Phenomenology of the Triple-Regge Region

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A two-term triple-Regge formula, which follows from a simple theoretical picture of the single-particle inclusive cross section, is presented and studied. A comparison is made with recent data for $pp \to p$ +anything, and $p\pi^- \to p$ +anything. The description of the data offered by this simple form is found to be remarkably good over a large region of phase space.

In the two-body reaction $a+b\to c+d$ there are two independent kinematic variables, say $s=(p_a+p_b)^2$ and $t=(p_a-p_c)^2$. For fixed t and large s, it has proved to be both possible and instructive to describe this process in terms of the exchange of a few Regge poles.[1] It has recently been suggested[2] that a generalized Regge analysis will also be useful as a framework in which to study inclusive reactions $a+b\to c$ +anything. In this case there are three independent variables, say $s=(p_a+p_b)^2$, $t=(p_a-p_c)^2$, and $M^2=(p_a+p_b-p_c)^2$. The essential feature of this generalized Regge analysis is that the Regge trajectories which appear are the same ones which were previously studied in two-body reactions. Thus we have an entirely new region of physics in which to test our ideas about Regge poles and related objects, in particular the Pomeranchukon. In this paper we shall report on an attempt to establish the validity of this generalized Regge picture by direct comparison with data. We shall focus our attention on the triple-Regge (TR) limit where t is fixed and both M^2 and s/M^2 are large. In this limit we expect the appropriate differential cross section to be given by

$$\frac{d^2\sigma}{dtdM^2} \underset{s/M^2\to\infty;\,M^2\to\infty}{\sim} \frac{m_0^{\,2}}{16\pi} s^{-2} \sum_{ijk} G_{ijk}(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)}\left(\frac{M^2}{m_0^{\,2}}\right)^{\alpha_k(0)} . \tag{1}$$

The α_i's are the usual trajectory functions. The TR residue G_{ijk} is a product of three particle-particle-Reggeon couplings β and the triple-Reggeon coupling g_{ijk}, i.e.,

$$G_{ijk}(t)=\beta_{iac}(t)\beta_{jac}(t)\beta_{kbb}(0)g_{ijk}(t,t,0) .$$

Regge analysis is useful for studying two-body reactions because only a few exchanges are required in order to describe the essential features of two-body scattering data. To demonstrate the existence of a similar role for the TR formula [Eq. (1)] in the case of inclusive processes, it is necessary to show that again only a few terms in the infinite sum are required to adequately describe the data. It is our purpose here to attempt such a demonstration. Consideration of the general features of the data plus limited theoretical input leads to the suggestion that at least two terms are necessary in Eq. (1). We shall show by comparison with experiment that these two terms seem also to be sufficient to describe the essential features of the existing data. Further, the agreement holds over an unexpectedly large kinematical region. We caution the reader, however, that the present study is not intended as a precise fit to the data, but rather as an initial test of the basic triple-Regge picture.

Let us review the general features of the data. We will limit ourselves to the case $a=c$ so that vacuum quantum exchange is possible in the $a\bar{c}$ channel. In particular, we have studied the data for $pp\to p+X^+$ (Refs. 3 and 4) and $p\pi^-\to p+X^-$ (Ref. 5) (here and below the symbol X stands for "anything").

The data show three main features: (1) For low M^2 ($M^2 \lesssim 4$ GeV^2) there is resonance structure in M^2 and the production cross section for these resonances seems to be independent of s. (2) In this same M^2 region there is a background contribution which seems to behave essentially as $1/s$. (3) As a function of M^2, $d\sigma/dtdM^2$ is first decreasing just above the resonances and then starts to rise for $M^2 \gtrsim 10$ GeV^2.

This behavior can be easily interpreted in terms of the usual Regge ideas. We say that (a) the resonances are being produced via Pomeranchukon exchange; (b) the background results from the usual Reggeized meson (ρ, f, A_2, ω) exchange. Thus we expect at least the two contributions pictured in Fig. 1(a).

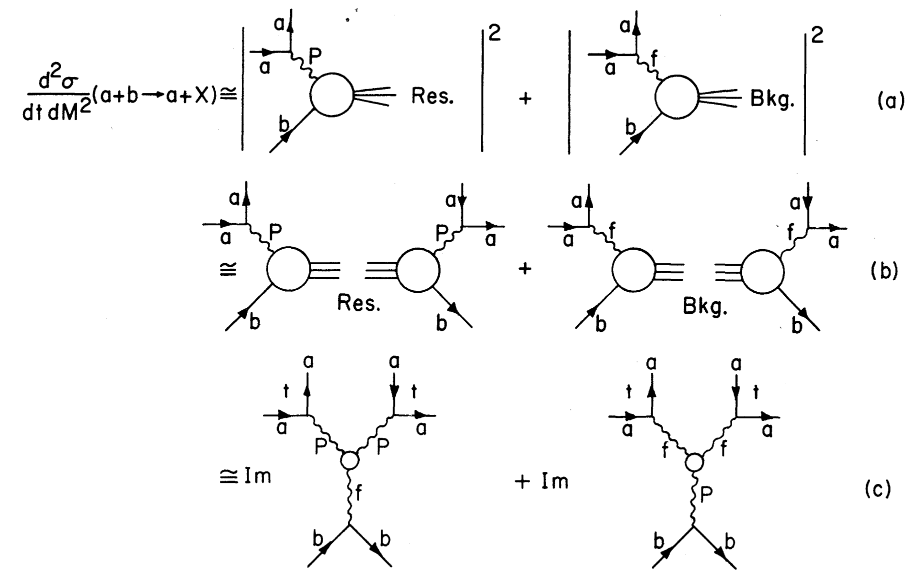

FIG. 1. (a) Minimum contributions needed to describe the general features of the $pp\to p+X^+$ data. (b) The absorptive part of the six-point function which gives the contribution shown in Fig. 1(a). (c) Triple-Regge diagrams.

The question still remains as to how to describe these contributions in the language of the triple-Regge formalism. In principle this is a question which can be answered by the data as more become available. However, at the present time we shall use theoretical input in order to arrive immediately at a unique, simple answer. This we may then compare to the data. Specifically, we shall apply duality in the form of the Freund-Harari[6] conjecture to Fig. 1(b). We interpret this to mean that the resonances are dual to the appropriate combination of ordinary meson trajectory exchanges (henceforth labeled simply as f) and the background to Pomeranchukon exchange. Hence we are led to try to describe the data in terms of the diagrams shown in Fig. 1(c). We shall call them the PPf and ffP contributions, respectively.[7] It must be pointed out, however, that, in principle, such a description is only expected to work over a very limited range of M^2. The major point of this paper is that with only these two terms all the essential features of the data are well described over a large range of M^2, in fact, from M^2 in the resonance region ($\propto 3$ GeV^2), up to M^2 being a sizeable fraction ($\sim\frac{1}{4}$) of s.

Limiting the summation of Eq. (1) to the two terms discussed, we have[8]

$$\frac{d^2\sigma}{dtdM^2} \cong \frac{m_0^2}{16\pi}s^{-2}\left[G_{PPf}(t)\left(\frac{s}{M^2}\right)^{2\alpha_P(t)}\left(\frac{M^2}{m_0^2}\right)^{\alpha_f(0)} + G_{ffP}(t)\left(\frac{s}{M^2}\right)^{2\alpha_f(t)}\left(\frac{M^2}{m_0^2}\right)^{\alpha_P(0)}\right]. \qquad (2)$$

In what follows we shall use $\alpha_P(t)=1.0+0.5$ (GeV/c)^{-2}t, $\alpha_f(t)=0.5+1.0$ (GeV/c)^{-2}t, and $m_0^2=1.0$ GeV^2 for definiteness.[9] Having written down Eq. (2), we can divorce ourselves from any specific theoretical picture and ask simply whether or not it adequately describes the data. It is important to note that this simple two-term TR formula makes very strong predictions. At a fixed value of t, it specifies both the M^2 and s dependence of the cross section in terms of just two constants (G_{PPf} and G_{ffP}).

In order to test all of these features of our simple TR formula, we have studied the reaction $pp\to p+X^+$ at two values of s,[3,4] and many values of t [$0.15<|t|<1.5$ (GeV/c)^2] for which data are available over a wide range in M^2. In order to discuss all these data we chose specific forms for G_{PPf} and G_{ffP} as functions of t. In the absence of a complete theory, we have chosen the simplest reasonable parametrizations of these couplings consistent with a preliminary study of the data. Using $G_{PPf}(t)=2.2\times10^2e^{0.6t}$ mb/(GeV/c)^2 and $G_{ffP}(t)=1.6\times10^3$ mb/(GeV/c)^2,[10] we have evaluated Eq. (2) and compared it to the data in Fig. 2. One observes remarkable agreement over a large region of phase space.

To further test the TR picture, we have exploited the property of factorization, i.e., the fact that $G_{PPf}(t)=\beta_{Paa}{}^2(t)\beta_{fbb}(0)g_{PPf}(t)$ and $G_{ffP}(t)=\beta_{faa}{}^2(t)\times\beta_{Pbb}(0)g_{ffP}(t)$. Our procedure was to utilize our results for $pp\to p+X$ to predict the cross section for $p\pi\to p+X$. This was possible since the only further inputs needed to convert our two-term TR

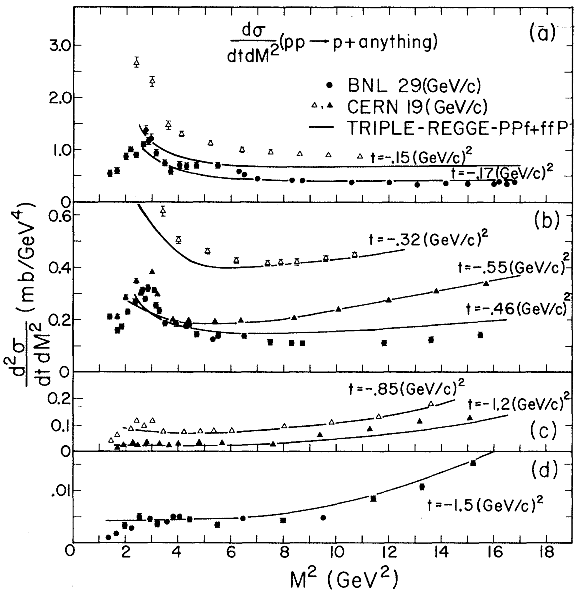

FIG. 2. $pp \to p + X^+$ data from Refs. 3 and 4 (only a fraction of the data is shown; in particular, the elastic peak is not included). The solid line is the two-term triple-Regge description with $PPf + ffP$ only. Similar curves result from $PPP + ffP$ only.

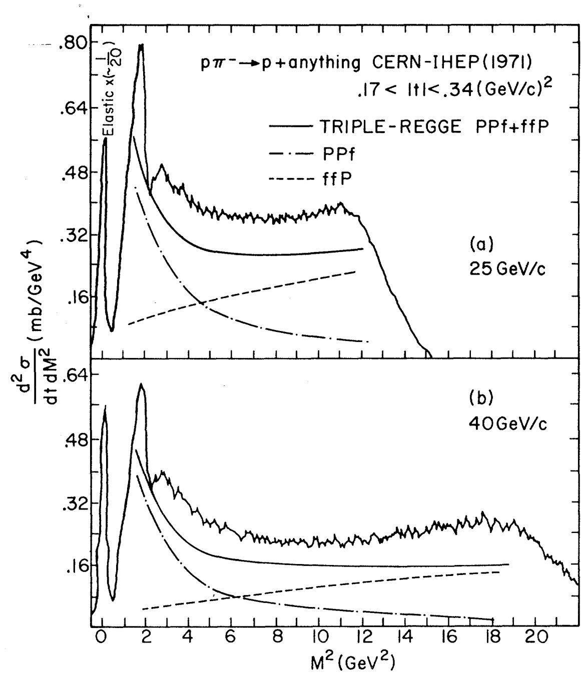

FIG. 3. $p\pi^- \to p + X^-$ data from Ref. 5. The solid line is the two-term triple-Regge description normalized to the pp data. Individual PPf and ffP contributions are shown; (a) $P_L = 25$ GeV/c, (b) $P_L = 40$ GeV/c.

formula for the pp reaction to the one for $p\pi$ were the ratios $\beta^{(0)}_{f\pi\pi}/\beta^{(0)}_{fpp}$ and $\beta^{(0)}_{P\pi\pi}/\beta^{(0)}_{Ppp}$. Having no well-established, specific values for these ratios and in keeping with the qualitative nature of the present work, we have taken both ratios to be $\frac{2}{3}$, as in a naive quark model and in reasonable agreement with two-body scattering data. Comparison with the new results of Ref. 5 is shown in Fig. 3 where the individual PPf and ffP contributions are explicitly indicated. Since the data included a finite range of t we have performed an integral over t using the specific forms of the G's given above. Considering the uncertainty of the relative normalization of the $p\pi$ and pp data and the simplicity of the present model, we regard the prediction to be in satisfactory agreement with the data.

In summary, we have considered a simple triple-Regge picture in which only the PPf and ffP terms (see Fig. 1) make important contributions. We find the agreement of this picture, Eq. (2), with the existing data most encouraging. This agreement holds over a surprisingly large range of the variables, e.g., from M^2 as small as 3 GeV^2 up to $M^2 \approx \frac{1}{4}s$. It is by no means trivial that the theoretical cross section is essentially flat in the intermediate M^2 region, but rises at both ends as do the data. The origin of this behavior lies in the characteristics of the two terms included. The PPf term decreases faster, as a function of M^2, than any other obvious TR term, whereas the ffP term[11] is the only obvious increasing function of M^2 (for $t<0$). We have also noted that the success of this two-term picture can be easily interpreted in terms of the general features of the data and a generalization of the Freund-Harari conjecture. It must be emphasized, however, that our analysis is intended to show only sufficiency of the TR terms PPf and ffP. Certainly one expects, at present energies, to have finite but small contributions from nonleading terms such as $\rho\rho f$, corresponding to Δ production. The more intriguing question is the role of the theoretically interesting PPP term. In terms of s and M^2 dependence, PPP differs only by a factor of M from PPf. We have tried describing the data with only PPP and ffP. We find that in the true TR region (e.g., $M^2 > 6$ GeV^2, $s/M^2 > 6$), $PPP + ffP$ and $PPf + ffP$ give equally acceptable descriptions of the data. Thus, the separation of the PPP and PPf contributions purely on the basis of their large-M^2 behavior does not seem possible at present.[12] Presumably this problem will be solved either by the advent of larger-s, larger-M^2 data or by the application of more powerful theoretical tools such as finite-energy sum rules.[13] The general results of the present analysis certainly suggest that the triple-Regge formalism is a useful structure within which to study inclusive reactions over a large kinematical region. Specific details, particularly the consistency with zero triple-Pomeranchukon contribution to all t, indicate that further theoretical and experimental studies encompassing larger ranges of s, M^2, and t will be most informative.

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[1]We include in this group of exchanges the Pomeranchukon or vacuum exchange although its detailed character is still in doubt.

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[3]E. W. Anderson *et al.*, Phys. Rev. Letters 19, 198 (1967); 16, 855 (1966); see also the discussion in R. M. Edelstein *et al.*, Phys. Letters 35B, 408 (1971).

[4]J. V. Allaby *et al.*, CERN Report No. CERN-TH-70-16, 1970 (unpublished).

[5]Y. M. Antipov *et al.*, Phys. Letters 40B, 147 (1972).

[6]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, *ibid.* 20, 1395 (1968).

[7]Such a simple interpretation of the data has also been suggested by J.-H. Ting and H. J. Yesian, Phys. Letters 35B, 321 (1971); J.-M. Wang and L.-L. Wang, Phys. Rev. Letters 26, 1287 (1971).

[8]There is a technical question about the variable M^2 in Eq. (2) if one wants to continue it down to $M^2 \lesssim 6$ GeV^2 as for example in conjunction with a finite-energy sum rule. With this in mind the calculations presented in this paper correspond to replacing M^2 by $\overline{M}^2 = M^2 - t - m_b^2$, a symmetrical variable much like the well-known $\nu = (s-u)/2m$ of πN scattering. This change plays an important role only in the resonance region. In principle this question also arises for the variable s when s/M^2 is not too large (i.e., $s/M^2 \lesssim 4$) but such effects have not been included here.

[9]We have also studied the case $\alpha_P = 1$, independent of t. The results are quite similar to those appearing in the text except the shape of the theoretical curve at small M^2 ($M^2 \lesssim 5$ GeV^2) is now t independent and will be much larger than the data at large t.

[10]The coupling $G_{ffP}(t)$ does not, in fact, seem to be independent of t nor does it seem to be a simple exponential. Since the variation is limited over our range of t, we were able to simply represent it as a constant and still allow a reasonable comparison with the data.

[11]It is also interesting to note that the ffP term is of the appropriate form to allow continuation from the TR region into the usual scaling region ($M^2/s = 1 - x$, where x is the usual Feynman variable). Such behavior was suggested by R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969). Note that a finite PPf contribution leads to "nonscaling" behavior for x near 1.

[12]If PPP is, in fact, large and there is *no* large s-independent background in the data, as is presently observed, then we will be led to discard the Freund-Harari conjecture as interpreted here.

[13]S. D. Ellis and A. I. Sanda, Phys. Letters (to be published).