\gamma A(x'')\psi(x'')(dx)(dx')(dx'').$$

We now wish to apply this to the case where one of the photons is soft. Since we are interested only in logarithmically singular terms, it suffices to replace either $A(x)$ or $A(x'')$ by the soft-photon field. In this way we obtain

$$\langle 0_+|0_-\rangle=ie^3\int id\omega_p\, id\omega_{p'}\, id\omega_k\sum_{\{e\}}\left(\frac{pe}{pk}-\frac{p'e}{p'k}\right)e^\nu J_\nu(-k)\,\psi(p)\gamma^0 q e^{ipy}\gamma A(y)G_+(y-y')\gamma A(y')e^{ip'y'}\psi(p')(dy)(dy'),$$

where k^μ and e^μ are the momentum and polarization of the soft photon. Then we have (apart from a phase factor depending on the definition of in and out states)

$$\langle\text{out}, ke|\text{in}\rangle=e(d\omega_k)^{1/2}\left(\frac{pe}{pk}-\frac{p'e}{p'k}\right)\langle\text{out}|\text{in}\rangle, \tag{16}$$

so summing over the soft photon gives

$$\left(\frac{d\sigma}{d\Omega}\right)(\text{extra soft photon})=J\frac{d\sigma}{d\Omega}(\text{no extra photon}), \tag{17}$$

where

$$J=4\pi\alpha\int d\omega_k\left(\frac{p}{pk}-\frac{p'}{p'k}\right)^2. \tag{18}$$

If the minimum detectable energy (in the lab frame) is mk^0_{min}, then J is, in the Compton channel,[11]

$$J=-\frac{\alpha}{\pi}\left[2(1-2y\coth 2y)\left(\ln\frac{2k^0_{\text{min}}}{\lambda}-\frac{1}{2}\right)+4y\coth 2y[h(2y)-1]\right]. \tag{19}$$

When the appropriate elastic and inelastic (extra soft photon) cross sections are added together, the dependence on $\ln\lambda$ disappears.

D. Low-Energy Limit

The low-energy form of the unpolarized differential cross section was obtained by BF (the $\omega^2 \ln\omega$ term, where $m\omega$ is the photon energy) and by Mitra[12] (including terms of order ω^2). Since it is nontrivial to obtain the ω^2 term, two methods were used to calculate this limit. They were carried out (i) by straightforward expansion of Eq. (15) and (ii) by using the low-energy approximation in the spectral forms before integration. The former required the cancellation of terms independent of and linear in ω, which complicates the calculation. The latter is simpler since the ω^2 dependence comes out automatically from the kinematical factors of the helicity amplitudes. We find (in the c.m. system)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} \to -\frac{\alpha^3\omega^2}{2\pi m^2}\left[-\tfrac{4}{3}(1-\cos\theta)(1+\cos^2\theta)\ln\lambda + 4(1+\cos\theta+\cos^2\theta-\tfrac{1}{3}\cos^3\theta)\ln 2\omega - \tfrac{11}{3}(1-2\cos\theta+\cos^2\theta-\tfrac{2}{3}\cos^3\theta)\right], \tag{20}$$

which disagrees with the result of Ref. 12. It is noteworthy that, in the individual helicity cross sections, lower powers of ω would occur, but when they are summed, only terms of order ω^2 and higher remain.

At this point we remark on a slight test of consistency. The M_i's can be calculated in terms of the basis $\mathcal{L}_i$ mentioned in Ref. 11 of paper I. If the basis is in fact irrelevant, as long as certain basic conditions (see paper I) are satisfied, either should yield the same results. And as far as the low-energy limit is concerned, we have verified that both give identical results.

Finally, including the low-energy form of Eq. (17) we have for the physical cross section in this limit[10]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} \to -\frac{\alpha^3\omega^2}{2\pi m^2}\left[-\tfrac{4}{3}(1-\cos\theta)(1+\cos^2\theta)\ln 2k^0_{\min} + 4(1+\cos\theta+\cos^2\theta-\tfrac{1}{3}\cos^3\theta)\ln 2\omega - \tfrac{1}{9}(23-56\cos\theta+23\cos^2\theta-12\cos^3\theta)\right]. \tag{21}$$

E. Pair-Creation Process

All of the previous calculations can be carried out for the pair-creation case by using the helicity amplitudes given in Eq. (3). Essentially the same integrals occur here as for the Compton case except that the following crossed definitions are used:

$$m^2\kappa = m^2 + (p_1 - k_2')^2 > 0\,,$$

$$m^2\tau = m^2 + (p_1 - k_2)^2 > 0\,,$$

$$m^2 t = -(k_2 + k_2')^2 > 4m^2\,.$$

The functions $G_0(\kappa)$ and $\ln\kappa$ are now real while y, $2yh(y)$, and $4yh(2y)$ are complex. The integrals presented in Appendixes A and B remain the same except the definitions of y, $h(y)$, and $h(2y)$ are now

$$y = y' - i\tfrac{1}{2}\pi = \tfrac{1}{2}\ln\frac{(t)^{1/2}+(t-4)^{1/2}}{(t)^{1/2}-(t-4)^{1/2}} - \tfrac{1}{2}i\pi\,,$$

$$2yh(y) = y\ln|4\sinh^2 y| - 4y'^2 - \tfrac{1}{3}\pi^2 - \tfrac{3}{2}f(e^{2y}) - \tfrac{5}{2}f(e^{-2y})\,,$$

where the form of $2yh(y)$ is written in such a fashion that the function $4yh(2y)$ can be obtained from it by letting $y \to 2y$, $y' \to 2y'$. Using the above in the crossed version of Eq. (I53), the various helicity amplitudes can be calculated, which again contain all the necessary polarization information. The total unpolarized cross section is the crossed version of Eq. (15).

The appearance of $\ln\lambda$ in the various helicity cross sections can be handled in exactly the same manner as in part C of this section. There we did not specify which process we were considering. The calculation of J for this case requires the replacement

$$4yh(2y) - 4y \to 4yh(2y) - 4y + \pi^2\,,$$

in the expression given in Eq. (19). The imaginary parts generated by these formal crossing operations are to be deleted.

The fact that the soft-photon contribution depends on $\ln\lambda$ disagrees with a remark made in BF. They state (in Ref. 13): "The two quantum pair processes are symmetric with respect to interchange of p_1 and p_2 (in our notation p and p') so that (37) [our Eq. (16)] vanishes." They then attribute the compensation of the infrared divergence to "the effect of Coulomb interaction." However, the quantity which actually occurs is

$$q\left(\frac{pe}{pk} - \frac{p'e}{p'k}\right).$$

Because the charge matrix is explicitly included, we see this quantity is symmetric, and so preserves the statistics. In fact, there is no real difference between the situation in the two channels. Physically, the factor describing the additional soft-photon radiation does not depend on the details of the charged particle process. It is surprising that this misinterpretation by BF has remained unchallenged for 20 years.[13]

IV. LEFT-RIGHT ASYMMETRY

As another application of the amplitude, Eq. (2), we consider the left-right asymmetry for Compton scattering. This effect is well known in nuclear scattering[14] and for e-e, e-μ scattering.[15] For Compton scattering with polarized electron target, the asymmetry was studied a decade ago[2,3] with an experimental measurement carried out two years ago.[16] However, none agree with each other and therefore we perform an independent calculation to check the theoretical result.

The calculation is quite easy if we use the two basic equations[16]

$$P=\mathrm{Tr}\,\mathfrak{M}^\dagger\vec{\sigma}\cdot\vec{s}\,\mathfrak{M}/\mathrm{Tr}\,\mathfrak{M}^\dagger\mathfrak{M} \tag{22}$$

and

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{unpol}}(1+\xi P), \tag{23}$$

where P is the polarization, $\vec{s}$ is the spin direction of the target electron, ξ is the degree of polarization, and $\mathfrak{M}$ is the scattering amplitude, Eq. (I52). From Eq. (22), to order α, we have

$$P=N/D, \tag{24}$$

where

$$N=2i\,\mathrm{Tr}\,\mathfrak{M}^{(2)\dagger}\vec{\sigma}\cdot\vec{s}\,\mathrm{Im}\,\mathfrak{M}^{(4)},$$

$$D=\mathrm{Tr}\,\mathfrak{M}^{(2)\dagger}\mathfrak{M}^{(2)}.$$

In the rest frame of the target electron with the scattering taking place in the x-z plane, and the spin vector, $\vec{s}$, oriented at an angle ϕ with the y axis, we have, for N,

$$N=2\,\mathrm{Tr}\,\mathfrak{M}^{(2)}(\sigma_+-\sigma_-)\,\mathrm{Im}\,\mathfrak{M}^{(4)}\cos\phi, \tag{25}$$

where

$$\sigma_\pm=\tfrac{1}{2}(\sigma_x\pm i\sigma_y).$$

Equation (25) can be conveniently evaluated by using the helicity amplitudes, Eq. (2). With the notation

$$\mathrm{Im}\,f_i^{(4)}=G_i,\quad f_i^{(2)}=f_i^{(2)*}=g_i,$$

where the lower index i, $i=1,\ldots,6$, labels the helicity amplitudes according to the order of appearance in Eq. (2), we obtain[17]

$$N=4\cos\phi[-g_2(G_1+G_3)+(g_1+g_3)G_2 + g_5(G_4+G_6)-(g_4+g_6)G_5], \tag{26}$$

$$D=8(4\pi\alpha)^2U,$$

with U given in Eq. (14).

The G_i's are easily extracted from Eqs. (6)–(11) by noting that only $\ln\kappa$ and $G_0(\kappa)$ have imaginary parts, while the g_i's are given in Eq. (5). We find[18]

$$P=-\frac{2\alpha}{\kappa^3\tau U}\,\vec{s}\cdot\vec{k}_2\times\vec{k}_1\left(\frac{\kappa}{2(\kappa-1)^2}[\kappa(6\kappa^2-11\kappa+6)-\tau(\kappa^2-3\kappa+2)] + \frac{\kappa^2}{d}[\kappa(3\kappa^2-14\kappa+12)+(3\kappa^2-8\kappa+12)\tau-2\tau^2]y\,\mathrm{csch}\,2y - \frac{1}{d}[\kappa^2(3\kappa^2-8\kappa+3)-\kappa\tau(\kappa-4)-\tau^2(\kappa-1)]\ln|1-\kappa|\right). \tag{27}$$

The result of Ref. 2 is reproduced if we write the result in terms of N instead of P. (The apparent sign difference may be due to the use of a different convention to describe the outgoing photon.) Therefore, there seems to be a real experimental-theoretical difference and it would be worthwhile redoing the experiment.

Note added in proof. After the completion of this work, three relevant references were brought to our attention:

(1) W. A. Bardeen and Wu-Ki Tung [Phys. Rev. D 4, 3229 (1971)] have published an erratum in which they note there are certain sign errors in the helicity amplitudes of Ref. 8. However, these are not the sign errors we point out following Eqs. (2) and (3).

(2) I. Harris and L. M. Brown [Phys. Rev. 105, 1656 (1957)] consider the annihilation channel explicitly. Besides presenting the special functions for this case, they also correctly handle the infrared problem.

(3) G. V. Frolov [Sov. J. Nucl. Phys. 13, 731 (1971)] has recalculated the asymmetry, obtaining our result, Eq. (27), which, as noted, is opposite in sign to that of Ref. 2. Also, he has analyzed other types of polarization coefficients.

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We wish to thank Professor Julian Schwinger for suggesting this problem, and for reading drafts of the papers in this series. He also kindly made available to us the manuscript of Ref. 10, where the analogous

spin-0 Compton scattering problem is treated.

APPENDIX A: x INTEGRALS

Considering Eq. (I53), we have one type of denominator structure for the M^2 integration with two possible parameters, namely,

$$M^2 - m^2 s = m^2(x+\kappa),$$

$$M^2 - m^2 u = m^2(x+\tau).$$

Here we will give the answers with the κ parameter and will ignore the imaginary part of the integrals. Recalling that

$$D = \tfrac{1}{16} y(y+4)(x^2 - x_0^2),$$

$$x_0 = 2\lambda\left(1 - \frac{4}{y+4}\right)^{-1/2},$$

we have the following types of x integrals:

$$E_i = \int_{x_0}^{\infty} \frac{dx}{x+\kappa} \frac{1}{(x^2 - x_0^2)^{1/2}} \left(1, \frac{x}{\delta}, \frac{1}{\delta}, \frac{x}{\delta^2}, \frac{1}{\delta^2}, \frac{x}{\delta^3}, \frac{1}{\delta^3}\right),$$

where

$$\delta = x^2 + (y+4)(x+1).$$

Defining

$$\beta(x) = \kappa^2 - (y+4)(\kappa - 1), \qquad \xi = \frac{(y+4)^{1/2} + y^{1/2}}{(y+4)^{1/2} - y^{1/2}},$$

we find

$$E_1 = \frac{1}{2\kappa}\left(\ln\frac{4(y+4)}{x_0^2} + \ln\frac{\kappa^2}{y+4}\right),$$

$$E_2 = -\frac{1}{2\beta}\left(\ln\frac{\kappa^2}{y+4} + \frac{y+4-2\kappa}{[y(y+4)]^{1/2}}\ln\xi\right),$$

$$E_3 = -\frac{1}{\kappa}E_2 + \frac{1}{2\kappa(y+4)}\left[\ln\frac{4(y+4)}{x_0^2} - \left(\frac{y+4}{y}\right)^{1/2}\ln\xi\right],$$

$$E_4 = -\frac{1}{2\beta^2 y(y+4)}\left(2\beta(y+2-\kappa) + y(y+4)\ln\frac{\kappa^2}{y+4} - \frac{y+4-2\kappa}{[y(y+4)]^{1/2}}[2\kappa^2 - 2(y+4)\kappa - (y+4)(y-2)]\ln\xi\right),$$

$$E_5 = -\frac{1}{\kappa}E_4 + \frac{1}{2\kappa y(y+4)^2}\left[-2(y+2) - (y-2)\left(\frac{y+4}{y}\right)^{1/2}\ln\xi + y\ln\frac{4(y+4)}{x_0^2}\right],$$

$$E_6 = -\frac{1}{2\beta^3 y^2(y+4)^2}\Big(\beta[-(y+6)\kappa^3 + (2y^2+15y+36)\kappa^2 - (y+4)(y^2+8y+18)\kappa + 3(y+4)(y^2+3y+4)]$$

$$-\frac{\beta(y+4-2\kappa)}{[y(y+4)]^{1/2}}[-2(y^2+4y-3)\kappa^2 + 2(y+4)(y^2+4y-3)\kappa$$

$$+(y+4)(y^3+2y^2-8y+6)]\ln\xi + y^2(y+4)^2\ln\frac{\kappa^2}{y+4}\Big),$$

$$E_7 = -\frac{1}{\kappa}E_6 + \frac{1}{4\kappa y^2(y+4)^3}\left[-3(2y^2+2y-8) - 2(y^2-2y+6)\left(\frac{y+4}{y}\right)^{1/2}\ln\xi + 2y^2\ln\frac{4(y+4)}{x_0^2}\right].$$

APPENDIX B: y INTEGRALS

Here we will make use of the special functions defined in Eq. (12). The following are then the integrals that occur:

$$I(\lambda)=-t\int_0^\infty \frac{dy}{y+4-t}\,\frac{1}{[y(y+4)]^{1/2}}\,\frac{y+2}{y+4}\ln\frac{4(y+4)}{x_0^{\,2}}-4yh(y)\coth 2y\;,$$

$$F_i=\int_0^\infty \frac{dy}{y+4-t}\,\frac{1}{[y(y+4)]^{1/2}}\left(1,\frac{1}{\beta},\frac{1}{\beta^2}\right),$$

$$F_1=y\,\mathrm{csch}2y\;,$$

$$F_2=\frac{1}{d}\left(y\,\mathrm{csch}2y+\frac{\kappa-1}{\kappa(\kappa-2)}\ln|1-\kappa|\right),$$

$$F_3=\frac{1}{d}\left[\frac{1}{d}\,y\,\mathrm{csch}2y+\frac{\kappa-1}{\kappa^3(\kappa-2)}\left(\frac{\kappa^2-2\kappa+2}{(\kappa-2)^2}+\frac{\kappa^2}{d}\right)\ln|1-\kappa|-\frac{\kappa-1}{\kappa^2(\kappa-2)^2}\right].$$

$$G_i=\int_0^\infty \frac{dy}{y+4-t}\,\frac{1}{[y(y+4)]^{1/2}}\left(\frac{1}{y+4},1,\frac{1}{\beta},\frac{1}{\beta^2},\frac{1}{\beta^3}\right)\ln\frac{\kappa^2}{y+4}\,,$$

$$G_1=\frac{2}{t}[-yh(y)+y\ln\kappa]\,\mathrm{csch}2y+\frac{1}{t}(1-\ln\kappa)\,,$$

$$G_2=2[-yh(y)+y\ln\kappa]\,\mathrm{csch}2y\;,$$

$$G_3=\frac{2}{d}[-yh(y)+y\ln\kappa]\,\mathrm{csch}2y+\frac{\kappa-1}{\kappa d(\kappa-2)}[E(\kappa)+\kappa G_0(\kappa)]\,,$$

$$G_4=\frac{2}{d^2}[-yh(y)+y\ln\kappa]\,\mathrm{csch}2y+\frac{\kappa-1}{d\kappa^3(\kappa-2)}\left(\frac{\kappa^2-2\kappa+2}{(\kappa-2)^2}+\frac{\kappa^2}{d}\right)[E(\kappa)+\kappa G_0(\kappa)]-\frac{\kappa-1}{d\kappa^2(\kappa-2)^2}\left(2\ln\kappa-\frac{\kappa-2}{\kappa}\ln|1-\kappa|\right),$$

$$G_5=\frac{2}{d^3}[-yh(y)+y\ln\kappa]\,\mathrm{csch}2y+\frac{\kappa-1}{d\kappa^5(\kappa-2)}\left(\frac{\kappa^4-4\kappa^3+10\kappa^2-12\kappa+6}{(\kappa-2)^4}+\frac{\kappa^2(\kappa^2-2\kappa+2)}{(\kappa-2)^2 d}+\frac{\kappa^4}{d^2}\right)[E(\kappa)+\kappa G_0(\kappa)]$$
$$+\frac{\kappa-1}{d\kappa^4(\kappa-2)^2}\left[-\left(\frac{3(\kappa^2-2\kappa+2)}{(\kappa-2)^2}+\frac{2\kappa^2}{d}\right)\ln\kappa+\frac{\kappa-2}{2\kappa}\left(\frac{3\kappa^2-8\kappa+8}{(\kappa-2)^2}+\frac{2\kappa^2}{d}\right)\ln|1-\kappa|-\frac{1}{2}\right].$$

$$H_i=\int_0^\infty \frac{dy}{y+4-t}\left(\frac{1}{y+4},\frac{1}{\beta},\frac{1}{\beta^2},\frac{1}{\beta^3}\right)\ln\xi\,,$$

$$H_1=-\frac{2}{t}\,y^2\,,$$

$$H_2=-\frac{1}{d}[2y^2-E(\kappa)]\;,$$

$$H_3=-\frac{1}{d^2}[2y^2-E(\kappa)]-\frac{1}{d\kappa(\kappa-2)}\ln|1-\kappa|\;,$$

$$H_4=-\frac{1}{d^3}[2y^2-E(\kappa)]-\frac{1}{2d\kappa^3(\kappa-2)}\left(\frac{\kappa^2-2\kappa+2}{(\kappa-2)^2}+\frac{2\kappa^2}{d}\right)\ln|1-\kappa|+\frac{1}{2d\kappa^2(\kappa-2)^2}\;,$$

where

$$E(x)=\tfrac{1}{2}\ln^2|x-1|+\begin{cases}0\,, & x<1\\ -\tfrac{1}{2}\pi^2\,, & x>1\,.\end{cases}$$

Note that for the various helicity amplitudes, $\ln|1-\kappa|$ and $E(\kappa)$ do not contribute.

One final note is necessary about the integrals with τ instead of κ. If $\tau>1$, then there exists a value of y (say y') such that $\beta(\tau)=0$. The E_i integrals are perfectly well defined at this point but the individual pieces are not. The simplest method to handle the y integral for these individual pieces is to remove a small neighborhood around y', i.e., $y'-\epsilon<y<y'+\epsilon$. Then some of the above integrals will depend on ϵ^{-1} but this dependence will cancel out in the final answer and so is ignored from the beginning.

APPENDIX C: INTEGRATED FORM OF M_i's

In this appendix, we present the integrated form of the invariant amplitudes M_i, Eq. (I52). The necessary integrals are given in Appendixes A and B. We obtain (apart from a factor of α^2)

$$M_1 = \frac{16}{\kappa\tau} I(\lambda) + \frac{32}{\kappa\tau} - \frac{4(\kappa+\tau)}{\kappa\tau} + \frac{16}{t} + 16\left(\frac{1}{d} + \frac{4}{t^2}\right)y^2 + 8\left[\left(-\frac{3\kappa-4}{\kappa(\kappa-1)} + \frac{8}{\kappa t} + \frac{\kappa(\kappa-2)}{d(\kappa-1)}\right)\ln\kappa + (\kappa \leftrightarrow \tau)\right] y\,\mathrm{csch}2y$$
$$-8\left[\left(\frac{4}{\kappa t} + \frac{1}{\kappa(\kappa-1)}\right)\ln\kappa + (\kappa\leftrightarrow\tau)\right] + 4\left[\left(\frac{1}{d} + \frac{1}{\kappa^2}\right)\kappa G_0(\kappa) + (\kappa\leftrightarrow\tau)\right],$$

$$M_2 = \frac{16}{\kappa\tau} I(\lambda) + \frac{48}{\kappa\tau} + \frac{4(\kappa+\tau)}{\kappa\tau} - \frac{16}{t} - 16\left(\frac{1}{d}\coth^2 y + \frac{4}{t^2}\right)y^2 + 8\left[\left(\frac{1}{\kappa(\kappa-1)} - \frac{8}{\kappa t}\right)\ln\kappa + (\kappa\leftrightarrow\tau)\right]$$
$$-4\left\{\left[\frac{1}{d}\left(\frac{\kappa-2}{\kappa}\right)^2 + \frac{1}{\kappa^2}\right]\kappa G_0(\kappa) + (\kappa\leftrightarrow\tau)\right\} + 8\left[\left(-\frac{5\kappa-4}{\kappa(\kappa-1)} + \frac{16}{\kappa t} - \frac{(\kappa-2)^3}{d\kappa(\kappa-1)}\right)\ln\kappa + (\kappa\leftrightarrow\tau)\right] y\,\mathrm{csch}2y,$$

$$M_3 = -\frac{4(\kappa-\tau)}{\kappa\tau} + \frac{16(\kappa-\tau)}{td}y^2 + 4\left[\left(\frac{1}{d} - \frac{2}{d\kappa} - \frac{1}{\kappa^2}\right)\kappa G_0(\kappa) - (\kappa\leftrightarrow\tau)\right] + 8\left[\left(\frac{1}{\kappa(\kappa-1)} - \frac{2y}{d}\coth y\right)\ln\kappa - (\kappa\leftrightarrow\tau)\right],$$

$$M_4 = -\frac{16}{\kappa\tau} I(\lambda) - \frac{16}{d^2}[2d + t(t-4)]y\coth y - 8\left(-\frac{1}{d} + \frac{t-2}{2d(\kappa-1)(\tau-1)} + \frac{2}{\kappa\tau}\right)$$
$$-\frac{8}{d^3}[2d^2 + 2d(t-4)(t-1) + t(t-2)(t-4)^2]y^2 + \frac{16}{d^3}[2d^2 - 2td(t-4) - t^2(t-4)^2]yh(y)\coth y$$
$$-\frac{4}{d}\left[\left(\frac{2\kappa^2-5\kappa+4}{(\kappa-1)^2} - \frac{(\kappa-2)^2(2\kappa^2-3\kappa+2)}{(\kappa-1)^2 d} + \frac{\kappa^2(\kappa-2)^4}{d^2(\kappa-1)^2}\right)\kappa G_0(\kappa) + (\kappa\leftrightarrow\tau)\right]$$
$$-\frac{4}{d}\left[\frac{\kappa-2}{\kappa(\kappa-1)^2}\left((3\kappa^2-4\kappa+4) - \frac{2\kappa^2(\kappa-2)^2}{d}\right)\ln\kappa + (\kappa\leftrightarrow\tau)\right]$$
$$+\frac{8}{d^3}\left[\left(\frac{4\tau^2(\tau-2)}{\kappa} - \kappa(\kappa^4-7\kappa^3+13\kappa^2+4\kappa-21) - (\tau-1)(3\kappa^4-17\kappa^3+26\kappa^2-5\kappa-8)\right.\right.$$
$$\left.\left. - (\tau-1)^2\kappa(4\kappa^2-16\kappa+17) - (\tau-1)^3(2\kappa^2-7\kappa+8)\right)\ln\kappa + (\kappa\leftrightarrow\tau)\right] y\,\mathrm{csch}2y,$$

$$M_5 = -\frac{32(\kappa+\tau)}{d\kappa\tau} + \frac{256}{d^3}[3d + t(t-4)]yh(y)\sinh 2y - \frac{64(t-2)}{d^3}[3d + t(t-4)]y^2 + \frac{64}{d^2}[2d + t(t-4)]y\,\mathrm{csch}2y$$
$$+\frac{32}{d^3}\left([d - \kappa^2(\kappa-2)^2]\frac{\kappa}{\kappa-1}G_0(\kappa) + (\kappa\leftrightarrow\tau)\right) + \frac{32}{d^2}\left(\frac{1}{\kappa-1}[d + 2\kappa(\kappa-2)]\ln\kappa + (\kappa\leftrightarrow\tau)\right)$$
$$-\frac{64}{d^3}\{[(\kappa^4-4\kappa^3+2\kappa^2+4\kappa-1) + (\tau-1)(\kappa-1)(\kappa^2-2\kappa+2) - (\tau-1)^2]\ln\kappa + (\kappa\leftrightarrow\tau)\}y\,\mathrm{csch}2y,$$

$$M_6 = -\frac{32}{d^3}(\kappa-\tau)t^2(t-4)yh(y)\coth y + \frac{16}{d^2}(\kappa-\tau)t(t-4)y\,\mathrm{csch}2y - \frac{8(\kappa-\tau)}{d(\kappa-1)(\tau-1)} - \frac{16}{d^3}(\kappa-\tau)[t(t-2)(t-4) + 2d]y^2$$
$$+\frac{16}{d^3}\{[-(\kappa^5-5\kappa^4+3\kappa^3+12\kappa^2-7\kappa-6) - (\tau-1)(3\kappa^4-13\kappa^3+14\kappa^2+\kappa-10)$$
$$- (\tau-1)^2(2\kappa^3-6\kappa^2+7\kappa-2) + (\kappa-2)(\tau-1)^3]\ln\kappa - (\kappa\leftrightarrow\tau)\}y\,\mathrm{csch}2y$$
$$-\frac{8}{d^3}\left(\frac{\kappa-2}{(\kappa-1)^2}[d^2 - \kappa d(2\kappa^2-5\kappa+5) + \kappa^3(\kappa-2)^2]\kappa G_0(\kappa) - (\kappa\leftrightarrow\tau)\right)$$
$$-\frac{8}{d^2}\left(\frac{1}{(\kappa-1)^2}[(3\kappa^2-8\kappa+8)d - 2\kappa^2(\kappa-2)^2]\ln\kappa - (\kappa\leftrightarrow\tau)\right).$$

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[4]See Ref. 2, footnote 9.

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[6]For these explicit representations see, for example, J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970), subsections 2.6 and 3.13.

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$$-\tfrac{47}{36}+\tfrac{31}{18}\cos\theta-\tfrac{47}{36}\cos^2\theta+\tfrac{4}{3}\cos^3\theta\,.$$

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[17]Another way to derive this result is to explicitly represent the spin state in terms of the helicity states (in the lab frame); for example

$$\langle \text{out}\,|\,\text{up},\lambda_i\rangle=\frac{1}{\sqrt{2}}[f(+,\lambda_i;\text{out})+e^{i(\pi/2-\phi)}f(-,\lambda_i;\text{out})]\,.$$

Squaring and summing over undetected helicities yields Eq. (26).
[18]Another way to evaluate Eq. (24) is to note that it can be simplified [by using Eq. (2)] to the form

$$N=-\frac{4\pi\alpha(-td)^{1/2}}{\kappa T}\,[-4t\,\text{Im}\,M_3-(\kappa-\tfrac{1}{2}t)d\,\text{Im}\,M_5+2d\,\text{Im}\,M_6]\,.$$

The imaginary part of M_3, M_5, and M_6 can be easily extracted from the integrated form of the M_i's given in Appendix C.

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Deep-Inelastic Contribution to Baryon Electromagnetic Mass Differences*

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Unsubtracted dispersion relations for $t_1(q^2,\nu)$ and $t_2(q^2,\nu)$ are proposed even in the case of the $\Delta I=1$ mass difference, with the requirement of the absence of divergences worse than logarithmic ones. By the use of the experimental data on the inelastic nucleon structure functions, the possibility is shown that the deep-inelastic effect leads to the correct sign of the observed $n-p$ mass difference, under the condition that

$$\lim_{q^2\to\infty}\int_0^{2M}(1-2q^2R/\omega^2)G_2d\omega>0\,,$$

where $\omega=q^2/\nu$, $R=\sigma_l/\sigma_t$ is the ratio of virtual-photon cross sections, and G_2 stands for $\nu W_2(q^2,\nu)$ in the Bjorken limit. The sufficient condition is then found to be either $R\simeq k\omega^2/q^2$ ($k<\frac{1}{2}$) or $R\propto 1/q^{2+\delta}$ ($\delta>0$), as $q^2\to\infty$. In consideration of the experimental fact that the ratio of structure functions, $W_{2,n}/W_{2,p}$, in the range where the greater part of the contribution to the relevant integral results is not far away from the threshold value as $\omega/2M\to 1$ predicted by Bloom and Gilman, the deep-inelastic part of the $n-p$ mass difference is effectively written in the form of the magnetic-moment-type self-energy. It is also shown that if this is similarly applicable to other baryons, and if the SU(3) magnetic-moment relations hold, the correct signs and right orders of magnitude of the mass differences $\Sigma^- -\Sigma^+$, $\Xi^- -\Xi^0$, as well as $\frac{1}{2}(\Sigma^+ +\Sigma^-)-\Sigma^0$, are reproduced by the theory with no adjustable parameter except an input of the observed $n-p$ mass difference.

I. INTRODUCTION

It was first pointed out by the author[1] in 1953 that the observed $n-p$ mass difference might be explained in terms of the predominance of the magnetic-moment self-energy over the electric-charge self-energy. The mass difference was given by

$$\Delta M(n-p)\simeq\Delta(Q^2)\langle e^2/r\rangle-\Delta(\mu^2)\langle(e/2M)^2/r^3\rangle\,, \tag{1.1}$$

where Q is the electric charge of the nucleon in units of the elementary charge e, μ is the magnetic moment of the nucleon in units of the nucleon magneton $e/2M$, M is the nucleon mass, and then $\Delta(Q^2)=Q_n^{\,2}-Q_p^{\,2}=-1$, $\Delta(\mu^2)=\mu_n^{\,2}-\mu_p^{\,2}=-4.14$ for the $n-p$ mass difference. The field-theoretic calculation of the mass difference at an early stage[1,2] seemed to support the above possibility, because cutoff factors could at that time be suitably chosen to make the magnetic self-energy dominant. After-