Damped Pseudoscalar- and Vector-Meson Model for Nucleon-Nucleon Scattering

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Recently, a pseudoscalar–vector-meson (PV) model for elastic nucleon-nucleon scattering was presented by Barker and Haracz in which phase parameters are defined by a perturbative method which is similar to the geometric unitarization scheme used in other models. These phase parameters are dominated by the two-pion-exchange (TPE) contribution for the low values of L, and TPE is unreasonably large for the phases associated with the S state. In order to suppress this large contribution, relativistic Heitler damping or K-matrix unitarization is applied to the PV model. It is found that the low-partial-wave phase parameters ${}^1K_0$, ${}^3\theta^S{}_1$, ${}^3\theta^D{}_1$, and ρ_1 are drastically reduced and made much more reasonable by damping. Moreover, the P- and D-state phase shifts are reduced by as much as 20% at the higher scattering energies. The resulting damped phase parameters are in good agreement with the phenomenological values for $L \geq 2$ and energies up to 200 MeV, excluding ${}^3\theta^D{}_1$. Coupling constants more consistent with experiment are used contrasting with our previous usage. It is also found that the presence of a strongly coupled scalar ϵ resonance does not improve this model.

I. INTRODUCTION

Recently, a model for elastic nucleon-nucleon scattering was presented by Barker and Haracz[1] which is based on the interaction energy density

$$
\begin{aligned}
H_{PV} = {} & ig_\pi :\bar\psi\gamma_5\tau_i\psi U_{\pi i}: \\
& + ig_\eta :\bar\psi\gamma_5\psi U_\eta : \\
& + ig_\omega :\bar\psi\gamma_\mu\psi U_{\omega\mu}: \\
& + ig_\rho :\bar\psi\gamma_\mu\tau_i\psi U_{\rho i\mu}: \\
& + \frac{f_\omega}{4\kappa} :\bar\psi\sigma_{\mu\nu}\psi\left(\frac{\partial U_{\omega\nu}}{\partial x_\mu} - \frac{\partial U_{\omega\mu}}{\partial x_\nu}\right): \\
& + \frac{f_\rho}{4\kappa} :\bar\psi\sigma_{\mu\nu}\tau_i\psi\left(\frac{\partial U_{\rho i\nu}}{\partial x_\mu} - \frac{\partial U_{\rho i\mu}}{\partial x_\nu}\right): \,, \qquad (1)
\end{aligned}
$$

with $\sigma_{\mu\nu} = (1/2i)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. Here, ψ is the nucleon, U_π the pion, U_η the η resonance, $U_{\omega\mu}$ the ω isoscalar vector, and $U_{\rho i\mu}$ the ρ isovector-vector field operator, and $\kappa = Mc/\hbar$ with $M = 938.903$ MeV the average of the proton and neutron masses. The exact relativistic contribution of H_{PV} to the nucleon-nucleon scattering phase parameters corresponding to the virtual exchanges of π, 2π, $\pi+\eta$, η, ω, ρ, and the radiative corrections to one-pion-exchange (OPE) are given in Ref. 1. The mesons are regarded as particles with the respective masses of $m_\pi = 138$ MeV, $m_\eta = 548.8$ MeV, $m_\omega = 783.3$ MeV, and $m_\rho = 767$ MeV. It is found that if the phase parameters associated with the S state are excluded, this pseudoscalar-vector-meson (PV) model produces phases in reasonably good agreement with the Yale phenomenological values[2] for values of $L \geq 2$ and scattering energies $E \leq 200$ MeV. The values of the coupling constants for this fit are $g_\pi^2/4\pi c\hbar = 13.2$, $g_\eta^2/4\pi c\hbar = 4.6$, $g_\omega^2/4\pi c\hbar = 1.6$, $g_\rho^2/4\pi c\hbar = 0.2$, $f_\rho/g_\rho = 6.6$, and $f_\omega/g_\omega = 2.0$.

The phase parameters derived in the PV model are defined by a perturbative method as described in Haracz and Sharma,[3] which is similar to the

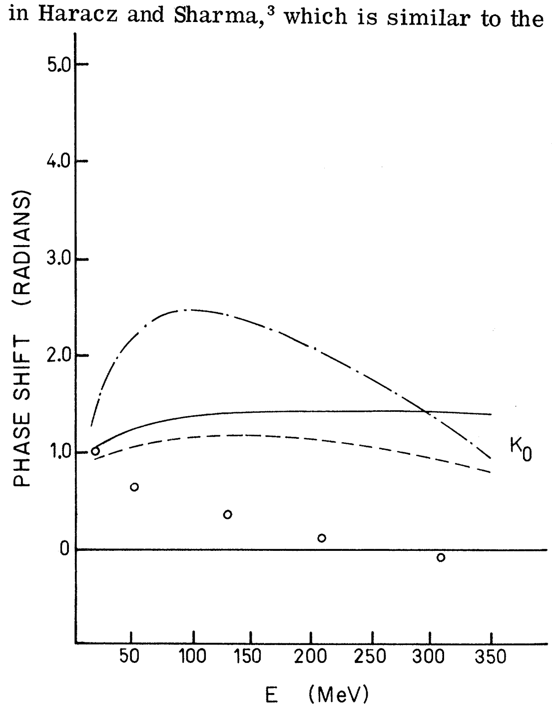

FIG. 1. The singlet phase shift K_0 in radians. The solid line corresponds to the damped OPED + TPED model, the dashed-and-dotted line corresponds to the undamped PV model, and the dashed line corresponds to the damped PVD model. The Yale phenomenological phase shift is shown at 10, 50, 130, 210, and 310 MeV.

geometric unitarization scheme used by Binstock,[4] and Binstock and Bryan,[5] in their two-pion-exchange (TPE) model. This method of unitarization breaks down when the phase shifts are of the order of 1 rad, and an anomalously large TPE contribution results for the S- and P-state phase parameters.

For this reason, the effect of radiation damping[6,7] on OPE and TPE was investigated by Bock and Haracz.[8] The introduction of this effect amounts to adding to the scattering matrix the contributions from all the real intermediate processes arising from elastic nucleon-nucleon scattering from any model interaction, and a damped and unitary approximation to the scattering matrix is assured. The effect on OPE and TPE is found in Ref. 8 to be as large as an order of magnitude in the S state and 20% in the P and D states.

As the PV model of Ref. 1 is strongly influenced by the P- and D-state contributions, the present investigation pays special attention to the influence of damping on the model parameters. Damping is very large for the core parameters, and hence these are presented to complete the description of the model. Finally, comment is made on the possibility of bringing a strongly coupled ϵ scalar resonance into the PV model.

II. RELATIVISTIC K-MATRIX UNITARIZATION

It is shown in Ref. 7 that if the scattering operator S is expressed in terms of the Hermitian operator K as

$$S=(1-iK/2)/(1+iK/2), \tag{2}$$

and K is expanded in powers of meson-nucleon coupling constants as

$$K=\sum_i K_i, \tag{3}$$

the resulting approximation to S on terminating this series at any point is unitary. The various orders of K are related to those of S in Ref. 7. Introducing the operator $U=(S-1)/2i$, rearranging Eq. (2), and taking the matrix element of the resulting expression between the initial and final nucleon states Ψ_i and Ψ_f gives

$$-2\Psi_f^\dagger U\Psi_i=\Psi_f^\dagger K\Psi_i+i\sum_n \Psi_f^\dagger K\Psi_n\Psi_n^\dagger U\Psi_i, \tag{4}$$

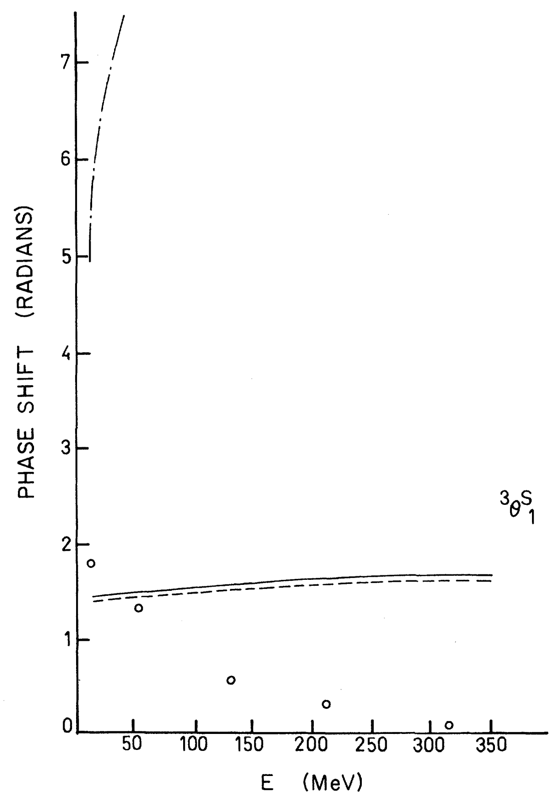

FIG. 2. The triplet phase shift $^3\theta^S{}_1$ corresponding to the OPED+TPED, PV, and PVD models, with the Yale values.

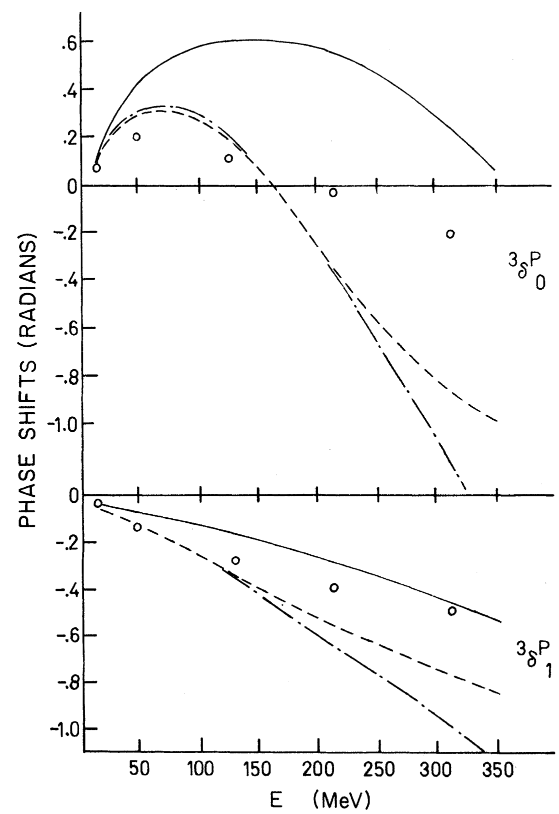

FIG. 3. The triplet phase shifts $^3\delta^P{}_0$ and $^3\delta^P{}_1$ from the OPED+TPED, PV, and PVD models, with the Yale values.

where Ψ_n is a real two-nucleon intermediate state, and the sum extends over all such intermediate states.

Equation (8) is reduced in Ref. 8 to

$$ {}^{S}\alpha_J^{L,L'} = {}^{S}\bar{\alpha}_J^{L,L'} + i \sum_{\bar{L}=|J-S|}^{\bar{L}=J+S} {}^{S}\bar{\alpha}_J^{L,\bar{L}}\, {}^{S}\alpha_J^{\bar{L},L'} . \tag{5} $$

The quantities α are the damped partial-wave amplitudes, and they satisfy the unitarity conditions

$$ \begin{aligned} {}^{0}\alpha_J^{L=L'=J} &= (1/2i)[\exp(2iK_J)-1] , \\ {}^{1}\alpha_J^{L=L'=J} &= (1/2i)[\exp(2i\,{}^{3}\delta^{J}{}_{J})-1] . \\ {}^{1}\alpha_J^{L=L'=J\pm 1} &= (1/2i)\{[1-\rho_J{}^2]^{1/2}[\exp(2i\,{}^{3}\theta^{J\pm 1}{}_{J})]-1\} , \\ {}^{1}\alpha_J^{L=J\pm 1, L'=J\mp 1} &= \tfrac{1}{2}\rho_J \exp[i({}^{3}\theta^{J+1}{}_{J} + {}^{3}\theta^{J-1}{}_{J})] , \end{aligned} \tag{6} $$

where K_J is the singlet phase shift, ${}^{3}\theta^{J}{}_{J}$ the uncoupled triplet phase shift, ${}^{3}\theta^{J\pm 1}{}_{J}$ a triplet coupled phase shift, and ρ_J the coupling parameter in the Yale notation. The real quantities $\bar{\alpha}$ are the undamped amplitudes of Ref. 1, and they are related to the PV model phase parameters $\delta(PV)$ as

$$ \begin{aligned} {}^{0}\bar{\alpha}_J^{L=L'=J} &= K_J(PV) , \\ {}^{1}\bar{\alpha}_J^{L=L'=J} &= {}^{3}\delta^{J}{}_{J}(PV) , \\ {}^{1}\bar{\alpha}_J^{L=L'=J\pm 1} &= {}^{3}\theta^{J\pm 1}{}_{J}(PV) , \\ {}^{1}\bar{\alpha}_J^{L=J\pm 1, L'=J\mp 1} &= \tfrac{1}{2}\rho_J(PV) . \end{aligned} \tag{7} $$

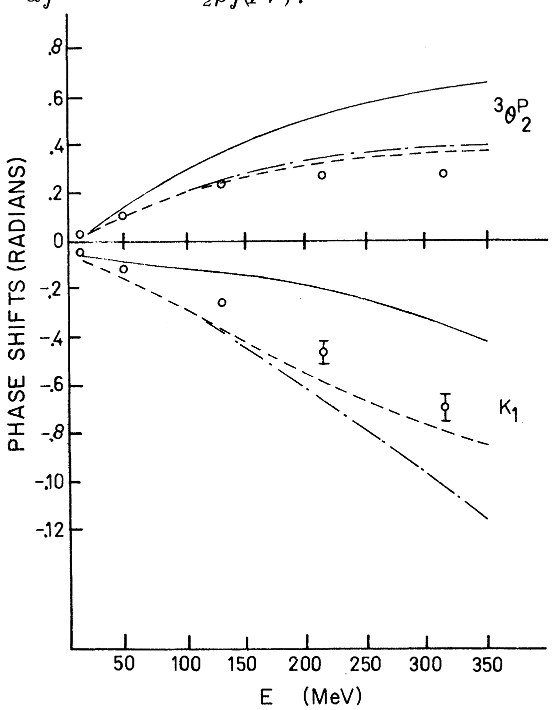

FIG. 4. The phase shifts ${}^{3}\theta^{P}{}_{2}$ and K_1 from the OPED +TPED, PV, and PVD models, with the Yale values. The phase-shift uncertainties from the energy-independent solution of MAW, Ref. 9, are shown at 50, 130, and 210 MeV, while the uncertainties of SH, Ref. 9, are shown at 310 MeV.

The undamped amplitudes are used in Eq. (5) to obtain the damped amplitudes, and the damped phase parameters are obtained from these by the inversion of Eqs. (6). The damping effect is evident from Eqs. (13) and (14) of Ref. 8.

III. RESULTS

The scattering phase parameters for the undamped and damped models are shown in Figs. 1–9. The undamped values (PV) are shown as dashed- and dotted-lines, the damped values (PVD) are shown as dashed lines, and the damped OPE + TPE values (OPED + TPED) are shown as solid lines. The Yale phenomenological phase parameters of Ref. 2 are given at 10, 50, 130, 210, and 310 MeV.[9]

It is found that the PVD model is not strongly affected by the values of the meson-nucleon coupling constants as long as these values are within the experimental limits set by the available data and their interpretation. A model value of $g_\pi{}^2/4\pi c\hbar = 14$ is used in PVD, which is consistent with the latest evaluations.[10] The η coupling constant

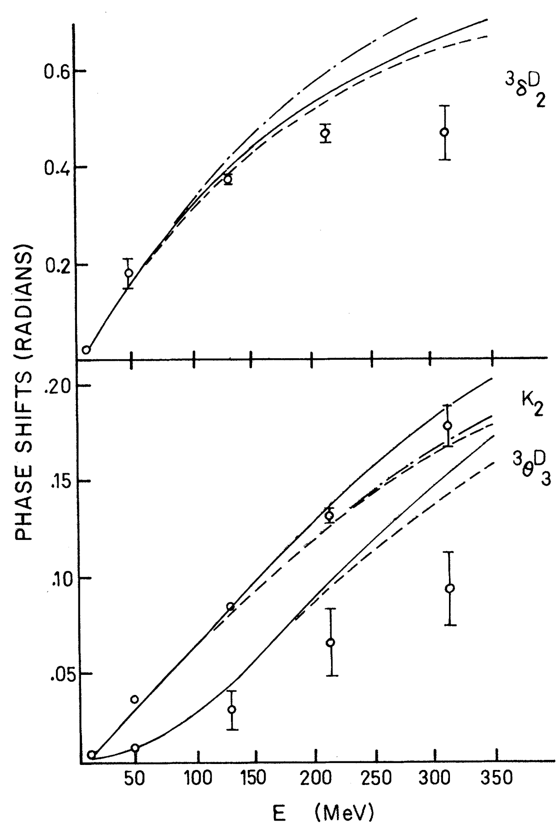

FIG. 5. The D-state phase shifts ${}^{3}\delta^{D}{}_{2}$, K_2, and ${}^{3}\theta^{D}{}_{3}$ from the OPED+TPED, PV, and PVD models, with the Yale values and the MAW and SH uncertainties.

is not known experimentally, but SU(3) predictions place its value at unity or less.[11] The experimental values for the vector resonances coupling constants given by Ting and his co-workers [12] are $g_\omega^2/4\pi c\hbar = 4.69^{+1.24}_{-0.81}$, with $f_\omega/g_\omega = -0.12$ and $g_\rho^2/4\pi c\hbar = 0.53 \pm 0.04$, with $f_\rho/g_\rho = 3.7$. The analysis of Biggs *et al.*[13] yields $g_\omega^2/4\pi c\hbar = 3.5 \pm 1.2$ and $g_\rho^2/4\pi c\hbar = 0.5^{+0.12}_{-0.10}$. Using these results, the *PVD* model is constructed from the following coupling constants:

$$g_\pi^2/4\pi c\hbar = 14\,,$$

$$g_\eta^2/4\pi c\hbar = 0.1\,,$$

$$g_\omega^2/4\pi c\hbar = 5.5, \quad f_\omega/g_\omega = -0.1\,,$$

$$g_\rho^2/4\pi c\hbar = 0.53, \quad f_\rho/g_\rho = 3.7.$$

These values differ significantly from the searched values used in the undamped *PV* model of Ref. 1. The *PV* and OPED + TPED models shown in Figs. 1–9 are also constructed from the above coupling constants.

The *S*-state phase shifts are shown in Figs. 1 and 2, and they are included to show the extreme damping effect. The singlet phase shift K_0 in Fig. 1 for *PVD* is reduced by a factor of 2 relative to result for the *PV* model, and the damping is an improvement. The damping effect on ${}^3\theta^S{}_1$, shown in Fig. 2, is even larger and in the correct direction. Needless to say, there is no agreement with experiment as many additional mechanisms will contribute significantly in this region.

The *P*-state phase shifts ${}^3\delta^P{}_0$ and ${}^3\delta^P{}_1$ are shown in Fig. 3 and ${}^3\theta^P{}_2$ and K_1 in Fig. 4. It is noted that fair agreement with the experimental values is achieved up to an energy of 100 MeV, and the vector contributions play an important role in correcting the OPED + TPED values. The damping effect is as large as 20% at the higher energies, but there is much room for improvement. It is noted that the effect of the ϵ scalar resonance would be to add a positive contribution to all of these phases. This would worsen the over-all agreement at the low energies, especially if the ϵ couples strongly to the nucleon.

The *D*-state phase shifts ${}^3\delta^D{}_2$, K_2, and ${}^3\theta^D{}_3$ are shown in Fig. 5, and the agreement with the phenomenological values is good up to about 200 MeV.

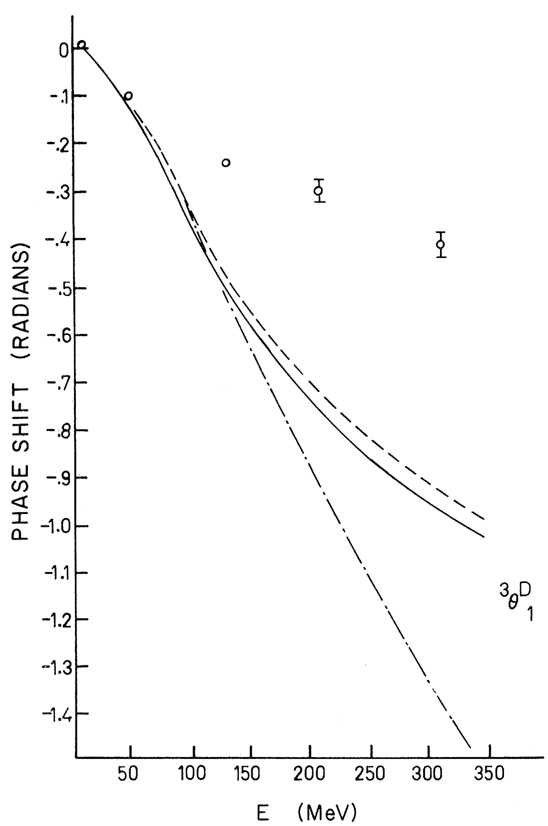

FIG. 6. The triplet phase shift ${}^3\theta^D{}_1$ from the OPED + TPED, *PV*, and *PVD* models, with the Yale values and MAW and SH uncertainties.

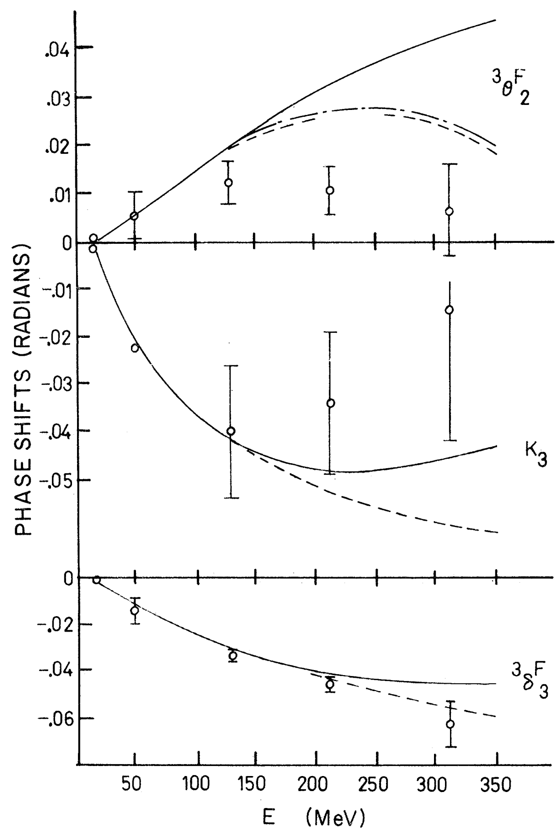

FIG. 7. The *F*-state phase shifts ${}^3\theta^F{}_2$, K_3, and ${}^3\delta^F{}_3$ from the OPED + TPED, *PV*, and *PVD* models, with the Yale values and MAW and SH uncertainties.

The phase shift ${}^3\delta^D{}_2$ is significantly affected by damping at the higher energies, and radiation damping brings about the good agreement at 200 MeV. Damping is smaller for the phases K_2 and ${}^3\theta^D{}_3$, but it does improve the situation. We also note that the vector contributions provide a general improvement to OPED + TPED. The phase shift ${}^3\theta^D{}_1$ is shown in Fig. 6. Since this phase is coupled to the ${}^3\theta^S{}_1$ phase shift, it is affected by the core of the interaction. Hence, the agreement to only 50 MeV is expected. Damping reduces ${}^3\theta^D{}_1$ by about 50% at the higher energies, but it is obviously incapable of compensating for the lack of information in the *PVD* model. The effect of the ϵ resonance with a mass centered about 750 MeV is small in the D state, and its positive contribution would slightly worsen the *PVD* values.

The F-state phase shifts are shown in Fig. 7. The damping effect is small here, but it is observed that vector resonances produce a large correction to the pion contribution which is an improvement.

The coupling parameters for the above phase shifts are shown in Figs. 8 and 9. The core parameter ρ_i is greatly affected by damping, and, although agreement with experiment is poor, the theoretical results are made sensible by damping. Damping provides a small improvement to the parameters ρ_2 and ρ_3.

IV. CONCLUSIONS

The presence of radiation damping allows the construction of a pion- and vector-meson-exchange model that uses reasonable coupling constants to achieve essentially the same results as in Ref. 1. Namely, the pion and vector-meson model produces elastic scattering phase parameters in reasonable agreement with the experimental values for values of $L \geq 2$, excluding ${}^3\theta^D{}_1$, and for scattering energies up to 200 MeV. However, the work of Ref. 1 relied on a search for the optimum values of the coupling constants, and a somewhat distorted set of values was needed to obtain the agreement. Actually, the damped model provides better agreement with the phenomenological values in the P-state relative to the results of Ref. 1,

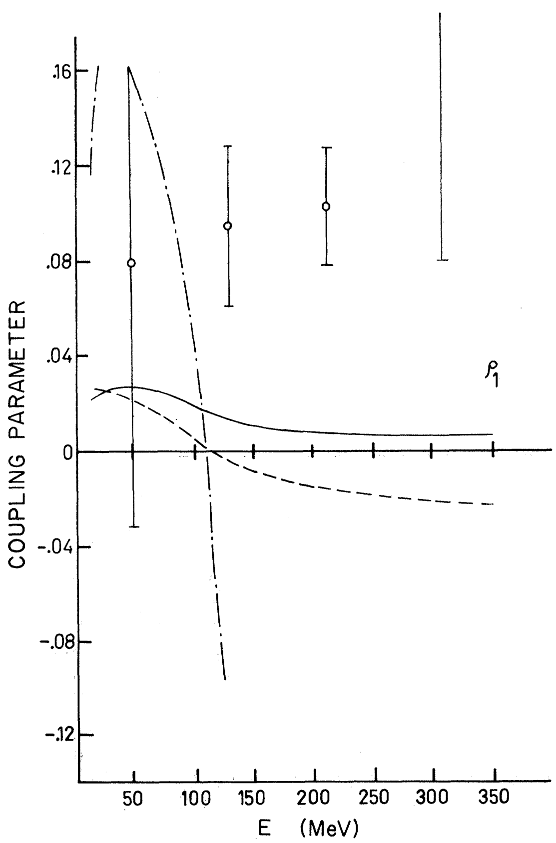

FIG. 8. The coupling parameter ρ_1 from the OPED +TPED, *PV*, and *PVD* models, with the Yale values and MAW and SH uncertainties.

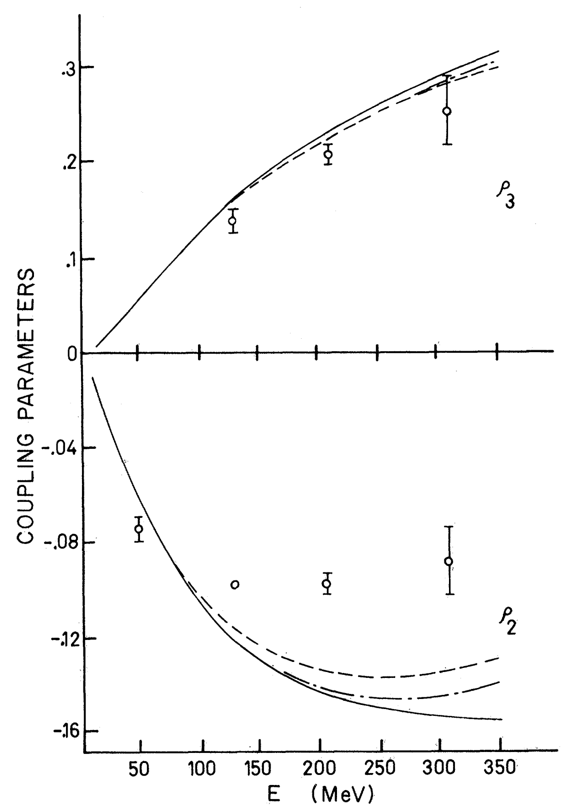

FIG. 9. The coupling parameters ρ_2 and ρ_3 from the OPED+TPED, *PV*, and *PVD* models, with the Yale values and the MAW and SH uncertainties.

and the S-state contributions are much more sensible. Naturally, damping does not cure any failings of the model. In fact, the presence of damping clarifies the need for additional effects that are as important as the pseudoscalar TPE effect in the P state. Additional effects should also be of some significance in the D state and higher-L states for energies greater than 200 MeV.

The PVD model presented here is not improved by the addition of the ϵ scalar resonance if it interacts strongly with the nucleon. This contrasts with the TPE model of Ref. 5 and other more recent investigations.[14,15] The reason seems to be that the ϵ exchange and TPE effects are in competition especially in the P state, and our field-theoretical TPE effect is larger in the P state than the TPE effects in these other models.[16] It is felt, however, that the P state is far from being theoretically established by the simple models discussed here, and any judgment about the role of the ϵ scalar resonance is premature.

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[9]The uncertainties in the phenomenological values of the phase parameters shown in Figs. 1–9 at the energies 50, 130, and 210 MeV are taken from the energy-independent analysis of M. H. MacGregor, R. A. Arndt, and R. W. Wright (MAW), Phys. Rev. 182, 1714 (1969), Table VIII. The MAW uncertainties are also shown at 310 MeV for ${}^3\theta^F{}_2$, ${}^3\delta^F{}_3$, K_2, and ρ_2. The uncertainties of P. Signell and J. Holdeman, Jr. (SH), Phys. Rev. Letters 27, 1393 (1971), solution 2 in Table I, are shown at 310 MeV.

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